ON THE COMBINATORICS OF TABLEAUX — A NOTEBOOK OF OPEN PROBLEMS

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Abstract. Inspired by the the Kourovka Notebook of unsolved problems in group theory[KhukhMaz2024], this is a notebook of unsolved problems in the combinatorics of tableaux. Contributions to the notebook are invited.

> Mathematicians can be subdivided into two types: problem solvers and theorizers. Most mathematicians are a mixture of the two although it is easy to find extreme examples of both types.
> — Gian-Carlo Rota, "Problem Solvers and Theorizers" in [Rot1997b]

*Version *** (***)*

Contributions to the notebook are invited. Please send all contributions, comments, and corrections to the author. Individual problems have HTML anchors of the form "`nameddest=problem-`n", so you can reference an individual problem with a URL like "`https://.../problem-notebook.pdf#nameddest=problem-10`".

1. Problems of 2024

1. [Stan1988a, sec. 6 Prob. 1] Classify all differential posets. [Fom1988][Stan2012, sec. 3.21][Byrn2012, Ch. 6]

2. [Dale R. Worley] Consider the tree $SSYT$ of shifted standard Young tableaux, with the tree structure generated by containment. Computation suggests that the only automorphism of $SSYT$ (as an unlabeled tree) is the identity. Computation further suggests that each node of $SSYT$ is uniquely determined by the census (cardinalities) of the ranks of the subtree rooted at the node.

3. [Dale R. Worley] Consider the tree SYT of (unshifted) standard Young tableaux, with the tree structure generated by containment. For any node with a symmetric shape, there is an automorphism of SYT (as an unlabeled tree) that transposes all nodes in the subtree rooted at the node and leaves all other nodes fixed. Computation suggests that all automorphisms of SYT are compositions of these transpose automorphisms (with the identity being the composition of zero transpose automorphisms). Computation further suggests that each node of SYT is determined up to transpose by the census (cardinalities) of the ranks of the subtree rooted at the node.

2. Problems of 2025

4. [Fom1994, Exam. 2.2.4] notes that lattice of Young diagrams with $\leq r$ rows is differential with degree r. Thus the general theory produces an RSK algorithm from r-colored permutations into pairs of such tableaux with suitable weighting. This should be expandable into similar theories as are developed for Young diagrams and shifted Young diagrams. In particular, (1) Can jeu de taquin be defined for these diagrams? (2) Can a Greene invariant be defined for r-colored permutations that groups permutations that produce the same P tableau?

5. [Dale R. Worley] Consider the process of randomly choosing a Young diagram of size n. If we let $n \to \infty$ and scale the diagram down by $\sqrt{n}$ in both dimensions, the average of the diagrams converges to a limit shape; points within the shape have probability 1 of being in the random diagram and points outside the shape have probability 0. Surprisingly, the same limit shape results under three distributions on the set of diagrams: uniform (weight 1), uniform on Young tableaux (weight f_λ, the number of tableaux of shape λ), and the Plancherel measure (weight f_λ^2).

Perform the parallel analysis on shifted Young diagrams, the set of partitions into distinct parts. Since the RSK algorithm for shifted diagrams has a weighting ($c_\lambda = 2^{|\lambda|-\ell(\lambda)}$), there are now six plausible distributions: 1, g_λ, g_λ^2, c_λ, $g_\lambda c_\lambda$, and $g_\lambda^2 c_\lambda$, where g_λ is the number of shifted Young tableaux (without coloring) of shape λ.

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6. [Dale R. Worley] Develop a similar theory of limit shapes for Young diagrams with $\leq r$ rows. The initial challenge is studying the general asymptotic behavior of the $r-row$ Young diagrams generated by r-colored permutations so that a proper scaling can be defined so that the required limit is defined.

7. [Dale R. Worley] Develop a similar theory of limit shapes for Young–Fibonacci diagrams. The initial challenge is defining a concept of scaling between smaller and larger diagrams. E.g., if the scaling is a doubling, we need to define an embedding D of $\mathbb{YF}_r$ into $\mathbb{YF}_r$ that maps words (diagrams) of size k into words of size $2k$, $\mathbb{YF}_r^{(k)} \to \mathbb{YF}_r^{(2k)}$. The embedding should be "nearly isometric" in that it maps words that are "near" each other in $\mathbb{YF}_r$ (in the distance metric of the Hasse diagram) into words that are near each other $\mathbb{YF}_r$, and vice-versa. And every word in $\mathbb{YF}_r$ must be near an element of the image of D.

Given such a doubling map, it defines a limit space within which the question can be asked whether randomly-chosen r-colored permutations have a limit shape. If there is a limit shape, determine it. Since $\mathbb{YF}_r$ as dual graded graphs has a unique weighting 1, there are no colorings of the tableaux, so there are only three distributions of diagrams to be considered.

8. [Dale R. Worley] Consider the tree k-SYT of standard Young tableaux with at most k rows, with the tree structure generated by containment. What are its automorphisms, when it is considered an unlabeled tree?

9. [Dale R. Worley] Consider the tree $SYFT$ of standard "Young–Fibonacci tableaux", with the tree structure generated by containment, or equivalently, the tree of upward paths in the Young–Fibonacci lattice. What are the automorphisms of this tree, when it is considered an unlabeled tree?

11. [Stan2017, sec. 1] via [Linus Setiabrata] Define $\nu_w = \mathfrak{S}_w(1, \ldots, 1)$ which is the sum of the coefficients of the Schubert polynomial $\mathfrak{S}_w$ of the permutation w. How large can ν_n be for $w \in S_n$? Define $u(n)$ to be this maximum. We have some rather crude bounds which show that,

$$\frac{1}{4} \leq \liminf_{n\to\infty} \frac{\log_2 u(n)}{n^2} \leq \limsup_{n\to\infty} \frac{\log_2 u(n)}{n^2} \leq \frac{1}{2}.$$

Determine better bounds. That is, can we find $1/4 \leq \alpha \leq 1/2$ such that $u(n) \sim 2^{\alpha n^2}$?

[MoraPakPan2019, Th. 1.3] improves the lower bound to ≈ 0.2932362762. [MoraPanPet2025, Rem. 6.8] improves the upper bound to 0.37.

12. [Stan2017, sec. 1][MerSmir2015, Ques. 5.6] via [Linus Setiabrata] Define $\nu_w = \mathfrak{S}_w(1, \ldots, 1)$ which is the sum of the coefficients of the Schubert polynomial $\mathfrak{S}_w$ of the permutation w. For a given n, define $u(n)$ to be the maximum ν_w for $w \in S_n$. For a given n, what permutations $w \in S_n$ maximize ν_w? As $n \to \infty$, is there some kind of "limiting shape" of the permutations $w \in S_n$ for which $\nu_w = u(n)$?

13. [MerSmir2015, Conj. 5.7] via [Linus Setiabrata] Define $\nu_w = \mathfrak{S}_w(1, \ldots, 1)$ which is the sum of the coefficients of the Schubert polynomial $\mathfrak{S}_w$ of the permutation w. For a given n, prove that the permutation $w \in S_n$ that maximizes ν_w is "Richardson" or "layered" in that it has the form [Def. 5.5] $(i_1, i_1{-}1, \ldots, 2, 1, i_2, i_2{-}1, \ldots, i_1{+}1, i_3, \ldots, i_2{+}1, \ldots)$ for some $i_1 < i_2 < i_3 < \cdots < i_k = n$.

14. [MoraPakPan2019, Conj. 4.1] via [Linus Setiabrata] Define $\Upsilon_w = \mathfrak{S}_w(1, \ldots, 1)$ which is the sum of the coefficients of the Schubert polynomial $\mathfrak{S}_w$ of the permutation w. For permutations w and v, define $w \otimes v$ to be the permutation whose permutation matrix is the Kronecker product of the permutation matrices of w and v. Define 1^n as the identity permutation in S_n. Is it true that for any integer $k \geq 2$, $\Upsilon_{w\otimes 1^k} \geq \Upsilon_w^{k^2}$?

15. [Byrn2012, Conj. 6.6] Is it true that an r-differential poset P is a lattice if and only if P does not contain a crown covering a crown? (This is known to be true for $r = 1$.)

16. [Byrn2012, Conj. 6.7] What are all the 1-differential posets that do not contain [a particular induced subposet]?

17. [Stan1988a, sec. 6 Prob. 6] Fix $r \in \mathbb{P}$. What is the least number of elements of rank n that an r-differential poset can have? It seems plausible that the minimum value is achieved by $\mathbb{Y}^{\times r}$, the r-th cartesian power of the lattice of partitions. (The maximum value is attained by $\mathbb{YF}_r$, the r-differential Young–Fibonacci lattice.[Byrn2012, Th. 1.2])

18. [Stan2012, Ch. 3 Exer. 29] A finite lattice L has n join-irreducibles. What is the most number $f(n)$ of meet-irreducible elements L can have?

19. [Stan2012, Ch. 3 Exer. 135(b)] Let Λ_n be the set of all $p(n)$ partitions of the integer $n \geq 0$. Order Λ_n by refinement. Determine the Möbius function $\mu(\lambda, \rho)$ of Λ_n. (This is trivial when $\lambda = \langle 1^n \rangle$ and easy when $\lambda = \langle 1^{n-2}2^1 \rangle$.)

20. [Stan2012, Ch. 3 Exer. 155(c)] Find all finite modular lattices for which every interval is self-dual. Do the same for finitary modular lattices with finite covers.

21. [Stan2012, Ch. 3 Exer. 183(f)] Let $\mathfrak{S}_n$ be the set of permutations of n ordered by the strong Bruhat order. Characterize the intervals of $\mathfrak{S}_n$ that are boolean algebras and compute their total number.

22. [Stan2012, Ch. 3 Exer. 185(j)] Let $\mathfrak{S}_n^W$ be the set of permutations of n ordered by the weak Bruhat order. Characterize the intervals of $\mathfrak{S}_n^W$ that are distributive lattices and compute their total number. The values for $1 \leq n \leq 8$ are 1, 2, 16, 124, 1262, 15898, 238572, 4152172.

23. [Stan2012, Ch. 3 Exer. 215(c)] Let P be an r-differential poset and let p_i be the size of rank i of P. Show that $p_i < p_{i+1}$ except for the case $i = 0$ and $r = 1$.

24. [Stan1999, Ch. 6 Exer. 25(i)] Let the symmetric group $\mathfrak{S}_n$ act on the polynomial ring $A = \mathbb{C}[x_1, \ldots, x_n, y_1, \ldots, y_n]$ by permuting both the $x_\bullet$ and the $y_\bullet$ simultaneously; $w \cdot f(x_1, \ldots, x_n, y_1, \ldots, y_n) = f(x_{w(1)}, \ldots, x_{w(n)}, y_{w(1)}, \ldots, y_{w(n)})$ for all $w \in \mathfrak{S}_n$. Let I be the ideal generated by all invariants of positive degree, i.e.,

$$I = \langle f \in A : w \cdot f = f \text{ for all } w \in \mathfrak{S}_n, \text{ and } f(0) = 0 \rangle .$$

Prove that the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$ is the dimension of the subspace of A/I affording the sign representation, i.e.,

$$C_n = \dim\{f \in A/I : w \cdot f = (\operatorname{sgn} w) f \text{ for all } w \in \mathfrak{S}_n\}.$$

25. [Stan1999, Ch. 7 Exer. 33(c)] Can anything be said in general about the number of distinct monomials in the Schur function $s_\lambda(x_1, \ldots, x_n)$ for arbitrary λ?

26. [Stan1999, Ch. 7 Exer. 38(b)] Fix $0 \leq k \leq \binom{n}{2}$, and let $\ell(w)$ denote the number of inversions of the permutation $w \in \mathfrak{S}_n$. Let λ and μ b partitions of length at most n, with $\mu \subset \lambda$. Define the symmetric function

$$t_{\lambda/\mu,k} = (-1)^k \sum_{\substack{w \in \mathfrak{S}_n \\ \ell(w) \geq k}} \varepsilon_w h_{\lambda+\delta-w(\mu+\delta)}.$$

Thus $t_{\lambda/\mu,k}$ is a "truncation" of the Jacobi–Trudi expansion [Stan1999, Ch. 7 eq. 7.69] of $s_{\lambda/\mu}$. Is there a "nice" combinatorial interpretation of the scalar product $\langle t_{\lambda/\mu,k}, s_\nu \rangle$?

27. [Stan1999, Ch. 7 Exer. 55(b)] Let $\rho^\lambda : \mathfrak{S}_n \to \mathrm{GL}(m, \mathbb{C})$ be an irreducible representation of $\mathfrak{S}_n$ with character χ^λ (so $m = f^\lambda$). Is it possible to count the number of λ's for which $\rho^\lambda(\mathfrak{S}_n) \subset \mathrm{SL}(m, \mathbb{C})$?

28. [Stan1999, Ch. 7 Exer. 68(b)] Let G be a finite group of order g. Given $w \in G$, let $f(w)$ be the number of pairs $(u, v) \in G \times G$ satisfying $w = uvu^{-1}v^{-1}$ (the commutator of u and v). Thus f is a class function on G and hence a linear combination $\sum c_\chi \chi$ of irreducible characters χ of G. The multiplicity c_χ of χ in f is equal to $g/\chi(1)$. Since $\chi(1)|g$, it follows that f is a character of G. Find an explicit G-module M whose character is f, which directly proves that f is a real character. Since in the equation $f = \sum \frac{g}{\chi(1)} \chi$ all the χ's are linearly independent, this would provide a new proof of the basic result that $\chi(1)$ divides the order of G.

29. [Stan1999, Ch. 7 Exer. 71(e)] Given a finite group G, define ψ_G as the character of the action of G on itself by conjugation. That is, the character of permutation representation $\rho : G \to \mathfrak{S}_G$ defined by $\rho(x)(y) = xyx^{-1}$, where $\mathfrak{S}_G$ is the group of permutations of G. Define $\kappa_{n\lambda} = \langle \psi_{\mathfrak{S}_n}, \chi^\lambda \rangle$, where χ^λ is the irreducible representation of $\mathfrak{S}_n$ indexed by the partition λ. $\kappa_{n\lambda} > 0$ with the sole exception of $n = 2, \lambda = (1, 1)$. Is there a "nice" combinatorial interpretation of the numbers $\kappa_{n\lambda}$?

31. [Bren2024, Conj. 1.4] Lusztig's "Combinatorial Invariance Conjecture" about the Kazhdan–Lusztig polynomials $P_{x,y}(q)$: Let $(W_1, S_1), (W_2, S_2)$ be two Coxeter systems, and $u \leq v$ in W_1, $w \leq z$ in W_2 (in the strong Bruhat order) be such that $[u, v] \simeq [w, z]$ (the intervals in the Strong Bruhat order are isomorphic as posets). Then prove

$$P_{u,v}(q) = P_{w,z}(q).$$

32. [Anders Björner] via [Bren2024, Prob. 1.9] Let (W, S) be a Coxeter system and $u \leq v$ in W. Is it true that then there are a Coxeter system (W', S') and $w' \in W'$ such that

$$P_{u,v}(q) = P_{e',w'}(q)$$

where e' is the identity in W'?

33. [Bren2024, Prob. 1.10] Find a combinatorial interpretation for Kazhdan-Lusztig polynomials.

34. [Haim1985a, sec. 1.0] Is the class of linear lattices self-dual?

35. [Jóns1953b, sec. 5] Is the class of linear lattices a variety? (That is, is it definable by a set of equations.)

36. [Wor2024b, quest. 3.65] Are there lattices that obey all orders of the Arguesian law but are not linear? Are there such lattices that are finite?

37. [Wor2024b, quest. 3.74] Is the class of lattices that are isomorphic to a sublattice of the lattice of normal subgroups of some group self-dual?

38. [Jóns1953b, sec. 5] Is the class of lattices that are isomorphic to a sublattice of the lattice of normal subgroups of some group a variety?

39. [Wor2024b, quest. 3.80] Is the class of lattices that are isomorphic to a sublattice of the lattice of subgroups of some Abelian group self-dual?

40. [Jóns1953b, sec. 5] Is the class of lattices that are isomorphic to a sublattice of the lattice of subgroups of some Abelian group a variety?

41. [Wor2024b, quest. 3.92] Given a characteristic p, is the class of lattices that are isomorphic to a sublattice of the lattice of subspaces of some vector space over a field of characteristic p a variety?

42. [Stan1990, sec. 2] For any vector $\mathbf{r}$ there is at most one $\mathbf{r}$-differential *distributive* lattice $L(\mathbf{r})$. It is probably hopeless to determine for which vectors $\mathbf{r}$ $L(\mathbf{r})$ exists. If the elements of the vector increase "sufficiently fast" then $L(\mathbf{r})$ exists. Is it possible to describe the necessary rate of growth?

43. A central point of [LiangSag2024, Th. 3.1] is that for any partition $\lambda = (\lambda_1, \lambda_2, \dots \lambda_{\ell(\lambda)})$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{\ell(\lambda)} > 0$, if we define $\lambda + i = (\lambda_1 + i, \lambda_2 + i, \dots, \lambda_{\ell(\lambda)} + i)$, then the set of partitions $(\lambda + i)_{i \geq 0}$ has the property that for every n, the set of intervals in the partition lattice $\{[\lambda + n, \lambda + (n + k)] \mid k \geq 0\}$ are all isomorphic. This series of elements allows their Order Ideal Lemma (Lem. 1.1) to be applied.

Similar series of elements can be found in other distributive Fomin lattices, the lattice of strict partitions, the lattice of k-row partitions, and the cylindric partitions. This is surprising since this property does not seem to be forced by the hypotheses of distributive Fomin lattices.

Can this observation be extended to the Young–Fibonacci lattices? That is, are there sequences of elements in $\mathbb{YF}_r$, $(p_i)_{i \geq 0}$ for which for every n, the set of intervals $\{[p_n, p_{n+k}] \mid k \geq 0\}$ are all isomorphic? If there are, can they be used to prove an analog of their Th. 3.1? If so, does this provide a clue to a "semi-distributive" property of $\mathbb{YF}_r$?

44. [Stan1990, sec. 2] For any vector $\mathbf{r}$ there is at most one $\mathbf{r}$-differential *distributive* lattice $L(\mathbf{r})$. If we consider the class of $\mathbf{r}$-vectors that are arithmetic progressions with a given difference Δ, then the class $L(\mathbf{r})$ of differential lattices that are differential posets with such $\mathbf{r}$-vectors is closed under cartesian products. Conversely, if the cartesian product of two sequentially differential posets each with at least two ranks is a sequentially differential poset, then both the factors and the product have $\mathbf{r}$-vectors that are arithmetic progressions and they all have the same difference Δ. Applying [Stan2012, Exer. 3.51(a) soln.] shows that $\Delta \leq 0$. If $\Delta = 0$, they are ordinary differential posets. But if $\Delta < 0$, they must be finite. Do there exist any finite distributive differential posts with $\mathbf{r}$-vectors that are arithmetic progressions other than the cartesian exponentials of the 2- and 3-element chains?[Stan1990, Exam. 2.5] Can the cases with $\Delta < 0$ be fully classified?

45. [Fom1994, sec. 2.3]. Given a graded vector space $V = \bigoplus_{i=0}^{\infty} V_i$ and a graded linear operator D on it where $D(V_0) = \{0\}$ and $D(V_{i+1}) \subset D(V_{i-1})$, is it always possible to define a bilinear product on V that turns it into a graded algebra with identity for which D is a derivation — $D(ab) = D(a)b + aD(b)$? It seems that one can always define such a product, but there is much freedom and no clear guarantee the product has any nice properties. So let us add that the product must be associative. If this product exists, it allows the graded linear operator D to be used to construct a dual graded graph by the construction in the citation.

46. [Gaetz2018, Conj. 1.6] Let $\mathfrak{G} : \{e\} = G_0 \subset G_1 \subset G_2 \subset \cdots$ be a tower of groups and let $r \in \mathbb{Z}_{>0}$. Define P to be the splitting diagram of the representations of the tower of groups, with weights that are the multiplicities of the splittings. The tower is *r-dual* if P is a self-dual graded graph with differential degree r. The tower is *r-differential* if the tower is r-dual and the weights of P are all 1, it is a differential poset. Show that: (a) If $\mathfrak{G}$ is r-differential then $P(\mathfrak{G}) \cong \mathbb{Y}^{\otimes r}$ and there exists an abelian group A of order r, not depending on n, such that $G_n \cong A \wr S_n$ for all $n \geq 0$. (b) If $\mathfrak{G}$ is r-dual then $P(\mathfrak{G}) \cong (d_1\mathbb{Y}) \times \cdots \times (d_k\mathbb{Y})$ for some $d_1, \dots, d_k$ and there exists a group H of order r, not depending on n, such that $G_n \cong H \wr S_n$ for all $n \geq 0$.

3. Problems of 2026

47. [ChoiNamSOh2019a, sec. 1] The "tableau switching" process has a correspondence with jeu de taquin.[BenSottStroom1996] There is a "shifted tableau switching" process that similarly corresponds to shifted jeu de taquin. However there is an alternative "modified shifting tableau switching" process with certain better properties. "It would be very nice to develop a modified version of shifted jeu de taquin which plays the same role as that of the shifted jeu de taquin in the shifted switching process."

48. [FagHerr1981] via [Jonathan Farley] shows that a finite modular lattice L with join-irreducible elements J is characterized by the partial ordering induced on J and a function $\rho : J \times J \to 2^J : (a,b) \mapsto \{c \in J \,|\, c \leq a \vee b\}$; if two modular lattices have isomorphic posets J and functions ρ, they are isomorphic. Can this be extended to a classification by describing necessary and sufficient criteria for ρ for it to correspond to a modular lattice?

50. [Stan2012, Exer. 3.51(a) soln.][Wor2026e] [Stan2012] shows that there exists exactly one distributive, d-differential lattice for any positive integer d. That analysis easily generalizes to an analysis of weighted differential distributive lattices, and when we constrain the differential degree and the weights to be positive integers, it can be used to demonstrate non-trivial constraints on possible lattices. Can this technique be extended to prove all of the "strong" hypotheses of the classification in [Wor2026b] to create a classification of all positive-weight differential distributive lattices with $\hat{0}$?

51. [Wor2026c] All locally-finite distributive lattices can be generated by choosing a poset and a fixed reference ideal of the poset. Then the generated lattice is the lattice of ideals of the poset whose symmetric difference from the reference ideal is finite. Find the characteristics of the poset and its reference ideal which determines whether the generated lattice has finite covers, that is, every element has only a finite number of upward and downward covers.

4. Solved Problems of 2025

10. [Stan2017, sec. 4 and Conj. 4.1] via [Linus Setiabrata] Define $\nu_w = \mathfrak{S}_w(1,\ldots,1)$ which is the sum of the coefficients of the Schubert polynomial $\mathfrak{S}_w$ of the permutation w. It is well-known that $\nu_w = 1$ iff w is 132-avoiding (or "dominant"). Conjecture 4.1.: We have $\nu_w = 2$ if and only if $w = a_1 \cdots a_n$ has exactly one subsequence $a_i a_j a_k$ in the pattern 132, i.e., $a_i < a_k < a_j$.

Proved by [Weig2018, Cor. 1.3].

30. [John Stembridge] via [Bren2024, Prob. 1.1] Let (W,S) be a Coxeter system. Let $T = \{wsw^{-1} : s \in S, w \in W\}$ be its set of reflections and for $w \in W$ let $\ell(w)$ be the length of w, the length of the shortest product of generators that $= w$. Is it true that

$$\sum_{t \in T} x^{\ell(t)}$$

is a rational generating function?

Via [Riccardo Biagioli]: Proved by [DeMan1999, Th. 4.1]. A more explicit formula for affine Coxeter systems is given by [BiagHohlSass2026, Th. 1.2].

5. Solved Problems of 2026

49. [Wor2026c] Prove that a locally-finite distributive lattice has a lattice of ideals of prime filters that contains at most three connected components (one isomorphic to the lattice, perhaps one containing the empty ideal, and perhaps one containing the ideal of all primes) iff the lattice has finite width (contains no infinite antichains).

Counterexample provided by Refine.ink: The lattice $\mathbb{Z} \times \mathbf{2}$ has no infinite antichains, but its lattice of ideals of prime filters has three connected components, one isomorphic to $\mathbb{Z} \times \mathbf{2}$, the other two isomorphic to $\mathbf{2}$.

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