THE COMBINATORICS OF TABLEAUX — A BIBLIOGRAPHY

DALE R. WORLEY

Bibliography

[Ab2026] Mohammed Abouziad, Andrew J. Blumberg, Martin Hairer, Joe Kileel, Paul D. Nelson, Daniel Spielman, Nihkil Srivastava, Rachel Ward, Shmuel Weinberger, and Lauren Kiyomi Williams, *First Proof* (2026), arXiv 2602.05192, primary class math.CO, DOI 10.48550/arXiv.2602.05192, available at `https://arxiv.org/abs/2602.05192`. GS 3999643655951135927

Abstract: To assess the ability of current AI systems to correctly answer research-level mathematics questions, we share a set of ten math questions which have arisen naturally in the research process of the authors. The questions had not been shared publicly until now; the answers are known to the authors of the questions but will remain encrypted for a short time.

[AdámSous2004] Jiří Adámek and Lurdes Sousa, *On reflective subcategories of varieties*, J. Algebra **276** (2004), 685–705, DOI 10.1016/j.jalgebra.2003.09.039, available at `https://www.sciencedirect.com/science/article/pii/S002186930300704X`. GS 14819722558156434104. MR2058463 Zbl 1056.08003

Abstract: Full reflective subcategories of varieties are characterized as the cocomplete categories with a regular generator, or as classes of algebras presented by "preequations." As a byproduct, a solution is presented to the problem of describing ω-orthogonality classes of locally finitely presentable categories in terms of closure properties.

Date: June 6, 2026.

[Adel1992] Arnold Adelberg, *On the degrees of irreducible factors of higher order Bernoulli polynomials*, Acta Arith. **62** (1992), 329–342, DOI 10.4064/aa-62-4-329-342, available at https://bibliotekanauki.pl/articles/1391903.pdf. GS 3579676981160173571. MR1199626 Zbl 0771.11013

Abstract: In this paper, we generalize the current results on the p-Eisenstein behavior of first and higher order Bernoulli polynomials [4], [6–9], using the machinery of [Adel1995]. In so doing, we provide a broader framework for the known results, all of which are either immediate consequences or special cases of our more general results. Because of an explicit formula for the coefficients in terms of falling factorials established in [Adel1995], the polynomials $A_n(x, -k)$ which we consider here are actually translates of the standard higher order Bernoulli polynomials $B_n^{(\omega)}(x)$, but *all* of our significant results apply equally well to the standard polynomials and their ordinary coefficients, with $\omega = n - k + 1$. The main results are summarized using standard notations in the research announcement [2].

[Adel1995] ——, *A finite difference approach to degenerate Bernoulli and Stirling polynomials*, Discrete Math. **140** (1995), 1–21, DOI 10.1016/0012-365X(93)E0188-A, available at https://www.sciencedirect.com/science/article/pii/0012365X93E0188A. GS 790452656721078438. MR1333708 Zbl 0841.11010

Abstract: Starting with divided differences of binomial coefficients, a class of multivalued polynomials (three parameters), which includes Bernoulli and Stirling polynomials and various generalizations, is developed. These carry a natural and convenient combinatorial interpretation. Calculation of particular values of the polynomials yields some binomial identities. Properties of the polynomials are established and several factorization results are proven and conjectured.

[Adel1996a] ——, *Higher order Bernoulli polynomials and p-adic analysis*, Ph.D. thesis, Princeton, 1996, https://www.proquest.com/openview/b84809927cc996799fb5533220c23413/1. GS 8580829322577926761.

Abstract: In this thesis we prove some new congruences for p-adic integer Bernoulli numbers, which can be thought of as generalizing the Kummer congruences for ordinary Bernoulli numbers (§3, 4). We apply these congruences to get the irreducible factorization of certain Bernoulli polynomials whose order is divisible by the first power of a rational prime p. We also give applications to Stirling numbers of the first and second kinds (§5).
In addition to this new material, we have provided an extended preface, which gives a non-technical overview and which relates this work to our previous publications on Bernoulli numbers and polynomials [1, 2, 13]. This preface also contains material on Newton polygons, which was implicit in our earlier work, and which marks a promising direction for further investigation.

[Adel1996b] ______, *Congruences of p-adic integer order Bernoulli numbers*, J. Number Theory **59** (1996), 374–388, DOI 10.1006/jnth.1996.0103, available at `https://www.sciencedirect.com/science/article/pii/S0022314X96901031`. GS 15163261087881264847. MR1402614 Zbl 0866.11013

In this paper we establish some new congruences of p-adic integer order Bernoulli numbers. These generalize the Kummer congruences for ordinary Bernoulli numbers. We apply our congruences to prove irreducibility of certain Bernoulli polynomials with order divisible by p and get new congruences for Stirling numbers.

[Adel1998] ______, *Higher order Bernoulli polynomials and Newton polygons*, Applications of Fibonacci numbers (G. E. Bergman, ed.), Vol. 7, Kluwer, 1998, DOI 10.1007/978-94-011-5020-0_1, available at `https://link.springer.com/chapter/10.1007/978-94-011-5020-0_1`. GS 7360630996615311785. MR1638423 Zbl 0922.11013

Abstract: In this paper, we are primarily concerned with factorization questions of the Bernoulli polynomials, both over the rational number field $\mathbf{Q}$ and over the field of p-adic numbers $\mathbf{Q}_p$. We proved several results in [1] on the powers of *pp* dividing the denominators of the Bernoulli numbers. We assumed that th order ℓ is an integer, in fact that $\ell \in \{1, \cdots, n\}$, but all the results remain true if ℓ is a p-adic integer, with no essential change in the proof.

Subsequent to the publication of that paper, it was pointed out to us independently by M. Filaseta and by B. Dwork that our results could be used to determine the Newton polygon of $B_n^{(\ell)}(x)$, or at least the down-sloping part. We carry this out in the current paper and get some strong factorizations results over $\mathbf{Q}_p$. In certain cases where $\ell \in \mathbf{Z}$, we get factorization results over $\mathbf{Q}$ as well.

[AdelFil2002] Arnold Adelberg and Michael Filaseta, *On the m-th order Bernoulli polynomials of degree m that are Eisenstein*, Colloquium Mathematicum **93** (2002), 21–26, DOI 10.4064/cm93-1-3, available at https://www.researchgate.net/profile/Arnold-Adelberg/publication/251067554_On_mTH_order_Bernoulli_polynomials_of_degree_m_that_are_Eisenstein/links/58f4f270aca27289c21ca4c7/On-mTH-order-Bernoulli-polynomials-of-degree-m-that-are-Eisenstein.pdf. 10776760501992231476. MR1930253 Zbl 1015.11006

This paper deals with the irreducibility of the mth order Bernoulli polynomials of degree m. As m tends to infinity, Eisenstein's criterion is shown to imply irreducibility for asymptotically $> 1/5$ of these polynomials.

[AdinRoich2014] Ron M. Adin and Yuval Roichman, *Enumeration of Standard Young Tableaux* (2014), arXiv 1408.4497, primary class math.CO, DOI 10.48550/arXiv.1408.4497, available at https://arxiv.org/abs/1408.4497. GS 2816273339785709559

Abstract: A survey paper, to appear as a chapter in a forthcoming Handbook on Enumeration.

[Ag2022] Mark Agrios, *A Very Elementary Introduction to Sheaves* (2022), arXiv 2202.01379, primary class math.AG, DOI 10.48550/arXiv.2202.01379, available at https://arxiv.org/pdf/2202.01379. GS 9564568987879991018

Abstract: This paper is a very non-rigorous, loose, and extremely basic introduction to sheaves. This is meant to be a a guide to gaining intuition about sheaves, what they look like, and how they work, so that after reading this paper, someone can jump into the extremely abstract definitions and examples seen in textbooks with at least some idea of what is going on.

Most of this material is inspired and built from the work of Dr. Michael Robinson, and that of Dr. Robert Ghrist and Dr. Jakob Hansen, as well as Dr. Justin Curry's PhD thesis, who are some of the only applied sheaf theorists out there and they do an amazing job of explaining sheaves in a concrete way through their research. The rest of this paper is populated by mathematical definitions found in textbooks that I have stretched from two lines into multiple pages, as well as some analogies for thinking of sheaves I have thought of myself. This paper only assumes knowledge of basic linear algebra, basic group theory, and the very fundamentals of topology. If there is anything in the setup that you do not understand it is probably a quick Wikipedia search away. I hope this paper provides insight, intuition, and helpful examples of why sheaves are such powerful tools in both math and science.

[Alex2021] Per Alexandersson, *The symmetric functions catalog; Border strip tableaux and the Littlewood map*, `https://www.symmetricfunctions.com/borderStripTableaux.htm`. Accessed July 20, 2024.

Abstract: The notion of border-strip tableaux, also known as rim-hook tableaux, show up in several areas, most notably the Murnaghan–Nakayama rule and the original definition of LLT polynomials.

[AliPoddNoor2012] M. Ayub Ali, Sompa Rani Podder, and A. S. A. Noor, *n-Distributive Lattice*, J. Phys. Sci. **16** (2012), available at `https://ir.vidyasagar.ac.in/handle/123456789/856`. GS 18179581271491700308. MR3011011

Abstract: J. C. Varlet has given the concept of 0-distributive and 1-distributive lattices. In this paper the authors have generalized the whole concept and introduced the notion of n-distributive lattices. They show that for a neutral element of a lattice L , the n-annihilator of any subset of L is an n-ideal if and only if L is n-distributive. Then the authors study different properties of these lattices. Finally, using the n-annihilators they generalize the well known prime separation theorem of distributive lattices with respect to annihilator n-ideal in a general lattice and produce an interesting characterization of n-distributive lattice.

[AlZeil2018] Judith M. Alcock-Zeilinger. *The symmetric group, its representations, and combinatorics.* Lecture notes, Tübingen, 2018. GS 4144100452088666076.

Abstract: In this course, we'll be examining the symmetric group and its representations from a combinatorial view point. We will begin by defining the symmetric group Sn in a combinatorial way (as a permutation group) and in an algebraic way (as a Coxeter group).
We then move on to study some general results of the representation theory of finite groups using the theory of characters.
Thereafter, we once again lay our focus on the symmetric group and study its representation. The method used here follows that of Vershik and Okounkov, and the central result is that the Bratteli diagram of the symmetric group (giving a relation between its irreducible representations) is isomorphic to the Young lattice. In doing so, we will be able to intruduce Young tableaux in a natural way, and we will see that the number of Young tableaux of a given shape λ is the dimension of the irreducible representation corresponding to λ.
Then, we discuss several results pertaining to the representation theory of the symmetric group from a combinatorial viewpoint. We will use a typical combinatorial tool, namely a proof by bijection, which was also already implemented in the Vershik-Okounkov method, without explicitly saying so. In particular, we discuss the Robinson-Schensted algorithm which allows us to proof that the sum of the (dimensions of the irreducible representations of the symmetric group)^2 is the order of the group. We use this result to discuss how one can arrive at a general formula for the number of Young tableaux of size n. Lastly, we focus on the famous hook length formula giving the number of Young tableaux of a certain shape λ. We will follow the bijective proof by Novelli, Pak and Stoyanovskii to prove this result.

[AnArbPer2014] Maribel Anacona, Luis Carlos Arboleda, and F. Javier Pérez-Fernández, *On Bourbaki's axiomatic system for set theory*, Synthese **191** (2014), 4069–4098, DOI 10.1007/s11229-014-0515-1, available at `https://link.springer.com/article/10.1007/s11229-014-0515-1`. GS 7080709106633060392. MR3265270 Zbl 1310.03048

Abstract: In this paper we study the axiomatic system proposed by Bourbaki for the Theory of Sets in the *Éléments de Mathématique*. We begin by examining the role played by the sign τ in the framework of its formal logical theory and then we show that the system of axioms for set theory is equivalent to Zermelo–Fraenkel system with the axiom of choice but without the axiom of foundation. Moreover, we study Grothendieck's proposal of adding to Bourbaki's system the axiom of universes for the purpose of considering the theory of categories. In this regard, we make some historical and epistemological remarks that could explain the conservative attitude of the Group.

[And1984] George E. Andrews. *Generalized Frobenius Partitions.* Memoirs AMS, vol. 49. AMS, Providence, RI, US, 1984, ISBN 978-0821839560. GS 12859368804672332380.

Abstract: This paper is devoted to the study of equilength two-line arrays of non-negative integers. These are called generalized Frobenius partitions. It is shows that such objects have numerous interactions with modular forms, Kloosterman quadratic forms, the Lusztig-Macdonald-Wall conjectures as well as with classical theta functions and additive number theory.

[ApMonKauers2012] Ainhoa Aparicio Monforte and Manual Kauers, *Formal Laurent series in several variables*, Expo. Math. **31** (2013), 350–367, DOI 10.1016/j.exmath.2013.01.004, available at `https://www.sciencedirect.com/science/article/pii/S0723086913000054`. GS 17552958152933882881. MR3133710 Zbl 1283.13018

Abstract: We explain the construction of fields of formal infinite series in several variables, generalizing the classical notion of formal Laurent series in one variable. Our discussion addresses the field operations for these series (addition, multiplication, and division), the composition, and includes an implicit function theorem.

[ArCano2001] Fuensanta Aroca and Jose Cano, *Formal Solutions of Linear PDEs and Convex Polyhedra*, J. Symb. Comput. **32** (2001), 717–737, DOI 10.1006/jsco.2001.0492, available at https://www.sciencedirect.com/science/article/pii/S0747717101904924. GS 1813582625656158179. MR1866713 Zbl 1077.35040

Abstract: The Newton polygon construction for ODEs, and Malgrange–Ramis polygon for partial differential equations in one variable are generalized in order to give an algorithm to find solutions of a linear partial differential equation at a singularity. The solutions found involve exponentials, logarithms and Laurent power series with exponents contained in a strongly convex cone.

[ArCanoJung2003] Fuensanta Aroca, Jose Cano, and F. Jung, *Power series solutions for non-linear PDE's*, ISSAC '03 Proceedings of the 2003 international symposium on symbolic and algebraic computation (Piladelphia, 2003), ACM, New York, 2003, DOI 10.1145/860854.860863, available at https://www.researchgate.net/profile/Jose-Cano-7/publication/221564440_Power_Series_Solutions_for_Non-Linear_PDE's/links/5566edf108aeccd77735c4bd/Power-Series-Solutions-for-Non-Linear-PDEs.pdf. GS 7766721448210565322. Zbl 1072.68644

Abstract: This paper describes an iterative method for searching power series solution of a partial differential equation. Power series expansions considered have support in some convex cone of $\mathbb{R}^N$. We do this by introducing a N-variables analog of the Newton polygon construction, used in the case of ordinary differential equations.

[ArDecRond2019] Fuensanta Aroca, Julie Decaup, and Guillaume Rond, *The Minimal cone of an algebraic Laurent series*, Math. Ann. **382** (2019), 1745–1773, DOI 10.1007/s00208-021-02338-9, available at https://arxiv.org/abs/1902.03961. GS 15529175428362621070. MR4403234 Zbl 1504.05308

Abstract: We study the algebraic closure of $\mathbb{K}((x))$, the field of power series in several indeterminates over a field $\mathbb{K}$. In characteristic zero we show that the elements algebraic over $\mathbb{K}((x))$ can be expressed as Puiseux series such that the convex hull of its support is essentially a polyhedral rational cone, strengthening the known results. In positive characteristic we construct algebraic closed fields containing the field of power series and we give examples showing that the results proved in characteristic zero are longer valid in positive characteristic.

[Arg1966] L. N. Argabright, *A note on invariant integrals on locally compact semigroups*, Proc. Am. Math. Soc. **17** (1966), 377–382, available at `https://www.ams.org/journals/proc/1966-017-02/S0002-9939-1966-0188341-7/S0002-9939-1966-0188341-7.pdf`. GS 3120722967737159954. MR188341 Zbl 0138.25602

Abstract: In this note we prove the following:

> THEOREM 1. Let S be a locally compact semigroup satisfying the condition (#). Then the following are equivalent.
> (a) S admits a right invariant integral.
> (b) S admits an r^*-invariant measure.
> (c) S contains a unique minimal left ideal (which is necessarily closed).

Thus we obtain a complete generalization of the theorem of W. G. Rosen [10] on the existence of invariant means on compact semigroups. In addition, we make some remarks concerning the existence and structure of r^*-invariant measures on semigroups which do not necessarily satisfy (#).

[ArmKais1972] Manfred Armbrust and Klaus Kaiser, *Some remarks on projective model classes and the interpolation theorem*, Math. Ann. **197** (1972), 5–8, DOI 10.1007/BF01427948, available at `https://link.springer.com/article/10.1007/BF01427948`. GS 15581391997674767407. MR297551 Zbl 0222.02059

[Art1957] Emil Artin. *Geometric Algebra*. Interscience tracts in pure and applied mathematics, vol. 3. Interscience, New York, 1957, ISBN 978-0471608394. GS 4772249499860859804.

Abstract: Linear algebra, topology, differential and algebraic geometry are the indispensable tools of the mathematician of our time. It is frequently desirable to devise a course of geometric nature which is distinct from these great lines of thought and which can be presented to beginning graduate students or even to advanced undergraduates. The present book has grown out of lecture notes for a course of this nature given at New York University in 1955. This course centered around the foundations of affine geometry, the geometry of quadratic forms and the structure of the general linear group. I felt it necessary to enlarge the content of these notes by including projective and symplectic geometry and also the structure of the symplectic and orthogonal groups.

[Baez2004] John C. Baez, *[review of] On Quaternions and Octonions: Their Geometry, Arithmetic, and Symmetry by John H. Conway and Derek A. Smith*, Bull. Am. Math. Soc. **39** (2002), 145–205, available at `https://math.ucr.edu/home/baez/octonions/conway_smith/conway_smith.pdf`.

Abstract: Conway and Smith's book is a wonderful introduction to the normed division algebras: the real numbers ($\mathbb{R}$), the complex numbers ($\mathbb{C}$), the quaternions ($\mathbb{H}$) and the octonions ($\mathbb{O}$). The first two are well-known to every mathematician. In constrast, the quaternions and especially the octonions are sadly neglected, so the authors rightly concentrate on these. They develop these number systems from scratch, explore their connections to geometry, and even study number theory in quaternionic and octonionic versions of the integers.

[Baez2023] ———, *Young Diagrams and Classical Groups* (2023), arXiv 2302.07971, primary class math.RT, DOI 10.48550/arXiv.2302.07971, available at `https://arxiv.org/abs/2302.07971`. GS 16524624212250277182

Abstract: Young diagrams are ubiquitous in combinatorics and representation theory. Here we explain these diagrams, focusing on how they are used to classify representations of the symmetric groups S_n and various "classical groups": famous groups of matrices such as the general linear group $\mathrm{GL}(n, \mathbb{C})$ consisting of all invertible $n \times n$ complex matrices, the special linear group $\mathrm{SL}(n, \mathbb{C})$ consisting of all $n \times n$ complex matrices with determinant 1, the group $\mathrm{U}(n)$ consisting

of all unitary $n \times n$ matrices, and the special unitary group SU(n) consisting of all unitary $n \times n$ matrices with determinant 1. We also discuss representations of the full linear monoid consisting of all linear transformations of $\mathbb{C}^n$. These notes, based on the column *This Week's Finds in Mathematical Physics*, are made to accompany a series of lecture videos.

[Band1981] Hans-J. Bandelt, *Tolerance relations on lattices*, Bull. Aust. Math. Soc. **23** (1981), 367–381, DOI 10.1017/S0004972700007255, available at https://www.cambridge.org/core/journals/bulletin-of-the-australian-mathematical-society/article/tolerance-relations-on-lattices/B7A754195C89ED13DB0FE16ACF761E80. GS 559901885079221523. Zbl 0449.06005

The lattice of all tolerance relations (that is, reflexive, symmetric compatible relations) on a lattice is investigated. For modular lattices some examples are given which show that such relations do naturally occur.

[Bart1979] Marcia Bartusiak, *The Effects of Proton Irradiation on the Absorption of Multi Component Glasses*, M.S. thesis, Old Dominion U., 1979, https://digitalcommons.odu.edu/physics_etds/205/. GS 17681897968606494264.

Abstract: Three Schott glasses typically used in space technology – LaK 21, KzFS N4, and LF 5 – were irradiated with 85 MeV protons. Induced absorption spectra were determined for each glass after varying fluences of radiation. It was found that the spectra for each glass can be fitted with three Gaussian shaped bands in the near ultraviolet-visible region, while a fourth Gaussian characterizes the absorption edge. For doses up to 10^7 rads, the dependence of the induced absorption α on total dose is accurately described by the saturating exponential function $\alpha(\phi) = \alpha_S(1-e^{-b\phi})$ where α_S and b are constants dependent on the wavelength and glass type. Similar analyses were performed on data obtained from a previous study into the effects of 7.0 MeV electron irradiation on those same three types of glass. For any one glass, it was found that electrons and protons produced absorption bands with peaks at the same energies but with different saturation levels. For the glasses and energy region investigated, protons form

low energy color centers more readily than do electrons, while electrons have a greater influence in creating centers at the higher energies.

[Bat2016] Batominovski, *Haar measure of a topological ring*, https://math.stackexchange.com/questions/1704993/haar-measure-of-a-topological-ring. Accessed August 4, 2024.

Is there a reasonable generalization of Haar measure to a topological ring?

[Bell1961] Eric Temple Bell. *The last problem.* Simon & Schuster, New York, 1961.

Abstract: The "last" problem of the title of Dr. Bell's entertaining volume is a problem in algebra so simple that any high-school freshman can understand it. And yet it is so difficult that no one in the whole history of the world has found a final answer.
The late, distinguished scholar wrote this as the last of his many highly readable contributions to the field of mathematics. It traces the history of the problem from ancient times down to Pierre Fermat, the seventeenth-century French mathematician and lawyer who claimed to have solved it but failed to leave his proof.
But while [the author] was tracing the history of the problem, he took many side excursions. In so many places and times did concern with the problem crop up that he used it as a peg on which to hang not only a whole history of mathematics but practically a social and cultural history of the world.

[BensConw1985] D. J. Benson and John H. Conway, *Diagrams for modular lattices*, J. Pure Appl. Algebra **37** (1985), 111-116, DOI 10.1016/0022-4049(85)90091-X, available at https://www.sciencedirect.com/science/article/pii/002240498590091X. GS 1310623670170217418. MR796403 Zbl 0575.06008

Abstract: The purpose of this paper is to describe a notation for arbitrary modular lattices satisfying an appropriate finiteness condition, in such a way that the lattice may always be recovered from the diagram, and if the lattice is finite then the diagram is also finite, and usually has significantly fewer vertices than the lattice has elements.

[BenSottStroom1996] Georgia Benkart, Frank Sottile, and Jeffrey Stroomer, *Tableau Switching: Algorithms and Applications*, J. Comb. Theory **Ser. A 76** (1996), 11–43, DOI 10.1006/jcta.1996.0086, available at `https://www.sciencedirect.com/science/article/pii/S0097316596900862`. GS 1813720954627375600.0858.05099

Abstract: We define and characterize *switching*, an operation that takes two tableaux sharing a common border and "moves them through each other" giving another such pair. Several authors, including James and Kerber, Remmel, Haiman, and Shimozono, have defined switching operations; however, each of their operations is somewhat different from the rest and each imposes a particular order on the switches that can occur. Our goal is to study switching in a general context, thereby showing that the previously defined operations are actually special instances of a single algorithm. The key observation is that switches can be performed in virtually any order without affecting the final outcome. Many known proofs concerning the jeu de taquin, Schur functions, tableaux, characters of representations, branching rules, and the Littlewood–Richardson rule use essentially the same mechanism. Switching provides a common framework for interpreting these proofs. We relate Schützenberger's evacuation procedure to switching and in the process obtain further results concerning evacuation. We define *reversal*, an operation which extends evacuation to tableaux of arbitrary skew shape, and apply reversal and related mappings to give combinatorial proofs of various symmetries of Littlewood–Richardson coefficients.

[Ber1986] Allan Berele, *A Schensted-type correspondence for the symplectic group*, J. Comb. Theory **Ser. A 43** (1986), 320–328, DOI 10.1016/0097-3165(86)90070-1, available at `https://www.sciencedirect.com/science/article/pii/0097316586900701`. GS 11001899118143708236. Zbl 0633.05009

UD. Topics: insertion algorithm, $Gl(n,\mathbb{C})$, $Sp(n,\mathbb{C})$. Constructs the UD insertion algorithm from representation-theoretic considerations.

Abstract: The Schensted correspondence is closely related to the decomposition of $V^{\otimes n}$ as a $GL(V)$-module. In this paper we obtain an analogous correspondence related to the decomposition of $V^{\otimes n}$ as an $Sp(V)$-module.

[Berk2024] Christine Berkesch, Benjamin Brubaker, Gregg Musiker, Pavlo Pylyavskyy, and Victor Reiner (eds.) *Open problems in algebraic combinatorics.* Proc. of Symposia in Pure Mathematics, vol. 110. AMS, Providence, RI, US, 2024, ISBN 978-1470473334.

Abstract: In their preface, the editors describe algebraic combinatorics as the area of combinatorics concerned with exact, as opposed to approximate, results and which puts emphasis on interaction with other areas of mathematics, such as algebra, topology, geometry, and physics. It is a vibrant area, which saw several major developments in recent years. The goal of the 2022 conference Open Problems in Algebraic Combinatorics 2022 was to provide a forum for exchanging promising new directions and ideas. The current volume includes contributions coming from the talks at the conference, as well as a few other contributions written specifically for this volume.
The articles cover the majority of topics in algebraic combinatorics with the aim of presenting recent important research results and also important open problems and conjectures encountered in this research. The editors hope that this book will facilitate the exchange of ideas in algebraic combinatorics.

[BiagHohlSass2026] Riccardo Biagioli, Christophe Hohlweg, and Elisa Sasso, *On generating functions and automata associated to reflections in Coxeter systems* (2026), arXiv 2602.16361, primary class math.GR, DOI 10.48550/arXiv.2602.16361, available at `https://arxiv.org/abs/2602.16361`

Abstract: In this article, we study two combinatorial problems concerning the set of reflections of a Coxeter system. The first problem asks whether the language of palindromic reduced words for reflections is regular, and the second is about finding formulas for the Poincaré series of reflections, namely the generating function of reflection lengths. These two problems were inspired by a conjecture of Stembridge stating that the Poincaré series of reflections is rational and by the solution provided by de Man.

To address the first problem, we introduce reflection-prefixes, arising naturally from palindromic reduced words. We study their connections with the root poset, dominance order on roots, and dihedral reflection subgroups. Using m-canonical automata associated with m-Shi arrangements, we prove that the language of reduced words for reflection-prefixes is regular. For the second problem, we focus on affine Coxeter groups. In this case, we derive a simple formula for the Poincaré series using symmetries of the Hasse diagram of the root poset.

[BillGaoPawl2025] Sara Billey, Yibo Gao, and Brendan Pawlowski, *Introduction to the cohomology of the flag variety*, March 10, 2025.

Abstract: One hundred years ago, Hilbert gave a list of big problems in mathematics in his day. Hilbert's 15th problem asked for the development of a rigorous calculus to explain the enumerative results derived by Hermann Schubert in the later 1800s with regard to intersecting varieties based on constraints imposed by rank conditions of vector spaces. Today by way of many contributions in algebraic topology, geometry, and combinatorics, we consider this problem to be fully solved. Yet, deep questions remain in terms of the subtleties of actually carrying out the process. In this chapter, we hope to summarize the contributions that have lead up to the rigorous development of what has become known as Schubert calculus. We will survey some of the vast literature development in the past few decades with an eye toward computation. We will discuss Grassmannian varieties and flag varieties. We will give a description of their cohomology rings in algebraic terms inspired by the Chow ring of a smooth variety. From this description, we will derive formulas for Schur polynomials and Schubert polynomials which are the cohomology classes of Schubert varieties in these cases. The Grassmannians, flag varieties, cohomology rings, Chow rings, Schubert polynomials, Schur polynomials, etc. can be generalized in many ways. In this chapter, we will hint at the vast literature in this area and point you to the other references in the Handbook for more information. Finally, we will identify some open problems that remain a challenge even with all of the modern tools at our fingertips in hopes of inspiring further contributions in this fascinating field.

[Birk1933] Garrett Birkhoff, *On the combination of subalgebras*, Math. Proc. Camb. Philos. Soc. **29** (1933), 44‘–464, DOI 10.1017/S0305004100011464, available at http://journals.cambridge.org/abstract_S0305004100011464. GS 5404777722122495360. Zbl 0007.39502

Abstract: The purpose of this paper is to provide a point of vantage from which to attack combinatorial problems in what may be termed modern†, synthetic, or abstract algebra. In this spirit, a research has been made into the consequences and applications of seven or eight axioms, only one [V] of which is itself new‡. The ‘lattice theory’ evolved from these leads directly to a number of interesting applications, scattered throughout the paper, in general point-set, group and ring theory. Perhaps the most novel are concerned with the completeness with which a small set of formal rules describes the implications of very general processes—as in Theorems 5·1 and 25·2.

[Birk1935] ______, *On the structure of abstract algebras*, Math. Proc. Camb. Philos. Soc. **31** (1935), 433–454, DOI 10.1017/S0305004100013463, available at https://www.cambridge.org/core/journals/mathematical-proceedings-of-the-cambridge-philosophical-society/article/abs/on-the-structure-of-abstract-algebras/D142B3886A3B7A218D8DF8E6DDA2B5B1. available at https://web.archive.org/web/20180330012312/https://pdfs.semanticscholar.org/a282/3f992ea5e2d2a1e989ce01844da71e4ec6a5.pdf GS 4998884063153058981.

Abstract: The following paper is a study of abstract algebras qua abstract algebras. ...
An outline of the material will perhaps tell the reader what to expect. In §§2–7, the. notion of abstract algebra is defined, and relations between abstract algebras of two kinds (groups and “lattices”) derived from a fixed abstract algebra are indicated.
In §8, abstract algebras are divided by a very simple scheme into self-contained “species”. Within each species, a perfect duality is found between families of formal laws and the families of algebras satisfying them; this occupies §§9–10. ...

In §§16–18, the "lattice" $E(C)$ of the equivalence relations between the objects of a fixed aggregate C is denned; in §§20–21 such lattices are shown to be interchangeable with lattices of Boolean subalgebras and lattices of subgroups. Other miscellaneous facts are proved in §19, §22, and §23. In §24, the interesting truth is established that, if C is an algebra, then the equivalence relations which are homomorphic are a "sublattice" of $E(C)$.
In §25 an open question is settled, and the paper concludes in §§26–31 with some observations on topology.

[Birk1937] ______, *Rings of sets*, Duke Math. J. **3** (1937), 443–454, DOI 10.1215/S0012-7094-37-00334-X, available at `https://projecteuclid.org/journals/duke-mathematical-journal/volume-3/issue-3/Rings-of-sets/10.1215/S0012-7094-37-00334-X.short`. GS 10180976689018188837. MR1546000

Proof of Birkhoff's representation theorem for finite distributive lattices.

[Birk1967] ______. *Lattice theory.* American Mathematical Society Colloquium Publications, vol. 25. American Mathematical Society, Providence, 3rd ed., 1967. Original edition 1940. GS 10180976689018188837.

Birkhoff's classic text on lattice theory.

[BlackKingWyb1983] G. R. E. Black, Ronald C. King, and B. G. Wybourne, *Kronecker products for compact semisimple Lie groups*, J. Phys. A **16** (1983), 1555–1589, DOI 10.1088/0305-4470/16/8/006, available at `https://iopscience.iop.org/article/10.1088/0305-4470/16/8/006/meta`. GS 474517853954770389. Zbl 0522.22012

Abstract: A review is given of the application of S-function techniques to the evaluation of Kronecker products of irreducible representations of compact semisimple Lie groups. Explicit formulae are derived for all irreducible representations of all such groups. Recent developments involving composite Young diagrams are brought to fruition and the vexed problem of SO_{2k} is dealt with completely. New branching rules for the classical groups are given in an appendix. These are exploited in the evaluation of Kronecker products by means of a technique which is applied to both classical and exceptional groups. A discussion is made of various modification rules which are needed to express the final results in standard form.

[Bod2021] Elijah Bodish, Ben Elias, David E. V. Rose, and Logan Tatham, *Decreasing subsequences and Viennot for oscillating tableaux* (2021), arXiv 2108.11528, primary class math.CO, DOI 10.48550/arXiv.2108.11528, available at `https://arxiv.org/abs/2108.11528`. GS 519427637904476303

Abstract: We establish an extension of Viennot's geometric (shadow line) construction to the setting of oscillating tableaux. We then use this to give a new proof of the Type C analogue of Schensted's theorem on longest decreasing subsequences. This pairs with our results from arXiv:2103.14997v1 [math.RT] on Type C webs to give a direct proof of a result of Sundaram and Stanley: that the dimension of the space of invariant vectors in a $2k$-fold tensor product of the vector representation of $\mathfrak{sp}_{2n}$ equals the number of $(n+1)$-avoiding matchings of $2k$ points.

[BogFreesKung1990] Kenneth P. Bogard, Ralph S. Freese, and Joseph P. S. Kung (eds.) *The Dilworth theorems: Selected papers of Robert P. Dilworth.* Contemporary Mathematicians. Springer, New York, 1990, ISBN 978-0817634346. GS 15660047109500941826.

The book is organized into chapters covering different subject areas of Dilworth's work. Each chapter begins with a background, written by Dilworth, for his papers reprinted in the chapter. In these backgrounds, he discusses how and why he approached the problems he solved. Following the background in each chapter are the reprints of Dilworth's papers, followed by articles about these papers we have solicited from experts in the appropriate field. At Dilworth's suggestion, these articles are not just commentaries on his papers, but are general surveys on the subsequent work in the area, bringing the reader up to the current state of the subject.

[Bred1993] Glen E. Bredon. *Topology and Geometry.* GTM, vol. 139. Springer, New York, 1993. GS 3991045028708386339.

Abstract: This book is intended as a textbook for a beginning (first-year graduate) course in algebraic topology with a strong flavoring of smooth manifold theory. The choice of topics represents the ideal (to the author) course. In practice, however, most such courses would omit many of the subjects in the book. I would expect that most such courses would assume previous knowledge of general topology and so would skip that chapter, or be limited to a brief run-through of the more important parts of it. The section on homotopy should be covered, however, at some point. I do not go deeply into general topology, but I do believe that I cover the subject as completely as a mathematics student needs unless he or she intends to specialize in that area.
I attempt to give the reader some glimpses into the beautiful and important realm of smooth manifolds along the way, and to instill the tenet that the algebraic tools are primarily intended for the understanding of the geometric world.
This book is not intended as a source book. There is no attempt to present material in the most general form, unless that entails no expense of time or clarity. Exceptions are cases, such as the proof of de Rham's Theorem, where generality actually improves both efficiency and clarity.

[Bren2024] Francesco Brenti, *Some open problems on Coxeter groups and unimodality*, Open problems in algebraic combinatorics (Christine Berkesch, Benjamin Brubaker, Gregg Musiker, Pavlo Pylyavskyy, and Victor Reiner, eds.), Proc. of Symposia in Pure Mathematics, vol. 110, AMS, Providence, RI, US, 2024, available at `https://www.mat.uniroma2.it/~brenti/65.pdf`. GS 5938034608988166121.

Abstract: In this paper I present some open problems on Coxeter groups and unimodality, together with the main partial results, and computational evidence, that are known about them.

[BritzMainPezz2001] T. Britz, M. Mainetti, and L. Pezzoli, *Some operations on the family of equivalence relations*, Algebraic Combinatorics and Computer Science: A Tribute to Gian-Carlo Rota (H. Crapo, D. Senato, Ralph S. Freese, and O. C. Garcia, eds.), Springer-Verlag, Milan, 2001, 2001, pp. 445–459, DOI 10.1007/978-88-470-2107-5_18, available at `http://www2.mat.dtu.dk/people/oldusers/T.Britz/papers/equrel.pdf`.

https://link.springer.com/chapter/10.1007/978-88-470-2107-5_18 GS 6682016611742498100. Zbl 0971.03046

Abstract: Throughout the history of mathematics, the notion of an equivalence relation has played a fundamental role. It dates back at least to the time when the natural numbers first were introduced: a non-negative integer may be thought of as a representative of the equivalence class of sets with the same cardinality. To express such a simple and "obvious" fact with equivalence relations may seem unnecessarily cumbersome. Nothing is further from the truth. Equivalence relations play a decisive role as building elements in every area of mathematics. For instance, algebra is firmly founded on equivalence relations: groups theory, rings theory, modules and fields would basically be impossible to define and use without equivalence relations.

[BumpSchil2017] Daniel Bump and Anne Schilling. *Crystal Bases: Representations and Combinatorics*. World Scientific, Singapore, 2017, ISBN 978-9814733441. GS 15106484894893345173.

Abstract: Crystal bases are purely combinatorial objects that are analogous to representations of Lie groups or Lie algebras. They appeared in the works of Kashiwara, Lusztig and Littelmann on quantum groups and the geometry of flag varieties. In retrospect, topics from the combinatorical theorv of tableaux such as the famous Robinson–Schensted-Knuth algorithm and the plactic monoid of Lascoux and Schützenberger fit into the crystal base theory. Crystal bases come up in many unexpected places, from mathematical physics to number theory.
This book originated from a plan to approach crystal base theory from a purely combinatorial point of view. It is aimed at graduate students and researchers who wish to delve into this subject.
It seems that every exposition of crystal base theory needs some powerful method behind the proofs. In existing expositions on crystal bases such as Hong and Kang (2002)], Kashiwara (2002)] and Littelmann Littelmann (1997)] this has come from either quantum groups or Littelmann paths. We have taken a different path, relying on ideas of Stembridge and Kashiwara for our foundations. Thus we were able to prove everything combinatorially. In our approach, the link

between crystals and representation theory is made through Demazure crystals.

[BurrSan1999] Stanley Burris and H. P. Sankappanavar. *A course in universal algebra*, Millennium, 1999. GS 5105398544941208764.

Abstract: Universal algebra has enjoyed a particularly explosive growth in the last twenty years, and a student entering the subject now will find a bewildering amount of material to digest. This text is not intended to be encyclopedic; rather, a few themes central to universal algebra have been developed sufficiently to bring the reader to the brink of current research. The choice of topics most certainly reflects the authors' interests. Chapter I contains a brief but substantial introduction to lattices, and to the close connection between complete lattices and closure operators. Chapter II develops the most general and fundamental notions of universal algebra–these include the results that apply to all types of algebras, such as the homomorphism and isomorphism theorems. In Chapter III we show how neatly two famous results–the refutation of Euler's conjecture on orthogonal Latin squares and Kleene's characterization of languages accepted by finite automata-can be presented using universal algebra. Chapter IV starts with a careful development of Boolean algebras, including Stone duality, which is subsequently used in our study of Boolean sheaf representations; however, the cumbersome formulation of general sheaf theory has been replaced by the considerably simpler definition of a Boolean product. The final chapter gives the reader a leisurely introduction to some basic concepts, tools, and results of model theory.

[Byrn2012] Patrick Byrnes, *Structural Aspects of Differential Posets*, Ph.D. thesis, Univ. of Minnesota, 2012, `https://conservancy.umn.edu/handle/11299/142992`. GS 37435333891573400. MR3130910

Abstract: This thesis investigates some structural properties of differential posets and answers some open questions. There are three main results.
First, a proof is given that the largest rank size of the nth rank of an r-differential poset is given by the nth rank of the Fibonacci r-differential poset. This solves a question of Stanley.

Second, a proof is given that the only 1-differential lattices are Young's lattice and the Fibonacci 1-differential poset. This also solves a question of Stanley.
Third, it is shown that any quantized r-differential poset has the Fibonacci r-differential poset as its underlying r-differential poset. This negatively answers a question of Lam.
Further, a method for computing all partial 1-differential posets up to a given rank is described. The results of this computation up to rank 10 are also included.

[Cam1999] Peter J. Cameron. *Permutation groups.* Lond. Math. Soc. Student Texts, vol. 45. Cambridge Univ. Press, Cambridge, UK, 1999, ISBN 978-1316696286. GS 11909429258034171620.

Abstract: Permutation groups are one of the oldest topics in algebra. However, their study has recently been revolutionised by new developments, particularly the classification of finite simple groups, but also relations with logic and combinatorics, and importantly, computer algebra systems have been introduced that can deal with large permutation groups. This book gives a summary of these developments, including an introduction to relevant computer algebra systems, sketch proofs of major theorems, and many examples of applying the classification of finite simple groups. It is aimed at beginning graduate students and experts in other areas, and grew from a short course at the EIDMA institute in Eindhoven.

[Cam2000] ______, *Projective spaces*, Projective and Polar Spaces, 2000, 2000, DOI 10.1007/978-88-470-2107-5_18, available at `https://webspace.maths.qmul.ac.uk/p.j.cameron/pps/pps1.pdf`.

Abstract: In this chapter, we describe projective and affine spaces synthetically, in terms of vector spaces, and derive some of their geometric properties.

[Cam2015] ______, *Projective and Polar Spaces*, 2015, available at `https://cameroncounts.wordpress.com/wp-content/uploads/2015/04/pps1.pdf`. GS 289809382116927580.

Abstract: These notes are about geometry, but by no means all or even most of geometry. I am concerned with the geometry of incidence of points and lines, over an arbitrary field, and unencumbered by metrics or continuity (or even betweenness). The major themes are the projective and affine spaces, and the polar spaces associated with sesquilinear or quadratic forms on projective spaces. The treatment of these themes blends the descriptive (*What do these spaces look like?*) with the axiomatic (*How do I recognize them?*) My intention is to explain and describe, rather than to give detailed argument for every claim. Some of the theorems (especially the characterisation theorems) are long and intricate. In such cases, I give a proof in a special case (often over the field with two elements), and an outline of the general argument.

[Can1996] Himmet Can, *Representations of the Generalized Symmetric Groups* **37** (1996), 289–307 pp., available at https://eudml.org/doc/233438. GS 16451971772850114946. Zbl 0870.20012

Abstract:
The main aim of this paper is to construct a full set of irreducible, inequivalent representations of the generalised symmetric groups $G(m, 1, n)$ in terms of m-sets of partitions of n and combinatorial concepts connected with generalised Young tableaux, etc. As a matter of fact, the irreducible representations of the generalised symmetric groups $G(m, 1, n)$ were studied by Osima [5], Puttaswamaiah [7] and Hughes [2], respectively. However there are well-known constructions of irreducible representations and of irreducible modules, called Specht modules, for the symmetric groups S_n which are based on elegant combinatorial concepts connected with Young tableaux, etc. (see, e.g. [3]). Therefore, in this paper we show how the Specht approach to the irreducible representations of the symmetric groups can be extended to deal with the generalised symmetric groups $G(m, 1, n)$.

[Car2006] Cartier Pierre, *A primer of Hopf algebras*, Frontiers in number theory, physics, and geometry II (Pierre Cartier, Pierre Moussa, Bernard Julia, and Pierre Vanhove, eds.), Springer, Berlin, Heidelberg, 2006. GS 4452484296623990029, available at https://ncatlab.org/nlab/files/CartierHopfAlgebras.pdf. GS 14988245623391773438. Zbl 1184.16031

Abstract: In this paper, we review a number of basic results about so-called Hopf algebras. We begin by giving a historical account of the results obtained in the 1930's and 1940's about the topology of Lie groups and compact symmetric spaces. The climax is provided by the structure theorems due to Hopf, Samelson, Leray and Borel. The main part of this paper is a thorough analysis of the relations between Hopf algebras and Lie groups (or algebraic groups). We emphasize especially the category of unipotent (and prounipotent) algebraic groups, in connection with Milnor–Moore's theorem. These methods are a powerful tool to show that some alge- bras are free polynomial rings. The last part is an introduction to the combinatorial aspects of polylogarithm functions and the corresponding multiple zeta values.

[Cel2023a] Kyle Celano, *Chromatic symmetric functions and RSK for (**3** + **1**)-free posets*, Ph.D. thesis, U. of Miami, 2023, https://scholarship.miami.edu/view/pdfCoverPage?instCode=01UOML_INST&filePid=13419056900002976&download=true. GS 12187791321317946238.

Abstract: In 1995, Stanley introduced the chromatic symmetric function of a graph, a symmetric function analog of the classical chromatic polynomial of a graph. The Stanley–Stembridge e-positivity conjecture is a long-standing conjecture that states that the chromatic symmetric function of a certain class of graphs, called incomparability graphs of $(\mathbf{3}+\mathbf{1})$-free posets, has nonnegative coefficients when expanded in the elementary symmetric function basis. In 1996, Gasharov described Schur expansion of the chromatic symmetric function for this class of graphs in terms of P-tableau, a generalization of a standard Young tableau. An open problem is to find a bijective proof of this expansion for all $(\mathbf{3}+\mathbf{1})$-free posets.
In the first part of the dissertation, we consider the problem of finding a bijection that looks like the classical *RSK* algorithm. The RSK algorithm takes (generalized) permutations and produces a pair of tableaux. Our approach is to view proper colorings as generalized permutations. This allows us to construct an algorithm that takes proper colorings and produce a pair of tableaux for certain classes of posets (**3**-free posets and beastly posets). This then provides a combinatorial proof of Gasharov's Schur expansion of the chromatic symmetric function as well as the

Shareshian–Wachs Schur expansion of the chromatic quasisymmetric function for these classes of posets. We also consider proper set colorings as well as dual algorithms. In fact, we show that any *RSK*-like algorithm on permutations that preserves descents and inversions can be extended to a bijection on colorings using our general framework of viewing proper colorings as generalized permutations.
In the second part of the dissertation, we provide new proofs of results surrounding the Shareshian–Wachs chromatic quasisymmetric function. First, we provide an elementary proof of a recurrence relation of Harada and Precup on the e-coefficients of the chromatic quasisymmetric function. This proof uses only the Shareshian–Wachs Schur expansion as well as a combinatorial interpretation of the inverse Kostka numbers due to Eğecioğlu and Remmel. Second, we discuss Hwang's acyclic chromatic quasisymmetric function, a refinement of the Shareshian–Wachs version. We show how the results of symmetry and Schur positivity for natural unit interval orders, which were first proven using noncommutative symmetric function theory, can be derived by much simpler means. We do the same for e-positivity of **3**-free posets.

[Cel2023b] ______, *RSK for 3-free posets*, Séminaire Lotharingien de Combinatoire **89B** (2023), art. 89B.82, available at `https://www.mat.univie.ac.at/~slc/wpapers/FPSAC2023/82.html`. MR4659590 Zbl 1572.05256

Abstract: A long-standing open problem is to find an RSK-like correspondence between permutations and pairs of tableaux coming from Gasharov's decomposition of Stanley's chromatic symmetric functions into Schur functions. In this, we present such a correspondence RSK_p for incomparability graphs of **3**-free posets P that moreover preserve the descent and inversion statistics. We then extend RSK_p to bijections from proper colorings and multicolorings providing new combinatorial proofs for the Schur expansions of Gasharov for the chromatic symmetric function, of Shareshian–Wachs for the chromatic quasisymmetric function, and of Hwang for the multichromatic quasisymmetric function, and its refinement to equivalence classes of acyclic orientations in the case that P is **3**-free.

[ChoKimBNamSohn2021] Hyunsoo Cho, Byungchan Kim, Hayan Nam, and Jaebum Sohn, *A survey on t-core partitions*, Hardy-Ramanujan J. **44** (2021), 81–101, DOI 10.46298/hrj.2022.8928, available at `https://hrj.episciences.org/8928`. GS 6207421837478673682. Zbl 1479.11176

Lem. 2.4 implies the multiple-of-hook-length rule.

Abstract: t-core partitions have played important roles in the theory of partitions and related areas. In this survey, we briefly summarize interesting and important results on t-cores from classical results like how to obtain a generating function to recent results like simultaneous cores. Since there have been numerous studies on t-cores, it is infeasible to survey all the interesting results. Thus, we mainly focus on the roles of t-cores in number theoretic aspects of partition theory. This includes the modularity of t-core partition generating functions, the existence of t-core partitions, asymptotic formulas and arithmetic properties of t-core partitions, and combinatorial and number theoretic aspects of simultaneous core partitions. We also explain some applications of t-core partitions, which include relations between core partitions and self-conjugate core partitions, a t-core crank explaining Ramanujan's partition congruences, and relations with class numbers.

[ChoiNamSOh2019a] Seung-Il Choi, Sun-Young Nam, and Young-Tak Oh, *Shifted tableau switchings and shifted Littlewood–Richardson coefficients*, J. Korean Math. Soc. **4** (2019), 947–984, DOI 10.4134/JKMS.j180488, available at `https://jkms.kms.or.kr/journal/view.html?doi=10.4134/JKMS.j180488`. GS 2106270081278871094.1423.05193

We provide two shifted analogues of the tableau switching process due to Benkart, Sottile, and Stroomer; the shifted tableau switching process and the modified shifted tableau switching process. They are performed by applying a sequence of elementary transformations called *switches* and shares many nice properties with the tableau switching process. For instance, the maps induced from these algorithms are involutive and behave very nicely with respect to the lattice property. We also introduce shifted generalized evacuation which exactly agrees with the shifted J-operation due to Worley when applied to shifted

Young tableaux of normal shape. Finally, as an application, we give combinatorial interpretations of Schur P-and Schur Q-function related identities.

[Con2009] Brian Conrad, *Math 248A. Completion of algebraic closure*, 2009, available at `http://virtualmath1.stanford.edu/~conrad/248APage/handouts/algclosurecomp.pdf`.

Theorem 1.1. The completion C_K of [the algebraic closure of] K is algebraically closed.

[ConwBurGoodStrauss2008] John H. Conway, Heidi Burgiel, and Chaim Goodman-Strauss. *The symmetries of things.* A. K. Peters, Wellesley, Mass., U.S., 2008. GS 13483869022321107874.

Abstract: Start with a single shape. Repeat it in some way—translation, reflection over a line, rotation around a point—and you have created symmetry.
Symmetry is a fundamental phenomenon in art, science, and nature that has been captured, described, and analyzed using mathematical concepts for a long time. Inspired by the geometric intuition of Bill Thurston and empowered by his own analytical skills, John Conway, with his coauthors, has developed a comprehensive mathematical theory of symmetry that allows the description and classification of symmetries in numerous geometric environments.
This richly and compellingly illustrated book addresses the phenomenological, analytical, and mathematical aspects of symmetry on three levels that build on one another and will speak to interested lay people, artists, working mathematicians, and researchers.

[ConwBurGoodStrauss2025] ———. *The symmetries of things.* A. K. Peters, Wellesley, Mass., U.S., 2025. GS 1879425585081746299.

Abstract: The Magic Theorem: a Greatly-Expanded, Much-Abridged Edition of The Symmetries of Things presents a wonderfully unique re-imagining of the classic book, The Symmetries of Things. Begun as a standard second edition by the original author team, it changed in scope following the passing of John Conway. This version of the book fulfills the original vision for the project: an elementary introduction to the orbifold signature notation and the theory behind it.

The Magic Theorem features all the material contained in Part I of The Symmetries of Things, now redesigned and even more lavishly illustrated, along with new and engaging material suitable for a novice audience. This new book includes hands-on symmetry activities for the home or classroom and an online repository of teaching materials avaialble at themagictheorem.com

[ConwSmith2003] John H. Conway and Derek A. Smith. *On quaternions and octonions: their geometry, arithmetic, and symmetry.* A. K. Peters, Wellesley, Mass., U.S., 2003. GS 11043206698192365601.

Abstract: This book investigates the geometry of quaternion and octonion algebras. Following a comprehensive historical introduction, the book illuminates the special properties of 3- and 4-dimensional Euclidean spaces using quaternions, leading to enumerations of the corresponding finite groups of symmetries. The second half of the book discusses the less familiar octonion algebra, concentrating on its remarkable "triality symmetry" after an appropriate study of Moufang loops. The authors also describe the arithmetics of the quaternions and octonions. The book concludes with a new theory of octonion factorization. Topics covered include the geometry of complex numbers, quaternions and 3-dimensional groups, quaternions and 4-dimensional groups, Hurwitz integral quaternions, composition algebras, Moufang loops, octonions and 8-dimensional geometry, integral octonions, and the octonion projective plane.

[Cox1973] H. S. M. Coxeter. *Regular Polytopes.* Dover, New York, 3rd ed., 1973, ISBN 978-0486614809. GS 11551642751560420801.

Polytopes are geometrical figures bounded by portions of lines, planes, or hyperplanes. In plane (two dimensional) geometry, they are known as polygons and comprise such figures as triangles, squares, pentagons, etc. In solid (three dimensional) geometry they are known as polyhedra and include such figures as tetrahedra (a type of pyramid), cubes, icosahedra, and many more; the possibilities, in fact, are infinite! H. S. M. Coxeter's book is the foremost book available on regular polyhedra, incorporating not only the ancient Greek work on the subject, but also the vast amount of information that has been accumulated on them since, especially in the last hundred years. The author, professor of Mathematics, University of Toronto, has contributed much valuable work himself on polytopes and is a well-known authority on them.

[CrawDil1973] J. P. Crawley and R. P. Dilworth. *Algebraic Theory of Lattices*. Prentice-Hall, Englewood Cliffs, N. J., 1973. available at `https://archive.org/details/algebraictheoryo0000craw`.

Abstract: Our purpose in this book is two-fold: first, to illustrate the depth and beauty of lattice theory by systematically developing a body of results at the *heart* of the subject including a representative sample if its most profound results; and second, as lattice theoretic techniques are useful in many branches of mathematics, to broadly illustrate the more important tools and techniques of lattice theory. A glance at the contents will reveal that this book is not encyclopedic. For example, it includes none of the theory of orthomodular lattices, pseudo-complemented lattices, multiplicative lattices, partially ordered systems such as lattice-ordered groups or rings, or lattices of continuous functions. Such topics as Boolean algebras, combinatorial geometries, and continuous geometries are treated very incompletely. Generally, our focus is on lattice structure theory, and we have tended to include those results that give insight into how lattices are put together and how they behave under certain assumptions. Also, the reader will note the absence of "applications;" we have involved other branches of mathematics only insofar as they give lattice theoretical insights.

[Dan1978] Seymour A. Danberg, *Evaluating queries concurrently in a shared database system*, Ph.D. thesis, Massachusetts Inst. of Tech., 1978, `https://hdl.handle.net/1721.1/153354`.

Includes Appendix D, "The Effect of Leisure Suits on Penguin Communities".

Abstract: In most shared database systems, when several database queries are to be evaluated concurrently, each query is processed as independently as possible. Interaction between queries only occurs when a situation arises which can potentially cause the data in the database to enter a bad state.

A database system need not constrain itself to follow this restriction. By evaluating concurrent queries in cooperation with each other as much as possible, a significant improvement in performance can be realized.
To this end, the concept of using a retrieved datum to solve more than just one query running concurrently is presented and explained. Its advantages and restrictions are investigated. An implementation (based on this concept) is described. Problems relating to the implementation are discussed and solutions presented. Possible optimizations for the implementation are considered.
An evaluation of the concept and a comparison to more traditional systems is made. It is found that, under some circumstances, the alternative approach is preferable to a more traditional architecture.

[DavPriest1990] Brian A. Davey and Hilary Ann Priestley. *Introduction to Lattices and Order*. Cambridge Math. Textbooks. Cambridge Univ. Press, Cambridge, UK, 1st ed., 1990. GS 17466900644257297586.

Abstract: This is the first textbook devoted to ordered sets and lattices and to their contemporary applications. It acknowledges the increasingly major role order theory is playing o the mathematical stage and is aimed at students of mathematics and at professionals in adjacent areas, including logic, discrete mathematics and computer science.

[Day1981] Alan Day, *In search of a Pappian lattice identity*, Can. Math. Bull. **24** (1981), 187–198, DOI 10.4153/CMB-1981-030-0, available at `https://www.cambridge.org/core/services/aop-cambridge-core/content/view/4FDC4136B6E81E491126C500A5666616/S0008439500063323a.pdf/in-search-of-a-pappian-lattice-identity.pdf`. `https://www.cambridge.org/core/journals/canadian-mathematical-bulletin/article/in-search-of-a-pappian-lattice-identity/4FDC4136B6E81E491126C500A5666616` GS 13192605395332536270. Zbl 0457.06005

Abstract: In [Jóns1954a] and subsequent papers, Jónsson (et al) developed a lattice identity which reflects precisely Desargues Law in projective geometry in that a projective geometry satisfies Desargues Law if and only if the geometry, qua lattice, satisfies this identity. This identity, appropriately called the Arguesian law, has become exceedingly important in recent investigations in the variety of modular lattices. In this note, we supply two possible lattice identities for the Pappus' Law of projective geometry.

[Day1982] ———, *Geometrical applications in modular lattices*, Universal algebra and lattice theory (Puebla, Mexico, 1982) (Ralph S. Freese and O. C. Garcia, eds.), Lecture Notes in Math. (LNM), vol. 1004, Springer-Verlag, Berlin and New York, 2006, 1982, DOI 10.1007/BFb0063433, available at `https://link.springer.com/chapter/10.1007/BFb0063433`. GS 14570042586739298811. Zbl 0516.06008

Abstract: [This paper] attempts to survey recent important results in modular lattices, due mainly to Freese, Herrmann, and Huhn, that have a strong geometric content in their ideas and proofs. These results (and others) represent a beautiful amalgamation of the classical results of Birkhoff and von Neumann with the newer disciplines (also due in part to Birkhoff) of universal algebra and model theory. Because the roots of the essential ideas lie in geometry, or perhaps more importantly in the lattice interpretation of projective geometry and the coordinatization thereof, we have attempted to present here a short (in fact too short) introductory course in these basic ideas.

[DayFrees1990] Alan Day and Ralph S. Freese, *The Role of Gluing Constructions in Modular Lattice Theory*, The Dilworth theorems: Selected papers of Robert P. Dilworth (Kenneth P. Bogard, Ralph S. Freese, and Joseph P. S. Kung, eds.), Contemporary Mathematicians, Springer, New York, 1990, pp. 251–260. GS 17122291477764575874.

Abstract: [R. P. Dilworth's and Marshall Hall's 1944 paper [HallDil1944]] used a construction that has become known as Hall-Dilworth gluing, but is now being called Dilworth gluing since it actually originated in an earlier paper of Dilworth, see below. With this construction Dilworth and Hall produced three examples of modular lattices, none of which can be embedded into a complemented modular lattice. Although other papers of Dilworth (and also Hall) contain deeper results, this paper has proved extremely important in the subsequent development of modular lattice theory. The examples themselves have proved useful in refuting various conjectures. The gluing technique used in constructing these lattices has turned out to be useful in settling some of the deeper questions of modular lattice theory. This gluing technique was the origin of more general gluing, which in turn has proved to be especially fruitful in solving some of the most stubborn problems of modular lattice theory.

[DayHerr1988] Alan Day and Christian Herrmann, *Gluings of modular lattices*, Order **5** (1988), 85–101, DOI 10.1007/BF00143900, available at `https://link.springer.com/article/10.1007/BF00143900`. GS 4405123069571633945. Zbl 0669.06007

See also the corrigendum in [Herr2006].

Abstract: The notions of gluing, tolerance relations, and Mal'cev products of varieties have been used by various authors to investigate varieties of lattices. In this paper the authors introduce a general framework for all these concepts and apply it to varieties of modular and Arguesian lattices.

[DayJóns1985] Alan Day and Bjarni Jónsson, *A structural characterization of non-Arguesian lattices*, Order **2** (1985), 335–350, DOI 10.1007/BF00367423, available at `https://link.springer.com/article/10.1007/BF00367423`. GS 14446929048083693725.

Note: References 5 is for [HallDil1944] but with the incorrect title *The embedding theorem for modular lattices*.

Abstract: This is the fist of a planned series of papers on the structure of non-Arguesian modular lattices. Apart from the (subspace lattices of) non-Arguesian projective planes, the best known examples of such lattices are obtained via the Hal-Dilworth construction by 'badly' gluing together two projective planes of the same order. Our principal result shows that every non-Arguesian modular lattice L retains some of the flavor of these examples: There exist in the ideal lattice of L 20 intervals, not necessarily distinct, that form non-degenerate projective [planes], and 10 points and 10 lines in these planes that constitute in a natural sense a 'classical' non-Arguesian configuration.

[DayPick1984] Alan Day and Douglas A. Pickering, *A note on the Arguesian lattice identity*, Conference on Universal Algebra (Visegrád, Hungary, May 1982), 1984, pp. 303–305, available at https://core.ac.uk/download/pdf/35157988.pdf#page=309. GS 15359437393910443487.

In a series of (sometimes joint) papers, Jónsson (et al.) introduced the Arguesian lattice identity, and proved it was equivalent to (the lattice theoretical formulation of) Desargues' implication. In this note we present two new equivalent formulations of the Arguesian law together with a simplified, complete proof of the aforementioned earlier results.

[Ded1900] Richard Dedekind, *Ueber die drei Moduln erzeugte Dualgruppe [The lattice generated by three subgroups]*, Math. Ann. **53** (1900), 371–403, DOI 10.1007/BF01448979, available at https://sci-hub.ru/https://doi.org/10.1007/BF01448979. https://link.springer.com/article/10.1007/BF01448979 GS 1843800222145966254.

Abstract: In tier vierten Auflage von Dirichlet's Vorlesungen über Zahlentheorie (die im Folgenden mit D. citirt werden soll) habe ich gelegentlich (in den Anmerkungen auf S. 499, 510, 556) die Dualgruppe erwähnt, die aus drei beliebigen Moduln durch fortgesetzte Bildung der gemeinsamen grössten Theiler und kleinsten Vielfachen erzeugt wird und im Allgemeinen aus 28 verschiedenen Moduln besteht. Da die Gesetze dieser Grupppe sich auf ganz andere Gebiete übertragen lassen und oft eine nützliche Hülfe gewähren, so sollen dieselben im Folgenden dargestellt werden; daran schliessen sich verschiedene Untersuchungen über allemeinere Dualgruppen.
Translation: In the fourth edition of Dirichlet's lectures on number theory (which will be quoted below as D.) I have occasionally (in the notes on pp. 499, 510, 556) mentioned the lattice which is generated by any three modules through continued formation of the common largest divisor and smallest multiple and in generally consists of 28 different modules. Since the laws of this lattice can be transferred to completely different areas and often prove useful, they will be presented below. This is followed by various studies regarding lattices.

[Def2024] Colin Defant, *Bender–Knuth Billiards in Coxeter Groups*, 2024-04-02, Brandeis Univ., Waltham, Mass., US, Brandeis Combinatorics Seminar

Based on *Bender–Knuth Billiards in Coxeter Groups* by Grant Barkley, Colin Defant, Eliot Hodges, Noah Kravitz, and Mitchell Lee.

[DeMan1999] Roald de Man, *The Generating Function for the Number of Roots of a Coxeter Group*, J. Symb. Comput. **27** (1999), 535–541, DOI 10.1006/jsco.1999.0280, available at `https://www.sciencedirect.com/science/article/pii/S0747717199902808`. GS 11217076539584158113. MR1701093 Zbl 0952.20030

Abstract: Using elementary roots and finite automata, we show that the generating function counting the depths of the roots of a Coxeter group of finite rank is rational.

[Dil1941] R. P. Dilworth, *The arithmetical theory of Birkhoff lattices*, The Dilworth theorems: Selected papers of Robert P. Dilworth (Kenneth P. Bogard, Ralph S. Freese, and Joseph P. S. Kung, eds.), Contemporary Mathematicians, Springer, New York, 1990, 1941, pp. 286–289. GS 9794479287713468342. Zbl 0025.10203

Abstract: In the development of lattice theory considerable work has been devoted to the study of the arithmetical properties of modular and distributive lattices. Indeed most of the decomposition theorems of abstract algebra have been extended to these more general domains. Nevertheless, there are lattices with very simple arithmetical properties which come under neither of these classifications. For example, the lattices with unique irreducible decompositions, which were studied by the author in a previous paper [3] satisfy the Birkhoff condition which is even less restrictive than the modular axiom. Furthermore, there are important algebraic systems which give rise to nonmodular, Birkhoff lattices. Thus, since every exchange lattice (Mac Lane [4]) is a Birkhoff lattice, the systems which satisfy Mac Lane's exchange axiom form lattices of the type in question. In this paper we shall study the arithmetical structure of general Birkhoff lattices and in particular determine necessary and sufficient conditions that certain important arithmetical properties hold.

In §§2–4 we characterize the irreducible decompositions in terms of the structure of the lattice and apply the results to determine necessary and sufficient conditions that the number of irreducible components be unique for each element of the lattice. The main result is the following:
Let $\mathfrak{S}$ be a Birkhoff lattice in which every quotient lattice is Archimedean. Then the number of irreducible components is unique for each element a of $\mathfrak{S}$ if and only if the sublattice generated by the elements covering a is a dense, modular sublattice of $\mathfrak{S}$.
§5 contains some methods for constructing Birkhoff lattices in which given arithmetical conditions hold. In §6 we treat the problem of determining the conditions that a set of irreducible components of an element must satisfy in order that it may be extended into a reduced representation. This problem is given a particularly simple solution in the case of a Birkhoff lattice in which the number of components is unique.

[Dil1961] ______, *Structure and decomposition theory of lattices*, Lattice Theory, Proc. of Symposia in Pure Math., vol. 2, AMS, Providence, 1961, pp. 3–16. GS 12542724803910434729.

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[DilFrees1976] R. P. Dilworth and Ralph S. Freese, *Generators of lattice varieties*, Algebra Univers. **6** (1976), 263–267, DOI 10.1007/BF02485834, available at `https://www.researchgate.net/profile/Ralph-Freese/publication/225381913_Generators_of_lattice_varieties/links/00b7d5361a3b54d2ea000000/Generators-of-lattice-varieties.pdf`. GS 4970906173934914648. Zbl 0381.06008

Abstract: Although it is well known that the variety of all lattices is generated by the subclass of finite lattices, there are lattice varieties which are not generated by their finite members. In fact, there are modular varieties which are not even generated by their finite dimensional members [3]. At the present time it is not known if the variety of all modular lattices is generated by its finite or even its finite dimensional members. This raises the question: Do there exist generators for lattice varieties which satisfy some kind of finiteness conditions? In this note we will be concerned with generators satisfying atomicity conditions. Since

any lattice variety is generated by its subdirectly irreducible members, we shall be particularly interested in generators which are also subdirectly irreducible. The main results are the following. The notation and terminology for this paper is taken from [CrawDil1973].

[Dob2026] Alexander Dobner, *An RSK correspondence for cylindric tableaux* (2026), arXiv 2603.09119, primary class math.CO, DOI 10.48550/arXiv.2603.09119, available at `https://arxiv.org/abs/2603.09119`. GS 12473567550904270012

Abstract: This paper establishes an analogue of the Robinson–Schensted correspondence for cylindric tableaux. In particular, for any pair of positive integers (d, L), we construct a bijection between permutations that avoid the patterns $d \cdots 1(d+1)$ and $1 \cdots (L+1)$ and pairs of (d, L)-cylindric standard Young tableaux with a common shape. This arises as a special case of a Knuth-type generalization involving cylindric semistandard tableaux and a further generalization involving oscillating tableaux. Using these results, we construct several other bijections and derive enumerative consequences involving cylindric tableaux and pattern-avoiding permutations. For example, we give asymptotics for the number of permutations in Sn that avoid the patterns $d \cdots 1(d+1)$ and $1 \cdots (L+1)$ as $n \to \infty$.

[Doš2003] Kosta Došen, *A note on the set-theoretic representation of arbitrary lattices* (2003), arXiv 0303005, primary class math.LO, DOI 10.48550/arXiv.math/0303005, available at `https://arxiv.org/abs/math/0303005`. GS 9435180622006250226

Abstract: Every lattice is isomorphic to a lattice whose elements are sets of sets, and whose operations are intersection and an operation extending the union of two sets of sets A and B by the set of all sets in which the intersection of an element of A and of an element of B is included. This representation spells out precisely Birkhoff's and Frinks's representation of arbitrary lattices, which is related to Stone's set-theoretic representation of distributive lattices.

[DulSag1995] Serge Dulucq and Bruce E. Sagan, *La correspondance de Robinson-Schensted pour les tableaux oscillants gauches [The Robinson-Schensted correspondence for skew oscillating tableaux]*, Discrete Math. **139** (1995), 129–142, DOI 10.1016/0012-365X(94)00129-7, available at https://www.sciencedirect.com/science/article/pii/0012365X94001297. GS 12645265544659743202.

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[EhrFabFajtMyc1973] A. Ehrenfeucht, V. Faber, S. Fajtlowicz, and J. Mycielski, *Representation of finite lattices as partition lattices on finite sets*, Univ. of Houston lattice theory conference (Houston, 1973), 1973, pp. 17–35, available at https://www.math.uh.edu/~hjm/1973_Lattice/p00017-p00035.pdf. GS 2737558652062410443.

Gives a number of techniques for constructing representations of finite lattices as partition lattices on finite sets and some lower bounds on the size of the partitioned sets.

[El2025] Sergi Elizalde, *Cylindric growth diagrams, walks in simplices, and exclusion processes* (2025), arXiv 2507.01097, primary class math.CO, DOI 10.48550/arXiv.2507.01097, available at https://arxiv.org/abs/2507.01097. GS 11905606782830664430

Abstract: We establish bijections between three classes of combinatorial objects that have been studied in very different contexts: lattice walks in simplicial regions as introduced by Mortimer–Prellberg, standard cylindric tableaux as introduced by Gessel–Krattenthaler and Postnikov, and sequences of states in the totally asymmetric simple exclusion process. This perspective allows us to translate symmetries from one setting into another, revealing unexpected properties of these objects.
Specifically, we show that a recent bijection of Courtiel, Elvey Price and Marcovici between certain simplicial walks with forward and backward steps is equivalent to a cylindric analogue of the Robinson–Schensted correspondence. Originally defined by Neyman by iterating an insertion operation, we provide an alternative description of this correspondence by

introducing a cylindric version of Fomin's growth diagrams. This natural description elucidates the symmetry of the correspondence, and it allows us to interpret the above walks as oscillating cylindric tableaux.

[EnMathArg] *Encyclopedia of Mathematics: Arguesian lattice*, https://encyclopediaofmath.org/wiki/Arguesian_lattice. Accessed December 3, 2023.

Desarguesian lattice. A lattice in which the Arguesian law is valid, i.e. for all a_i, b_i, $(a_0 + b_0)(a_1 + b_1)(a_2 + b_2) \leq a_0(a_1 + c) + b_0(b_1 + c)$, $c = c_0(c_1 + c_2)$, $c_i = (a_j + a_k)(b_j + b_k)$ for any permutation i, j, k.

[EnMathDes] *Encyclopedia of Mathematics: Desargues assumption*, https://encyclopediaofmath.org/wiki/Desargues_assumption. Accessed December 3, 2023.

Desargues theorem. If the corresponding sides of two triangles ABC and $A'B'C'$ intersect at points P, Q, R on the same straight line, then the straight lines which connect the corresponding vertices intersect at one point.

[EnMathMod] *Encyclopedia of Mathematics: Modular lattice*, https://encyclopediaofmath.org/wiki/Modular_lattice. Accessed December 13, 2023.

Dedekind lattice A lattice in which the modular law is valid, i.e. if $a \leq c$, then $(a + b)c = a + bc$ for any b.

[EnMathPapp] *Encyclopedia of Mathematics: Pappus axiom*, https://encyclopediaofmath.org/wiki/Pappus_axiom. Accessed October 22, 2024.

[ErdosProb1196] *Erdös problem 1196, with discussion*, https://www.erdosproblems.com/forum/thread/1196?order=oldest. Accessed April 27, 2026.

A problem concerning the bounds of $\sum_{a \in A} (a \log a)^{-1}$ where A ranges over prmitive sets of positive integers (no element divides any other). Solved by Liam Price prompting ChatGPT on April 13. By April 16, Math, Inc. had used its Gauss tool to formalize the proof and used Lean to verify the formalized proof. The prompt to ChatGPT from Price was:

don't search the internet.
This is a test to see how well you can craft non-trivial, novel and creative proofs given a "number theory and primitive sets" math problem. Provide a full unconditional proof or disproof of the problem.

Problem:
"Is it true that, for any x, if $A \subset [x, \infty)$ is a primitive set of integers (so that no distinct elements of A divide each other) then $\sum_{a \in A} \frac{1}{a \log a} < 1 + o(1)$, where the $o(1)$ term $\to 0$ as $x \to \infty$?"

information you may or may not need to help with the above problem
"It is proved that $\sum_{a \in A} \frac{1}{a \log a} < e^{\gamma} \frac{\pi}{4} + o(1) \approx 1.399 + o(1)$."
"It is proved that if A is the set of all integers with exactly k prime factors (so that $A \subset [2^k, \infty)$ and A is a primitive set) then $\sum_{a \in A} \frac{1}{a \log a} \geq 1 + O(k^{-1/2+o(1)})$, "
"It is proved that $\sum_{a \in A} \frac{1}{a \log a} = 1 - (c + o(1))k^2 2^{-k}$ where $c \approx 0.0656$ is an explicit constant."

[Eriks1996] Kimmo Eriksson, *Strong convergence and the polygon property of 1-player games*, Discrete Math. **153** (1996), 1005–122, available at `https://www.sciencedirect.com/journal/discrete-mathematics/vol/153/issue/1`. GS 8925994845081806896. Zbl 0849.90149

Refers to removing k-rim hooks as "the k-snake game".

Abstract: A one-player game is *strongly convergent* if the length of the game and the terminal position depends only on the starting position. We present several examples (the chips game, bubble-sort, extendable shelling, the chromatic polynomial, the k-core, jeu-de-tacquin) of how strong convergence can be shown using a technique of finding a 'polygon', a local convergence property.

[Et2011] Pavel Etingof. *Introduction to representation theory*, 2011. GS 17839036026498619658.

Abstract: The goal of this book is to give a "holistic" introduction to rep- resentation theory, presenting it as a unified subject which studies representations of associative algebras and treating the representation theories of groups, Lie algebras, and quivers as special cases. It is designed as a textbook for advanced undergraduate and beginning graduate students and should be accessible to students with a strong background in linear algebra and a basic knowledge of abstract algebra. Theoretical material in this book is supplemented by many problems and exercises which touch upon a lot of additional topics; the more difficult exercises are provided with hints.

[FagLehr1957] R. E. Fagen and Tom A. Lehrer, *The gambler's ruin with soft hearted adversary*, 1957, available at `https://www.nsa.gov/portals/75/documents/news-features/declassified-documents/gamblers-ruin.pdf`.

Abstract: The random walk on a one dimensional lattice with restraining barrier at the origin and absorbing barrier at N is discussed for the case where only steps of r units to the right and ℓ units to the left are allowed. Asymptotic expressions for the mean and variance of the duration of such a walk are derived, and, in some cases, the limiting distribution is found.

[FagHerr1981] Ulrich Faigle and Christian Herrmann, *Projective Geometry on Partially Ordered Sets*, Trans. Am. Math. Soc. **266** (1981), 319–332, DOI 10.1090/S0002-9947-1981-0613799-9, available at `https://www.ams.org/journals/tran/1981-266-01/S0002-9947-1981-0613799-9/`. GS 15804038105968954166. MR613799 Zbl 0466.51001

Abstract: A set of axioms is presented for a projective geometry as an incidence structure on partially ordered sets of "points" and "lines". The axioms reduce to the axioms of classical projective geometry in the case where the points and lines are unordered. It is shown that the lattice of linear subsets of a projective geometry is modular and that every modular lattice of finite length is isomorphic to the lattice of linear subsets of some finite-dimensional projective geometry. Primary geometries are introduced as the incidence-geometric counterpart of primary lattices. Thus the theory of finite-dimensional projective geometries includes the theory of finite-dimensional projective Hjelmslev-spaces of level n as a special case. Finally, projective geometries are characterized by incidence properties of points and hyperplanes.

[Far2000] Jonathan David Farley, *Quasi-Differential Posets and Cover Functions of Distributive Lattices; I. A Conjecture of Stanley*, J. Comb. Theory **Ser. A 90** (2000), 123–147, DOI 10.1006/jcta.1999.302, available at `http://latticetheory.net/media/pdf/Stanley1975.pdf`. GS 2636441975265066580.

A distributive lattice L with 0 is *finitary* if every interval is finite. A function $f : \mathbb{N}_0 \to \mathbb{N}_0$ is a *cover function* for L if every element with n lower covers has $f(n)$ upper covers. In this paper, all finitary distributive lattices with non-decreasing cover functions are characterized. A 1975 conjecture of Richard P. Stanley is thereby settled.

[Far2003] ———, *Quasi-Differential Posets and Cover Functions of Distributive Lattices; II: A Problem in Stanley's Enumerative Combinatorics*, Graphs Combin. **19** (2003), 475–491, DOI 10.1007/s00373-003-0525-0, available at `http://latticetheory.net/media/pdf/stanley_quasi_II.pdf`. GS 18144920561279469427.

A distributive lattice L with 0 is *finitary* if every interval is finite. A function $f : \mathbb{N}_0 \to \mathbb{N}_0$ is a *cover function* for L if every element with n lower covers has $f(n)$ upper covers. In this paper, all finitary distributive lattices with cover functions are characterized. A problem in Stanley's *Enumerative Combinatorics* is thus solved.

[FarKim2004] Jonathan David Farley and Sungsoon Kim, *The Automorphism Group of the Fibonacci Poset: A "Not Too Difficult" Problem of Stanley from 1988*, J. Algebr. Comb. **19** (2004), 197–204, available at `http://ftp.gwdg.de/pub/EMIS/journals/JACO/Volume19_2/r81p4r9620432166.fulltext.pdf`. GS 8897085809314013116.

Abstract: Stanley asks: What is the automorphism group $\mathrm{Aut}(Z(r))$ of the Fibonacci poset $Z(r)$? We solve the problem by explicitly determining all of the automorphisms of $Z(r)$.

[Feit1982-part] Walter Feit. *The Representation Theory of Finite Groups*. North-Holland, Amsterdam, 1982.

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[Fom1988] Sergey V. Fomin, *Generalized Robinson-Schensted-Knuth correspondence*, J. Sov. Math. **41** (1988), 979–991, DOI 10.1007/BF01247093, available at `https://link.springer.com/article/10.1007/BF01247093`. GS 2171217720861623851.

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[Fom1992] ——, *Dual graphs and Schensted correspondences*, Proceedings of the 4th International Conference on Formal Power Series and Algebraic Combinatorics (University of Montréal, 1992), available at `https://fpsac-archive.github.io/FPSAC92/ARTICLES/18Fomin.pdf`. GS 2860110408161983339.

Abstract: A graph is said to be graded if its vertices are divided into *levels* numbered by integers, so that the endpoints of any edge lie on consecutive levels.
The following three types of problems are considered:
(1) path counting in graded graphs, and related combinatorial identities;
(2) bijective proofs of these identities;
(3) design and analysis of algorithms establishing corresponding bijections.
R.P.Stanley's [St88, St90] linear-algebraic approach to (1) is extended to cover a wide range of graded graphs. The main idea is to consider the pairs of graded graphs with the common set of vertices and common rank function. Such graphs are said to be *dual* if the associated linear operators satisfy a certain *commutation relation* (the "Heisenberg" one). The algebraic consequences of these relations are then interpreted as combinatorial identities. (This idea is also implicit in [St90].)

[Fom1994] ———, *Duality of Graded Graphs*, J. Algebr. Comb. **3** (1994), 357–404, DOI 10.1023/A:1022412010826, available at https://link.springer.com/content/pdf/10.1023/A:1022412010826.pdf. GS 3401296478290474488. MR1293822 Zbl 0810.05005

Topics: growth diagram. General introduction to growth diagrams.

[Fom1995a] ———, *Schensted Algorithms for Dual Graded Graphs*, J. Algebr. Comb. **4** (1995), 5–45, DOI 10.1023/A:1022404807578, available at https://link.springer.com/content/pdf/10.1023/A:1022404807578.pdf. GS 9003315695694762360. MR1314558 Zbl 0817.05077

U, *S*. Topics: growth diagram, insertion algorithm. Applies growth diagrams to various insertion algorithms.

[Fom1995b] ———, *Schur Operators and Knuth Correspondences*, J. Comb. Theory **Ser. A 72** (1995), 277–292, DOI 10.1016/0097-3165(95)90065-9, available at https://www.sciencedirect.com/science/article/pii/0097316595900659. GS 820160330774353013. Zbl 0839.05093

Abstract: The paper presents a general combinatorial approach to the Schur functions and their modifications, respective generalized Cauchy identities, and bijective Knuth-type correspondences between matrices and pairs of tableaux. All of these appear whenever one has a pair of graphs with the same vertices such that the linear operators associated with these graphs satisfy a certain type of commutation relations. A parallel implementation of insertion-type algorithms is suggested that generalizes the sequential constructions of Sagan and Stanley [13, 14] and the earlier bijections of Knuth, Worley-Sagan, and Haiman. We use the linear-algebraic approach of [17, 2] and the algorithmic techniques of [3]. This paper is a revised version of [5].

[Fom1999] ———, *Knuth Equivalence, Jeu de Taquin, and the Littlewood–Richardson Rule*, Enumerative Combinatorics, Volume 2, Cambridge Studies in Advanced Mathematics, vol. 62, Cambridge University Press, Cambridge, 1999, 1990, pp. 413–439, available at https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=93339fb1678a0ac3f07c1875b0e94bf0924a5008. GS 267248130834265370.

U. Topics: Knuth equivalence, Greene invariant, jeu de taquin, growth diagram.

[Fom2024] ———, *Incidences and tilings*, 2024-06-03, Harvard Univ.,Cambridge, Mass., US, available at https://drive.google.com/file/d/19G6i7ic3fg7M7zUOB0yiPoAAf828ssum/view. video at https://www.youtube.com/watch?v=5fGEk9c5qlU

Presentation of [FomPyl2023].

[FomPyl2023] Sergey V. Fomin and Pavlo Pylyavskyy, *Incidences and tilings* (2023), arXiv https://doi.org/10.48550/arXiv.2305.07728, primary class 2305.07728, DOI 10.48550/arXiv.2305.07728, available at https://arxiv.org/abs/2305.07728. GS 12033594455168061630

One of those rare works which will be a foundation stone of its field. All future books on incidence theorems in projective geometry will include this work.

Abstract: We show that various classical theorems of real/complex linear incidence geometry, such as the theorems of Pappus, Desargues, Möbius, and so on, can be interpreted as special cases of a single "master theorem" that involves an arbitrary tiling of a closed oriented surface by quadrilateral tiles. This yields a general mechanism for producing new incidence theorems and generalizing the known ones.

[FomStant1997] Sergey V. Fomin and Dennis Stanton, *Rim hook lattices*, Algebra i Analiz **9** (1997), 140–150, available at https://www.mathnet.ru/php/archive.phtml?wshow=paper&jrnid=aa&paperid=876&option_lang=eng. GS 14918964870864232397. Zbl 0897.05086

Abstract: We consider the partial order on partitions of integers defined by removal of rim hooks of a given length. Isomorphism between this poset and a product of Young's lattices leads to rim hook versions of Schensted correspondences. Similar results are given for shifted shapes.

[FrankUrsch2025] Elsa Selin Frankel and John Urschel, *On the Frobenius norm of the inverse of a non-negative matrix*, Linear Algebra Its Appl. **708** (2025), 193–203, DOI 10.1016/j.laa.2024.11.030, available at `https://www.sciencedirect.com/science/article/abs/pii/S0024379524004579`. GS 5504152742464665110. MR4837839 Zbl 1567.15011

Abstract: We prove a new lower bound for the Frobenius norm of the inverse of an non-negative matrix. This bound is only a modest improvement over previous results, but is sufficient for fully resolving a conjecture of Harwitz and Sloane, commonly referred to as the S-matrix conjecture, for all dimensions larger than a small constant.

[FrayneMorScottD1962] T. Frayne, A. C. Morel, and D. S. Scott, *Reduced direct products*, Fundam. Math. **51** (1962), 195–228, available at `https://eudml.org/doc/213693`. GS 12760828016353255704. Zbl 0108.00501

Abstract: The *reduced direct product* of a sequence of relational systems is a generalization of the standard algebraic notion of the (complete) direct product; in fact, a reduced product is a special type of homomorphic image of a corresponding direct product—thought it is not always a quotient of this direct product except in the case of algebraic systems. . . . This paper is divided into two sections together with an appendix. In Section 1 the basic algebraic (or relation-theoretic) definitions are given, and several theorems are presented of a general algebraic and set-theoretic nature. In Section 2 the notions from the theory of models are recalled and the applications of reduced products to this theory are indicated. . . . The appendix introduces some topological notions, and it is shown how some of the results of Section 2 could have been derived in an abstract setting.

[Frees1984] Ralph S. Freese, *On Jónsson's theorem*, Algebra Univers. **18** (1984), 70–76, DOI 10.1007/BF01182248, available at https://www.researchgate.net/publication/265939915_On_Jonsson's_Theorem. GS 17075821032965119689. Zbl 0545.08012

Abstract: This paper surveys some results which are closely related to Jónsson's famous theorem. The theorem states that every subdirectly irreducible algebra in the variety generated by a class $\mathcal{K}$ of similar algebras is in $HSP_u(\mathcal{K})$ provided $V(\mathcal{K})$ is distributive (i.e. has distributive congruence lattices) [Jóns1967]. Of course $P_u(\mathcal{K})$ stands for ultraproducts of members of $\mathcal{K}$. This theorem is the main impetus for the resurgence in the study of varieties of algebras, in particular lattice varieties. The theorem is used so often that one often forgets to acknowledge it. There have been several important applications of Jónsson's theorem. Kirby Baker has shown that a finite algebra in a distributive variety has a finite basis for its equational theory [1]. Ralph McKenzie has given several applications to lattice varieties and lattice structure theory in [19].
The first section of this paper shows how Jónsson's theorem is applied to obtain an important but overlooked result of Christian Herrmann and Andras Huhn on the embeddability of modular lattices into complemented modular lattices. In the second section we give what now appears to be the correct generalization of Jńsson's theorem to modular varieties (=varieties of algebras with modular congruence lattices). This result is in terms of the "commutator" — a new binary operation on all congruence lattices of algebras in a modular variety. The definition and important facts about the commutator will be reviewed in §2.
If two finite, subdirectly irreducible algebras generate the same distributive variety, then Jńsson's theorem implies they are isomorphic. This is false for modular varieties. In §3 we investigate under what additional hypotheses it is true. For example, if one of the algebras is simple, it is true. The third section also investigates to what extent other consequences of Jónsson's theorem are true in modular varieties.

[Frees1994] ———, *Finitely based modular congruence varieties are distributive*, Algebra Univers. **32** (1994), 104–114, DOI 10.1007/BF01190818, available at https://math.hawaii.edu/~ralph/Preprints/modcv2.pdf.

https://link.springer.com/article/10.1007/BF01190818 GS 5796163421085917375.

Abstract: R. Dedekind introduced the modular law, a lattice equation true in most of the lattices associated with classical algebraic systems, see [4]. Although this law is one of the most important tools for working with these lattices, it does not fully describe the equational properties of these lattices. This was made clear in [8] where B. Jónsson and the author showed that if any modular congruence variety actually satisfies the (stronger) Arguesian law. (A *congruence variety* is the variety generated by all the congruence lattices of the members of a variety of algebras.) In this note we show that no finite set of lattice equations is strong enough to describe the equational properties of the lattices associated with classical algebraic systems in the following strong sense: *there is no modular, nondistributive congruence variety which has a finite basis for its equational theory.*

[Frees2015] ———, *Notes on congruence n-permutability and semidistributivity*, 2015, available at https://math.hawaii.edu/~ralph/Classes/619/n-perm.pdf. GS 10752015158753915816.

Abstract: In [1] T. Dent, K. Kearnes and Á. Szendrei define the *derivative*, Σ', of a set of equations Σ and show, for idempotent Σ, that Σ implies congruence modularity if Σ' is inconsistent ($\Sigma' \models x \approx y$). In this paper we investigate other types of derivatives that give similar results for congruence n-permutable for some n, and for congruence semidistributivity.

[FriedGrätzLaks1990] E. Fried, George Grätzer, and H. Lakser, *Projective geometries as cover-preserving sublattices*, Algebra Univers. **27** (1990), 270–278, DOI 10.1007/BF01182460, available at https://link.springer.com/article/10.1007/BF01182460. https://www.researchgate.net/publication/226195371_Projective_geometries_as_cover-preserving_sublattices GS 9260381081533728579. Zbl 0703.06005

Abstract: It is a well known result in the folklore of lattice theory that whenever M_3 (the five element modular nondistributive lattice) can be embedded into a finite modular lattice L, then M_3 also has a *cover-preserving* embedding into L, that is, an embedding Φ with the property that if a covers b in M_3 , then $\Phi(a)$

covers $\Phi(b)$ in L. Therefore, if L is a finite nondistributive modular lattice, then L contains M_3 as a cover-preserving sublattice.
We formalize this concept: a finite lattice K has the *cover-preserving embedding property*, abbreviated as CPEP, with respect to a variety $\mathbf{V}$ of lattices, if whenever K can be embedded into a finite lattice L in $\mathbf{V}$, then K has a cover-preserving embedding into L. In this note, we determine which finite projective geometries P satisfy the CPEP with respect to the variety $\mathbf{M}$ of modular lattices; from our point of view, a finite projective geometry is a finite complemented simple modular lattice.

[FriedGrätzSchmidt1993] E. Fried, George Grätzer, and Elegius Tamás Schmidt, *Multipasting of lattices*, Algebra Univers. **30** (1993), 214–261, DOI 10.1007/BF01196095, available at https://www.researchgate.net/profile/George-Graetzer/publication/226220396_Multipasting_of_lattices/links/00b49522494afaf61f000000/Multipasting-of-lattices.pdf. https://link.springer.com/article/10.1007/BF01196095 GS 6574279807542133939.

Abstract: In this paper we introduce a lattice construction, called multipasting, which is a common generalization of gluing, pasting, and S-glued sums. We give a Characterization Theorem which generalizes results for earlier constructions. Multipasting is too general to prove the analogues of many known results. Therefore, we investigate in some detail three special cases: strong multipasting, multipasting of convex sublattices, and multipasting with the Interpolation Property.

[Frink1946] Orrin Frink Jr., *Complemented modular lattices and projective spaces of infinite dimension*, Trans. Am. Math. Soc. **60** (1946), 451–467, DOI 10.2307/1990349, available at http://www.ams.org/journals/tran/1946-060-00/S0002-9947-1946-0018635-9/S0002-9947-1946-0018635-9.pdf. GS 15600796933218394803. Zbl 0060.05811

Abstract: The general projective spaces we consider are atomic; they contain points or atoms. Consequently the lattices associated with them are atomic. But complemented modular lattices need not be atomic, as is shown by the example of continuous geometries. Our procedure is to show that every complemented modular lattice determines a complete atomic

complemented modular lattice in which it is imbedded. This extension to an atomic lattice is accomplished by the use of maximal dual ideals. The resulting atomic lattice is then shown to be the direct union of irreducible projective spaces of a particular kind. The final characterization theorem we obtain states that every complemented modular lattice is the subdirect union of projective planes and irreducible projective coordinate spaces. A projective coordinate space is determined by an arbitrary cardinal number (its dimension) and an arbitrary division ring.

[Ful2012] William Fulton. *Young Tableaux With Applications to Representation Theory and Geometry.* Lond. Math. Soc. Student Texts, vol. 35. Cambridge Univ. Press, Cambridge, UK, 2012, ISBN 9780511626241. GS 9474957195503701443.

[FulHarr2004] William Fulton and Joe Harris. *Representation theory: a first course.* Graduate Texts in Mathematics, vol. 129. Springer, 2004, ISBN 978-0-387-97527-6. GS 1843019167906556077.

Abstract: The primary goal of these lectures is to introduce a beginner to the finite-dimensional representations of Lie groups and Lie algebras. Since this goal is shared by quite a few other books, we should explain in this Preface how our approach differs, although the potential reader can probably see this better by a quick browse through the book. ...
To put it another way, we intend this as a book for beginners to learn from and not as a reference. ...
This idea essentially determines the choice of material covered here. As simple as is the definition of representation theory given above, it fragments considerably when we try to get more specific. ...
By contrast, the present book focuses exactly on the simplest cases: representations of finite groups and Lie groups on finite-dimensional real and complex vector spaces. This is in some sense the common ground of the subject, the area that is the object of most of the interest in representation theory coming from outside. The intent of this book to serve nonspecialists likewise dictates to some degree our approach to the material we do cover. Probably the main feature ofour presentation is that we concentrate on examples, developing the general theory sparingly, and then mainly as a useful and unifying language to describe phenomena already encountered in concrete cases. By the same token, we for the most part introduce theoretical notions

when and where they are useful for analyzing concrete situations, postponing as long as possible those notions that are used mainly for proving general theorems.

[Gaetz2018] Christian Gaetz, *Dual graded graphs and Bratteli diagrams of towers of groups*, Electron. J. Comb. **26** (2019), P1.25, DOI 10.37236/7790, available at https://www.combinatorics.org/ojs/index.php/eljc/article/view/v26i1p25. GS 2020963036495729129.

Abstract: An r-dual tower of groups is a nested sequence of finite groups, like the symmetric groups, whose Bratteli diagram forms an r-dual graded graph. Miller and Reiner introduced a special case of these towers in order to study the Smith forms of the up and down maps in a differential poset. Agarwal and the author have also used these towers to compute critical groups of representations of groups appearing in the tower. In this paper the author proves that when $r = 1$ or r is prime, wreath products of a fixed group with the symmetric groups are the only r-dual tower of groups, and conjecture that this is the case for general values of r. This implies that these wreath products are the only groups for which one can define an analog of the Robinson-Schensted bijection in terms of a growth rule in a dual graded graph.

[GaetzPechPfannSS2023a] Christian Gaetz, Oliver Pechenik, Stephan Pfannerer, Jessica Striker, and Joshua P. Swanson, *Promotion permutations for tableaux* (2023), arXiv 2306.12506, primary class math.CO, DOI 10.48550/arXiv.2306.12506, available at https://arxiv.org/abs/2306.12506. GS 5127326674150559010

Abstract: In our companion paper, we develop a new SL_4-web basis. Basis elements are given by certain planar graphs and are constructed so that important algebraic operations can be performed diagrammatically. A guiding principle behind our construction is that the long cycle $(1\,2\ldots n) \in \mathfrak{S}_n$ should act by rotation of webs. Moreover, the bijection between webs and tableaux should intertwine rotation with the promotion action on tableaux.

In this paper, we develop necessary notions of promotion permutations and promotion matrices, which are new even for standard tableaux. To support inductive arguments in the companion paper, we must however work in the more general setting of fluctuating tableaux, which we introduce and which subsumes many classes of tableaux that have been previously studied, including (generalized) oscillating, vacillating, rational, alternating, and (semi)standard tableaux. Therefore, we also give here a full development of the basic combinatorics and representation theory of fluctuating tableaux.

[GaetzPechPfannSS2023b] ———, *Rotation-invariant web bases from hourglass plabic graphs* (2023), arXiv 2306.12501, primary class math.CO, DOI 10.48550/arXiv.2306.12501, available at `https://arxiv.org/abs/2306.12501`. GS 3841145929195145096

Abstract: Webs give a diagrammatic calculus for spaces of tensor invariants. We introduce hourglass plabic graphs as a new avatar of webs, and use these to give the first rotation-invariant $U_q(\mathfrak{sl}_4)$-web basis, a long-sought object. The characterization of our basis webs relies on the combinatorics of these new plabic graphs and associated configurations of a symmetrized six-vertex model. We give growth rules, based on a novel crystal-theoretic technique, for generating our basis webs from tableaux and we use skein relations to give an algorithm for expressing arbitrary webs in the basis. We also discuss how previously known rotation-invariant web bases can be unified in our framework of hourglass plabic graphs.

[GaetzPechPfannSS2024] ———, *Web bases in degree two from hourglass plabic graphs* (2024), arXiv 2402.13978, primary class math.CO, DOI 10.48550/arXiv.2402.13978, available at `https://arxiv.org/abs/2402.13978`. GS 16191674528180564564

Abstract: Webs give a diagrammatic calculus for spaces of $U_q(\mathfrak{sl}_r)$-tensor invariants, but intrinsic characterizations of web bases are only known in certain cases. Recently, we introduced hourglass plabic graphs to give the first such $U_q(\mathfrak{sl}_4)$-web bases. Separately, Fraser introduced a web basis for Plücker degree two representations of arbitrary $U_q(\mathfrak{sl}_r)$. Here, we show that Fraser's basis agrees with that predicted by the hourglass plabic graph framework and give an intrinsic characterization of the resulting webs. A further compelling feature with many applications is that our bases exhibit rotation-invariance. Together with the results of our earlier paper, this implies that hourglass plabic graphs give a uniform description of all known rotation-invariant $U_q(\mathfrak{sl}_r)$-web bases. Moreover, this provides a single combinatorial model simultaneously generalizing the Tamari lattice, the alternating sign matrix lattice, and the lattice of plane partitions. As a part of our argument, we develop properties of square faces in arbitrary hourglass plabic graphs, a key step in our program towards general $U_q(\mathfrak{sl}_r)$-web bases.

[GaoJ2025] Jiyang (Johnny) Gao, *Tilted Richardson Varieties*, 2025, Harvard Univ., Cambridge, Mass., US, Harvard Ph.D. thesis defense

Brief introduction to Schubert calculus and quantum Schubert calculus. Definition of tilted Richardson varieties and application to the tilted Deodhar decomposition and total positivity results.

[GarHaim1998] Adriano M. Garsia and Mark D. Haiman, *A random q, t-hook walk and a sum of Pieri coefficients*, J. Comb. Theory **Ser. A 82** (1998), 74–111, DOI 10.1006/jcta.1997.2842, available at `https://www.sciencedirect.com/science/article/pii/S0097316597928429https://math.berkeley.edu/~mhaiman/ftp/glenn-adriano/qt-walk.pdf`. GS 12404599432281597319.

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[GarMcLar1987] Adriano M. Garsia and Timothy J. McLarnan, *Robinson-Schensted Algorithms Obtained from Tableau Recursions* (1987), arXiv 2201.12908, primary class math.CO, DOI 10.48550/arXiv.2201.12908, available at `https://arxiv.org/abs/2201.12908`. GS 446442338380191517

U. Topics: Generic insertion algorithms. The part of [McLar1986] dealing with unshifted tableaux, but not including the proof regarding which insertion algorithms show inversion duality.

[Garv2001] Frank G. Garvan, *More cranks and t-cores*, Bull. Aust. Math. Soc. **63** (2001), 379–391, DOI 10.1017/S0004972700019481, available at https://www.cambridge.org/core/journals/bulletin-of-the-australian-mathematical-society/article/more-cranks-and-tcores/0062F3A42770256EAA42D38E2C5EC032. GS 7887583614261304163. Zbl 0985.11050

Abstract: In 1990, new statistics on partitions (called cranks) were found which combinatorially prove Ramanujan's congruences for the partition function modulo 5, 7, 11 and 25. The methods are extended to find cranks for Ramanujan's partition congruence modulo 49. A more explicit form of the crank is given for the modulo 25 congruence.

[GarvKimDStant1990] Frank G. Garvan, Dongsu Kim, and Dennis Stanton, *Cranks and t-cores*, Invent. Math. **101** (1990), 1–17, DOI 10.1007/BF01231493, available at https://link.springer.com/article/10.1007/BF01231493. GS 12861509396269905411. Zbl 0721.11039

Abstract: New statistics on partitions (called *cranks*) are defined which combinatorially prove Ramanujan's congruences for the partition function modulo 5, 7, 11, and 25. Explicit bijections are given for the equinumerous crank classes. The cranks are closely related to the t-core of a partition. Using q-series, some explicit formulas are given for the number of partitions which are t-cores. Some related questions for self-conjugate and distinct partitions are discussed.

[Geor2025] Bogdan Georgiev, Javier Gómez-Serrano, Terence Tao, and Adam Zsolt Wagner, *Mathematical exploration and discovery at scale* (2025), arXiv 2511.02864, primary class cs.NE, DOI 10.48550/arXiv.2511.02864, available at https://arxiv.org/abs/2511.02864

Abstract: AlphaEvolve [223] is a generic evolutionary coding agent that combines the generative capabilities of LLMs with automated evaluation in an iterative evolutionary framework that proposes, tests, and refines algorithmic solutions to challenging scientific and practical problems. In this paper we showcase AlphaEvolve as a tool for autonomously discovering novel mathematical constructions and advancing our understanding of long-standing open problems.

To demonstrate its breadth, we considered a list of 67 problems spanning mathematical analysis, combinatorics, geometry, and number theory. The system rediscovered the best known solutions in most of the cases and discovered improved solutions in several. In some instances, AlphaEvolve is also able to generalize results for a finite number of input values into a formula valid for all input values. Furthermore, we are able to combine this methodology with Deep Think [148] and AlphaProof [147] in a broader framework where the additional proof-assistants and reasoning systems provide automated proof generation and further mathematical insights.
These results demonstrate that large language model-guided evolutionary search can autonomously discover mathematical constructions that complement human intuition, at times matching or even improving the best known results, highlighting the potential for significant new ways of interaction between mathematicians and AI systems. We present AlphaEvolve as a powerful new tool for mathematical discovery, capable of exploring vast search spaces to solve complex optimization problems at scale, often with significantly reduced requirements on preparation and computation time.

[Gess1989] Ira M. Gessel, *Determinants, Paths, and Plane Partitions*, 1989, available at `https://people.brandeis.edu/~gessel/homepage/papers/pp.pdf`. GS 1316551195686960631.

Abstract:
In studying representability of matroids, Lindström [42] gave a combinatorial interpretation to certain determinants in terms of disjoint paths in digraphs. In a previous paper [25], the authors applied this theorem to determinants of binomial coefficients. Here we develop further applications. As in [25], the paths under consideration are lattice paths in the plane. Our applications may be divided into two classes: first are those in which a determinant is shown to count some objects of combinatorial interest, and second are those which give a combinatorial interpretation to some numbers which are of independent interest. In the first class are formulas for various types of plane partitions, and in the second class are combinatorial interpretations for Fibonomial coefficients, Bernoulli numbers, and the less-known Salié and Faulhaber numbers (which arise

in formulas for sums of powers, and are closely related to Genocchi and Bernoulli numbers).

[Gess2026] ———, *Generalized Schur functions*, 2026.

Examines the question of generalizing the determinant formula for $s_\bullet$ in terms of $h_\bullet$ for counting tableaux where two arbitrary relations R and S apply to consecutive row entries and consecutive column entries, respectively.

[GessKratt1997] Ira M. Gessel and Christian Krattenthaler, *Cylindric partitions*, Trans. Am. Math. Soc. **349** (1997), 429–479, DOI 10.1090/S0002-9947-97-01791-1, available at `https://www.ams.org/journals/tran/1997-349-02/S0002-9947-97-01791-1/`. GS 10012216907340663744. Zbl 0865.05003

Abstract: A new object is introduced into the theory of partitions that generalizes plane partitions: cylindric partitions. We obtain the generating function for cylindric partitions of a given shape that satisfy certain row bounds as a sum of determinants of q-binomial coefficients. In some special cases these determinants can be evaluated. Extending an idea of Burge (J. Comb. Theory Ser. A **63** (1993), 210–222), we count cylindric partitions in two different ways to obtain several known and new summation and transformation formulas for basic hypergeometric series for the affine root system $\tilde{A}_r$. In particular, we provide new and elementary proofs for two $\tilde{A}_r$ basic hypergeometric summation formulas of Milne (Discrete Math. **99** (1992), 199–246).

[GillGorGriff2024] Maria Gillespie, Eugene Gorsky, and Sean T. Griffin, *A combinatorial skewing formula for the Rise Delta Theorem* (2024), arXiv 2408.12543, primary class math.CO, DOI 10.48550/arXiv.2408.12543, available at `https://arxiv.org/abs/2408.12543`. GS 15049396937943085330

Abstract: We prove that the symmetric function $\Delta'_{e_{k-1}} e_n$ appearing in the Delta Conjecture can be obtained from the symmetric function in the Rational Shuffle Theorem by applying a Schur skewing operator. This generalizes a formula by the first and third authors for the Delta Conjecture at $t = 0$, and follows from work of Blasiak, Haiman, Morse, Pun, and Seelinger.

Our main result is that we also provide a purely combinatorial proof of this skewing identity, giving a new proof of the Rise Delta Theorem from the Rational Shuffle Theorem.

[GoodHarpeJones1989] Frederick M. Goodman, Pierre de la Harpe, and Vaughan F. R. Jones. *Coxeter Graphs and Towers of Algebras.* Math. Sci. Research Inst. Publications, vol. 14. Springer, New York, 1989. GS 8379473692311296423.

Abstract: Chapter 1 begins with a (slightly new guise of) a familiar combinatorial problem: to classify finite matrices over the non-negative integers which have Euclidean norm no greater than 2. These are classified by the ubiquitous *Coxeter graphs* of type A, D, or E (see [HHSV] for other occurrences of these graphs) and the set of possible norms is $\{2\} \cup \{2cos\pi/q : q \geq 2\}$.
The central theme of the book — the discussion of which begins in Chapter 2 — is the tower of algebras $M_O \subset M_1 \subset \cdots \subset M_k \subset \cdots$ determined by a pair $M_0 \subset M_1$ of algebras (with the same identity). The tower can be used to define various invariants of the pair, including the index $[M_1 : M_0]$
In Chapters 2 and 3, we study two cases of the tower construction in detail.
In Chapter 2, the algebras are finite direct sums of full matrix algebras over some field. A pair $M_0 \subset M_1$ is described, up to isomorphism, by an inclusion matrix Λ with non-negative integer entries. This matrix may be encoded as a graph, known as the Bratteli diagram of the pair. It turns out that the index $[M_1 : M_0]$ equals $\|\Lambda\|^2$; thus it follows from Chapter 1 that $[M_1 : M_0] \leq 4$ if and only if the Bratteli diagram is a Coxeter graph of type A, D, or E.
In Chapter 3, the algebras are finite von Neumann algebras with finite dimensional centers. Somewhat surprisingly, the results of Chapter 2 essentially extend to this setting. But now a pair $M_0 \subset M_1$ is (partially) described by an inclusion matrix Λ with entries in $\{2cos\pi/q : q \geq 2\} \cup \{r : r \geq 2\}$, and pairs with index no greater than 4 are associated to Coxeter graphs of arbitrary type, including types B,F,G,H,I.
Finally, Chapter 4 is a further analysis of pairs $N \subset M$ of finite factors of finite index. There are two main themes. The first is the notion of a commuting square, due to Popa [Pop1], and its use in approximating pairs of hyperfinite Π_1 factors simultaneously by finite dimensional von Neumann algebras. The second theme

is the derived tower of a pair of Π_1 factors, which is the chain of (necessarily finite dimensional) relative commutants $M_0{}' \cap M_k$ in the tower. All the information in the derived tower can be encoded in a (possibly infinite) graph, the principal graph of the pair. When the index is less than 4, the graph is a Coxeter graph of type A, D, or E.

[GoodKer2000] Frederick M. Goodman and Sergei V. Kerov, *The Martin boundary of the Young-Fibonacci lattice*, J. Algebr. Comb. **11** (2000), 17–48, DOI 10.1023/A:1008739619211, available at https://link.springer.com/article/10.1023/A:1008739619211. GS 2838607610531151777. Zbl 0959.06003

Abstract: In this paper we find the Martin boundary for the Young-Fibonacci lattice YF. Along with the lattice of Young diagrams, this is the most interesting example of a differential partially ordered set. The Martin boundary construction provides an explicit Poisson-type integral representation of non-negative harmonic functions on YF. The latter are in a canonical correspondence with a set of traces on the locally semisimple Okada algebra. The set is known to contain all the indecomposable traces. Presumably, all of the traces in the set are indecomposable, though we have no proof of this conjecture. Using an explicit product formula for Okada characters, we derive precise regularity conditions under which a sequence of characters of finite-dimensional Okada algebras converges.

[Gord2009] Iain Gordon, *Haiman's work on the n! theorem, and beyond*, 2009, available at https://webhomes.maths.ed.ac.uk/~igordon/pubs/grenoble3.pdf. GS 14262130796798650148.

Introduction to Haiman's proof of the n! conjecture and the Madconald positivity conjecture.

[Gow2000] William Timothy Gowers, *The two cultures of mathematics*, https://www.dpmms.cam.ac.uk/~wtg10/2cultures.pdf. Accessed March 19, 2025.

Abstract: I would like to argue that a similar sociological phenomenon can be observed within pure mathematics, and that this is not an entirely healthy state of affairs. The "two cultures" I wish to discuss will be familiar to all professional mathematicians. Loosely speaking, I mean the distinction between mathematicians who regard their central aim as being to solve problems, and those who are more concerned with building and understanding theories.

[Grätz1978] George Grätzer. *General lattice theory.* Lehrbücher und Monographien aus dem Gebiete der exakten Wissenschaften, Mathematische Reihe (LMW/MA), vol. 52. Birkhäuser, Basel, 1st ed., 1978. GS 14018131805847906905.

Abstract: The goal of the present volume can be stated very simply: to discuss in depth the basics of general lattice theory. In other words, I tried to include what I consider the most important results and research methods of all of lattice theory. To treat the rudimentary results in depth and still keep the size of the volume from getting out of hand, I had to omit a great deal. I excluded many important chapters of lattice theory that have grown into research fields on their own. Ordered algebraic systems and other applications were also excluded. The reader will find appropriate references to these throughout this book. It is hoped that even those whose main interest lies in areas not treated here in detail will find this volume useful by obtaining from this book the background in lattice theory so necessary in allied fields.

[Grätz1979] ———. *Universal Algebra.* Springer, New York, 1979. GS 9635876586085855118.

Abstract: Thus universal algebra is the study of finitary operations on a set, and the purpose of research is to find and develop the properties which such diverse algebras as rings, fields, Boolean algebras, lattices, and groups may have in common. ...
Because of the way in which universal algebras developed, many elementary results have never been published but have been used without any reference in the papers, sometimes only in the form of a "therefore". It is hoped that this book will give an adequate background for the explanation of the "therefore's".

The purpose of this book is to give a systematic treatment of the most important results in the field of universal algebras. We will consider generalizations of universal algebras only to the extend that they are necessary for the development of the theory of universal algebras themselves.

[Grätz1996] ______. *General lattice theory.* Birkhäuser, Basel, 2nd ed., 1996. GS 14018131805847906905.

Abstract: In 20 years, tremendous progress has been made in Lattice Theory. Nevertheless, the change is in the superstructure not in the foundation. Accordingly, I decided to leave the book unchanged and add appendices to record the change. In the first appendix: Retrospective, I briefly review developments from the point of view of this book, specifically, the major results of the last 20 years and solutions of the problems proposed in this book. It is remarkable how many difficult problems have been solved! I was lucky in getting an exceptional group of people to write the other appendices: Brian A. Davey and Hilary A. Priestley on distributive lattices and duality, Friedrich Wehrung on continuous geometries, Marcus Greferath and Stefan E. Schmidt on projective lattice geometries, Peter Jipsen and Henry Rose on varieties, Ralph Freese on free lattices, Bernhard Ganter and Rudolf Wille on formal concept analysis; Thomas Schmidt collaborated with me on congruence lattices. Many of these same people are responsible for the definitive books on the same subjects. I changed very little in the book proper. The diagrams have been redrawn and the book was typeset in LaTeX. To bring the notation up-to-date, I substituted Con$L$ for $C(L)$, IdL for $I(L)$, and so on. Almost 200 mathematicians helped me with this project, from correcting typos to writing long essays on the topics that should go into Retrospective. The last section of Retrospective lists the major contributors. My deeply felt thanks to all of them.

[Grätz2011] ______. *Lattice Theory: Foundation.* Birkhäuser, Basel, 2011. GS 5679706934102218841.

Abstract: The goal of the present volume can be stated very simply: to discuss in depth the foundation of lattice theory. I tried to include the most important results and research methods that form the basis of all the work in this field.

[GrätzJónsLaks1973] George Grätzer, Bjarni Jónsson, and H. Lakser, *The amalgamation property in equational classes of modular lattices*, Pac. J. Math. **49** (1973), 507–524, DOI 10.2140/pjm.1973.45.507, available at https://msp.org/pjm/1973/45-2/p13.xhtmlhttps://www.researchgate.net/profile/George-Graetzer/publication/38344944_The_amalgamation_property_in_equational_classes_of_modular_lattices/links/0deec5283d66a3afba000000/The-amalgamation-property-in-equational-classes-of-modular-lattices.pdf. GS 16349272661254660772.

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[GratzNat2010] George Grätzer and James Bryant Nation, *A new look at the Jordan–Hölder theorem for semimodular lattices*, Algebra Univers. **64** (2010), 39–311, DOI 10.1007/s00012-011-0104-9, available at https://link.springer.com/article/10.1007/s00012-011-0104-9. available at https://math.hawaii.edu/~jb/dedekindult.pdf GS 16740432643568493450. Zbl 1216.06006

We show that in a semimodular lattice L of finite length, from any prime interval we can reach any maximal chain C by an up- and a down-perspectivity. Therefore, C is a congruence-determining sublattice of L.

[Gre1974] Curtis Greene, *An extension of Schensted's theorem*, Adv. Math. **14** (1974), 254–265, DOI 10.1016/0001-8708(74)90031-0, available at https://www.sciencedirect.com/science/article/pii/0001870874900310. GS 8272534487423654886. MR50 #6874

U. Topics: Greene invariant. Introduces the Greene invariant.

[Grin2026] Darij Grinberg, Ekatrina A. Vassilieva, Sarah Brauner, Patricia Commins, and Franco Saliola, *Tales of the descent algebra*, 2026-03-20, MIT, Cambridge, Mass., US, Richard P. Stanley Seminar in Combinatorics, available at https://www.cip.ifi.lmu.de/~grinberg/algebra/da2026s.pdf

Introduction to the descent algebra.

[GrinRein2020] Darij Grinberg, *Hopf Algebras in Combinatorics* (2020), arXiv 1409.8356, primary class math.CO, DOI 10.48550/arXiv.1409.8356, available at https://arxiv.org/abs/1409.8356. GS 12645288413734990255

Abstract: These notes — originating from a one-semester class by their second author at the University of Minnesota — survey some of the most important Hopf algebras appearing in combinatorics. After introducing coalgebras, bialgebras and Hopf algebras in general, we study the Hopf algebra of symmetric functions, including Zelevinsky's axiomatic characterization of it as a "positive self-adjoint Hopf algebra" and its application to the representation theory of symmetric and (briefly) finite general linear groups. The notes then continue with the quasisymmetric and the noncommutative symmetric functions, some Hopf algebras formed from graphs, posets and matroids, and the Malvenuto–Reutenauer Hopf algebra of permutations. Among the results surveyed are the Littlewood–Richardson rule and other symmetric function identities, Zelevinsky's structure theorem for PSHs, the antipode formula for P-partition enumerators, the Aguiar–Bergeron–Sottile universal property of QSym, the theory of Lyndon words, the Gessel–Reutenauer bijection, and Hazewinkel's polynomial freeness of QSym. The notes are written with a graduate student reader in mind, being mostly self-contained but requiring a good familiarity with multilinear algebra and — for the representation-theory applications — basic group representation theory.

[Haim1984a] Mark D. Haiman, *The Theory of Linear Lattices*, Ph.D. thesis, Massachusetts Inst. of Tech., 1984, https://hdl.handle.net/1721.1/153355. GS 12728508576024553972.

[Haim1985a] contains most of the results of this thesis.

Abstract: We make a study of lattices representable by commuting equivalence relations, which we call linear lattices. We develop a proof theory for implications valid in linear lattices, which differs from classical proof theories in that its deductions transform representative graphs rather than well-formed formulas. We then use graph-theoretic arguments to establish a duality theorem and a normal form theorem for this proof theory.

We introduce a matroid analogue of the graph proof theory, and find the class of lattices to which it applies. This turns out to be the class of lattices representable by what we call pseudo-congruences on a Mal'cev algebra.
For completeness, we review some known results about lattices of ordinary congruences on a Mal'cev algebra, showing how they may be reformulated in the present terminology.

[Haim1984b] ———, *Linear lattice proof theory: an overview*, Universal algebra and lattice theory (Charleston, South Carolina, 1984) (Stephen D. Comer, ed.), Lecture Notes in Math. (LNM), vol. 1149, Springer-Verlag, Berlin and New York, 1985, 1984, DOI 10.1007/BFb0098460, available at `https://link.springer.com/chapter/10.1007/BFb0098460`.
GS 2326541148673430734.

Abstract: This paper concerns lattices representable by commuting equivalence relations (Jónsson's *type I* lattices), which we propose to call *linear lattices* in order to evoke their archetypal examples, the (coordinate) projective geometries. The results discussed here are taken from the author's doctoral thesis [Haim1984a] and will appear in full detail in a separate publication adapted from it [Haim1985a]. Therefore we give here only a brief historical introduction to the subject, followed by statements and proof sketches of the main results, with applications and examples.
...
It was soon realized that the distributive law was a defining property for lattices of sets. ...
For the other classical example—of quotient lattices in algebra—the situation was and remains not nearly so simple and satisfying. One problem is that Dedekind's "modular law" incompletely axiomatizes these lattices; some such stronger axiom as Jónsson's Arguesian law [Jóns1953b] [Jóns1954a] is needed even to prove Desargues' theorem of projective geometry lattice-theoretically. Another problem is the difficulty of the available proofs of Von Neumann's coordinatization theorem for complemented modular lattices [1, 7, 15, 23, 29]—the modular analog of Stone's Boolean algebra representation theorem. A final problem is that modular calculations are notoriously difficult, and in general impossible, in view of Freese's [9] result (later improved by Herrman [14]) that the free modular lattice word problem is unsolvable.

Since all the classical algebraic entities are Mal'eev algebras [27], their congruence lattices are linear (i.e., consist of commuting equivalence relations) by Mal'cev's theorem [25, 11]. Our guiding purpose in this paper and forthcoming ones is to advance the contention that linear lattices provide a better framework than modular lattices for the study of the classical quotient 130 structures, and in particular of projective geometry; that in this framework many of the difficulties that plague modular lattices can be reduced or eliminated.

While the importance in universal algebra of linear congruence lattices is universally rec ognized, linear lattices *per se* have gone curiously unstudied since Jónsson. ...

In this paper we resurrect the study of linear lattices and continue it with two theorems which together form a linear lattice proof theory. Theorem 1 gives a linear analog of the Hilbert- or Gentzen-type deductive systems which solve the distributive lattice word problem. Theorem 2, a normal form theorem for the proofs given by Theorem 1, can be viewed as the linear analog of Gentzen's Hauptsatz in propositional logic. The "formulas" with which our proof theory deals are edge-labelled series-parallel graphs, which should be understood as linear analogs of Venn diagrams. These diagrams lend to linear lattice calculations an ease and intuitiveness not shared by modular ones. (interestingly, Czedli [5] introduced similar diagrams for a like purpose in general lattice calculations.) Related to Theorems 1 and 2 are a number of observations, corollaries, and conjectures. The axiomatizability of linear lattices by finitary universal Horn sentences (originally due to Jónsson) is a corollary to Theorem 1. The proof theory of Theorem 1 also possesses a duality which explains why simple linear lattice theorems tend also to be dually valid (although most likely the dual of a linear lattice need not be linear). Theorem 2, while not solving the free linear lattice word problem, comes close and clearly isolates the difficulties. It also suggests a natural conjecture which if true would simultaneously solve the free linear lattice word problem and prove that the class of linear lattices is not self-dual and (hence) not equal to the variety of Arguesian lattices. Finally, to suggest the value of linear lattice theory as a tool in synthetic projective geometry, we produce a sequence

of progressively (strictly?) stronger linear lattice identities expressing “higher-dimensional” generalizations of Desargues’ theorem.

[Haim1985a] ———, *Proof theory for linear lattices*, Adv. Math. **58** (1985), 209–242, DOI 10.1016/0001-8708(85)90118-5, available at https://www.sciencedirect.com/science/article/pii/0001870885901185. GS 2116643225209972735.

Includes most of the results of [Haim1984a]; see sec. 3.0 for a description of what parts of are not included. Gives the full version of the results in [Haim1984b].

Abstract: We make a study of lattices representable by commuting equivalence relations, which we call *linear* lattices. We develop a proof theory for implications valid in linear lattices, which differs from classical proof theories in that its deductions transform representative graphs rather than well-formed formulas. Using graph- theoretic arguments, we establish a duality theorem and a normal form theorem for this proof theory.

[Haim1985b] ———, *Two notes on the Arguesian identity*, Algebra Univers. **21** (1985), 167–171, DOI 10.1007/BF01188053, available at https://link.springer.com/article/10.1007/BF01188053. GS 3496212099161506733.

Abstract: We find an explicitly self-dual lattice identity equivalent to the Arguesian law. We also show that any lattice identity equivalent to the Arguesian-law must necessarily involve at least six variables.

[Haim1987] ———, *Arguesian lattices which are not linear*, Bull. Am. Math. Soc. **16** (1987), 121–123, DOI 10.1090/S0273-0979-1987-15483-8, available at https://community.ams.org/journals/bull/1987-16-01/S0273-0979-1987-15483-8/S0273-0979-1987-15483-8.pdf. GS 1301710048995236821.

Elaborated in [Haim1991].

Abstract: A *linear* lattice is one representable by commuting equivalence relations. We construct a sequence of finite lattices A_n ($n \geq 3$) with the properties: (i) A_n is not linear, (ii) every proper sublattice of A_n is linear, and (iii) any set of generators for A_n has at least n elements. In particular, A_n is then Arguesian for $n \geq 7$. This settles a question raised in [Jóns1953b].

[Haim1989] ———, *On mixed insertion, symmetry, and shifted Young tableaux*, J. Comb. Theory **Ser. A 50** (1989), 196–225, DOI 10.1016/0097-3165(89)90015-0, available at `https://www.sciencedirect.com/science/article/pii/0097316589900150`. GS 14350300647709625382.

U, *S*. Topics: left-right insertion, mixed insertion, conversion, jeu de taquin, folding. A good catalog of insertion algorithms, including "mixed" and "left-right" versions. Discusses jeu de taquinand the "conversion" operation and their relationships to symmetry operations on generalized permutations.

[Haim1991] ———, *Arguesian lattices which are not type-1*, Algebra Univers. **28** (1991), 128–137, DOI 10.1007/BF01190416, available at `https://link.springer.com/article/10.1007/BF01190416`. GS 13577691971007877653. Zbl 0724.06004

Elaboration of [Haim1987].

Abstract: A "type-1" representation of a lattice, in the sense of Jónsson, means a representation by pairwise commuting equivalence relations on some set. Jónsson proved that type-1 representable lattices obey the Arguesian law, a strong lattice identity with important applications in coordinatization theory (see [1] for a survey). Jónsson asked whether, conversely, the Arguesian law might imply type-1 representability.
The research announcement [Haim1987] sketches the construction of certain Arguesian (for $n \geq 7$ and probably for $n \geq 4$) lattices A_n which do not have type-1 representations, settling Jónsson's question in the negative. In fact, these counterexamples allow one to conclude that no finite system of lattice identities or even universal Horn sentences can imply type-1 representability. By contrast, it is known that there is an infinite system of universal Horn sentences characterizing type-1 representability [Haim1985a], [Jóns1959a].

In this paper we explain the construction of the lattices A_n, including in full the parts of the reasoning omitted in [Haim1987]. For $n \geq 4$ (A_3 is not Arguesian), these lattices have skeletons which are not distributive. Prompted by this fact and by their result showing that an Arguesian lattice whose skeleton is a chain is type-1 representable, Nation and Pickering conjectured that having a distributive skeleton might be a sufficient condition for a finite-dimensional Arguesian lattice to be type-1 representable. At the end of this paper we show how to modify the examples A_n, to give counterexamples to this conjecture.

[HallDil1944] Marshall Hall and R. P. Dilworth, *The Imbedding Problem for Modular Lattices*, Ann. Math. **2nd Ser. 45** (1944), 450–456, DOI 10.2307/1969187, available at `https://www.jstor.org/stable/1969187`. GS 14523482054239126280.

Abstract: It is trivially true that an arbitrary lattice may be imbedded in a complemented lattice. We need only adjoin a unit and null elements if they do not already exist and a single element which is a complement of each of the elements not the unit or null element. For distributive lattices, the imbedding problem is not so trivial, but is contained in the representation theorem which asserts that any distributive lattice is isomorphic with a ring of sets (Birkhoff (1), Mac Neille (1)). The corresponding problem of imbedding a modular lattice in a complemented modular lattice is an outstanding problem in lattice theory. We exhibit here an example of a modular lattice which cannot be imbedded in any complemented modular lattice. However we will be concerned primarily with the isometric problem for finite dimensional modular lattices; that is, the prob- lem of imbedding a finite dimensional modular lattice in a complemented modu- lar lattice of the same dimensionality.

[Halm1973] Paul R. Halmos, *The legend of John von Neumann*, Am. Math. Mon. **80** (1973), 382–394, DOI 10.1080/00029890.1973.11993293, available at `https://gwern.net/doc/math/1973-halmos.pdf`. GS 4147532046574882503. MR317867 Zbl 0264.01018

Abstract: John von Neumann was a brilliant mathematician who made important contributions to quantum physics, to logic, to meteorology, to war, to the theory and applications of high-speed computing machines, and, via the mathematical theory of games of strategy, to economics. . . .

[HardCain2019] Godfrey H. Hardy and Alan J. Cain. *An Annotated Mathematician's Apology*, Lisbon, 2019.

Abstract: I propose to put forward an apology for mathematics; and I may be told that it needs none, since there are now few studies more generally recognized, for good reasons or bad, as profitable and praiseworthy. ... I shall ask, then, why is it really worth while to make a serious study of mathematics? What is the proper justification of a mathematician's life? And my answers will be, for the most part, such as are to be expected from a mathematician: I think that it is worth while, that there is ample justification.

[HarVincWor1982] Frank Harary, Andrew Vince, and Dale R. Worley, *A point-symmetric graph that is nowhere reversible*, SIAM J. Algebr. Disc. Methods **3** (1982), 285–287, DOI 10.1137/0603028, available at `https://epubs.siam.org/doi/abs/10.1137/0603028`. GS 14017475894817380504.

Abstract: It is the purpose of this note to investigate the relationships among four concepts relating to symmetry in graphs: point-symmetry, line-symmetry, arc-symmetry and reversibility; especially which of the first three properties do not imply reversibility. Holt has found a counterexample to one such question and we construct a counterexample to another using a Cayley graph. Both examples are nowhere reversible, a property which is stronger than nonreversibility

[HaskGudd1971] L. Haskins and S. Gudder, *Semimodular posets and the Jordan–Dedekind chain condition*, Proc. Am. Math. Soc. **28** (1971), 395–396, DOI 10.1090/S0002-9939-1971-0276144-6, available at `https://www.ams.org/journals/proc/1971-028-02/S0002-9939-1971-0276144-6/`. GS 1143172550768635107. Zbl 0231.06014

Abstract: In this paper it is shown that an upper semimodular poset without infinite chains satisfies the Jordan–Dedekind chain condition. This corrects an error in Theorem 14, p. 40 of [Birk1967] and generalizes that theorem.

[Hawr1996] Michael Hawrylycz, *Arguesian identities in invariant theory*, Adv. Math. **122** (1996), 1–48, DOI 10.1006/aima.1996.0056, available at https://www.sciencedirect.com/science/article/pii/S0001870896900560. https://core.ac.uk/reader/82133140 GS 18029858374324039613.

Abstract: Having been motivated by an example of Doubilet, Rota, and Stein [*Stud. Appl. Math.* **56** (1976), 185–216], we present a technique for constructing geometric identities in a Grassmann–Cayley algebra. Each identity represents a projective invariant closely related to the Theorem of Desargues in the plane and its generalizations to higher dimensional projective space. The construction employs certain combinatorial properties of matchings in bipartite graphs. We also prove a dimension independence result for Arguesian identities, thereby connecting the identities with lattice theory.

[Haz2007a] Michiel Hazewinkel, *Towards uniqueness of MPR, THE Malvenuto–Poitier–Reutenauer Hopf algebra of permutations*, Honam Math. J. **29** (2007), 119–192, DOI 10.5831/HMJ.2007.29.2.119, available at https://www.researchgate.net/publication/264133099_TOWARDS_UNIQUENESS_OF_MPR_THE_MALVENUTO-POITIER-REUTENAUER_HOPF_ALGEBRA_OF_PERMUTATIONS. GS 8724720211998100825. MR2334374 Zbl 1181.16030

Abstract: A very important Hopf algebra is the graded Hopf algebra Symm of symmetric functions. It can be characterized as the unique graded positive selfdual Hopf algebra with orthonormal graded distinguished basis and just one primitive element from the distinguished basis. This result is due to Andrei Zelevinsky. A noncommutative graded Hopf algebra of this type cannot exist. But there is a most important positive graded Hopf algebra with distinguished basis that is noncommutative and that is twisted selfdual, the Malvenuto–Poirier–Reutenauer Hopf algebra of permutations. Thus the question arises whether there is a corresponding uniqueness theorem for MPR. This prepreprint records initial investigations in this direction and proves that uniquenees holds up to and including the degree 4 which has rank 24.

[Haz2007b] ———, *Rigidity for MPR, the Malvenuto–Poirier–Reutenauer Hopf algebra of permutations*, Honam Math. J. **29** (2007), 495–509, DOI 10.5831/HMJ.2007.29.4.495, available at https://www.researchgate.net/publication/267074553_Rigidity_for_MPR_the_Malvenuto-Poirier-Reutenauer_Hopf_algebra_of_permutations. GS 3668416997823375795. MR2375962 Zbl 1180.16026

Abstract: It is proved that MPR is rigid as a Hopf algebra with distinguished basis. I.e. there are no nontrivial automorphisms that preserve the multiplication and comultiplication and take the distinguished basis of all permutations into itself (as a graded set).

[Haz2008] ———, *Corrigenda and addenda: "Rigidity for MPR, the Malvenuto–Poirier–Reutenauer Hopf algebra of permutations"*, Honam Math. J. **30** (2008), 205. MR2399789 Zbl 1181.16029

[HazGubKir2010] Michiel Hazewinkel, Nadiya Gubareni, and V. V. Kirichenko, *The Hopf algebra of permutations*, Algebras, Rings and Modules: Lie Algebras and Hopf Algebras, Math. Surveys and Monographs, vol. 168, AMS, Providence, RI, US, 2010. GS 8033108627104571537, pp. 263–276, DOI 10.1090/surv/168/07, available at https://www.researchgate.net/publication/299677985_The_Hopf_algebra_of_permutations.

Abstract: There is a beautiful, highly noncommutative and highly noncocommutative Hopf algebra, that generalizes both **NSymm** and **QSymm**. It is highly noncommutative and noncocommutative in that it is both free and cofree. It was invented and studied by Reutenauer, Malvenuto and Poirier. It will usually be referred to it as the ***MPR*** **Hopf algebra**. It is connected graded and it is self dual up to an isomorphism .

[Herr1973] Christian Herrmann, *S-verklebte Summen von Verbänden [S-glued sums of lattices]*, Math. Z. **130** (1973), 255–274, DOI 10.1007/BF01246623, available at https://link.springer.com/article/10.1007/BF01246623. GS 5554875835071000456 English translation in [Herr1973-en]. Zbl 0275.06007

Abstract: Bei vielen gleichungstheoretischen Fragen über modulare Verbände oder solchen Fragen, die von Einbettungsproblemen herrühren, erwies sich eine Konstruktion von Verbänden als nützlich, die von Hall und Dilworth in [4] angegeben wurde (vgl. Jónsson [6, 7] und Grätzer, Jónsson and Lakser [3]): Sei L_0 ein Verband mit größtem Element u_0, L_1 ein zu L_0 disjunkter Verband mit kleinstem Element v_1 und $a \in L_0$, $b \in L_1$ Elemente so, daß die Intervalle $[a, u_0]$ und $[v_1, b]$ zueinander isomorph sind. Dann erhält man nach Identifizierung der sich bei diesem Isomorphismus entsprechenden Elemente auf $L_0 \cup L_1$ eine Verbandsstruktur, deren Halbordnung gerade die von den Halbordnungen von L_0 und L_1 erzeugte transitive Relation ist. Sie ist modular, falls L_0 und L_1 modular sind.
Da in diese Konstruktion die Indexmenge $\{0, 1\}$ im wesentlichen als Kette eingeht, liegt es nahe danach zu fragen, ob man allgemeiner zu einer Familie L_x ($x \in S$) von Verbänden einen Verband angeben kann, der im Kleinen durch die Verbandsstruktur der L_x und im GroBen durch eine Verbandsstruktur auf S bestimmt wird. Ein Ansatz dazu, der hauptsächlich im Hinblick auf Anwendungen in der Theorie modularer Verbände konzipiert ist und als Spezialfall die Konstruktion von Hall-Dilworth enthält, soil in dieser Arbeit mit dem Begriff der S-verklebten Summe gegeben werden. Einen wesentlichen davon verschiedenen Ansatz findet man in der L-Summe yon Koh [9], die im allgemeinen von modularen Verbänden nicht wieder zu modularen Verbänden führt.
Als entscheidende Anwendung ergibt sich eine Möglichkeit, modulare Verbände endlicher Länge durch projektive Geometrien darzustellen, sie beruht auf dem in Abschnitt 6 zu beweisenden Hauptsatz dieser Arbeit und dem Satz von Birkhoff (vgl. [1; Chap. IV, §7]), daß die atomistischen modularen Verbände endlicher Länge gerade die Teilraumverbände endlichdimensionaler (möglicherweise reduzibler) projektiver Geometrien sind.
Hauptsatz. Jeder modulare Verband M endlicher Länge ist S-verklebte Summe seiner maximalen atomistischen Intervalle. Der Verband S kann als die Menge $S(M)$ der kleinsten Elemente dieser Intervalle gewählt werden, die ein Verbindungs-Unterhalbverband yon M ist.

Insbesondere wird das Rechnen in M durch die Aussagen (10) – (16) — und die dazu dualen — aus Abschnitt 2 konkret bestimmt. Der Verband $S(M)$ wird das *Skelett* von M genannt.

Translation: For many equation-theoretical questions about modular lattices or those arising from embedding problems, a construction of lattices given by Hall and Dilworth in [HallDil1944] has proven useful (cf. Jónsson [6, 7] and Grätzer, Jónsson and Lakser [3]): Let L_0 be a lattice with largest element u_0, L_1 a disjoint lattice to L_0 with smallest element v_1 and $a \in L_0$, $b \in L_1$ elements such that the intervals $[a, u_0]$ and $[v_1, b]$ are isomorphic to each other. Then, after identifying the elements corresponding to this isomorphism on $L_0 \cup L_1$, one obtains a lattice structure whose half-order is precisely the transitive relation generated by the half-orders of L_0 and L_1. It is modular if L_0 and L_1 are modular.

Since the index set $\{0, 1\}$ enters into this construction essentially as a chain, it makes sense to ask whether one can more generally specify a lattice for a family L_x $(x \in S)$ of lattices which is determined on a small scale by the association structure of L_x and on a large scale by an association structure on S. An approach to this, which is mainly designed with regard to applications in the theory of modular assemblies and includes the Hall-Dilworth construction as a special case, will be given in this work with the concept of the S-glued sum. A substantially different approach can be found in Koh's L sum [9], which generally does not lead from modular associations back to modular associations.

A crucial application is the possibility of representing modular assemblies of finite length using projective geometries; it is based on the main theorem of this work to be proven in Section 6 and Birkhoff's theorem (cf. [1; Chap. IV, §7]), that the atomistic modular assemblies of finite length are precisely the subspace assemblies of finite-dimensional (possibly reducible) projective geometries.

Main Theorem. Every modular lattice M of finite length is S-glued sum of its maximum atomistic intervals. The lattice S can be chosen as the set $S(M)$ of the smallest elements of these intervals, which is a connecting sublattice of M.

In particular, computing in M is specifically determined by the statements (10) – (16) — and the dual — from Section 2. The lattice $S(M)$ is called the *skeleton* of M.

[Herr1973-en] ______, *S-glued sums of lattices*, translated by Dale R. Worley, arXiv 2409.10738, primary class math.CO, DOI 10.48550/arXiv.2409.10738, available at https://arxiv.org/abs/2409.10738. English translation of [Herr1973] GS 11656565639169386626

English translation with various typos corrected and a few minor mathematical errors noted.

[Herr2006] ______, *Corrigendum: Gluings of Modular Lattices*, Order **23** (2006), 169–171, DOI 10.1007/s11083-006-9042-0, available at https://link.springer.com/article/10.1007/s11083-006-9042-0. GS 9123862131182993021.

Abstract: In Day and Hermann [DayHerr1988] it has been stated that for every S-cover of a lattice L there is an extension L' of L in the variety of L and a bounded S-cover of L' which restricts to the given S-cover and has each block $L'(x)$ in the variety of $L(x)$. A correct proof of this statement is given, here.

[Herr2013] ______, *A review of some of Bjarni Jónsson's results on representation of arguesian lattices*, Algebra Univers. **70** (2013), 163–174, DOI 10.1007/s00012-013-0240-5, available at https://www2.mathematik.tu-darmstadt.de/~herrmann/recherche/modularlattices/glueing/argAU.pdf. GS 5889273947199313238.

Sec. 2 gives a good introduction to subspace lattices of vector spaces and projective spaces and constructing lattices via the Dilworth-Hall "gluing" operation.

Abstract: We review (and slightly extend) Bjarni Jónsson's results on representations of arguesian lattices that are complemented, of low height, or of simple gluing structure. ... The focus of this review will be on lattices embeddable into subspace lattices of vector spaces.

[Herr2025] ______, *[Summary of Mal'cev's method of model correspondence]*, 2025.

A summary of the axiomatization properties that can be inferred using Ma'cev's method of model correspondence.

[HerrPickRodd1994] Christian Herrmann, Douglas A. Pickering, and Michael Roddy, *A geometric description of modular lattices*, Algebra Univers. **31** (1994), 365–396, DOI 10.1007/BF01221792, available at `https://link.springer.com/article/10.1007/BF01221792`. GS 14927782717538699583. Zbl 0816.06008

Abstract: Here, we consider, more generally, modular lattices in which every element is the join of completely join irreducible 'points'. We prove the isomorphy of an algebraic lattice of this kind and the associated subspace lattice and give a first order characterization of the associated 'ordered spaces' in terms of collinearity and order which appears more natural and powerful. The crucial axioms are a 'triangle axiom', which includes the degenerate cases, and a strengthened 'line regularity axiom', both derived from [7]. As a consequence, using Skolemization, we get that any variety of modular lattices is generated by subspace latices of countable spaces.

[HerrPog1974] Christian Herrmann and Werner Poguntke, *The class of sublattices of normal subgroup lattices is not elementary*, Algebra Univers. **4** (1974), 280–286, DOI 10.1007/BF02485739, available at `https://link.springer.com/article/10.1007/BF02485739`. GS 17782140340734611760. Zbl https://zbmath.org/0303.20031

Abstract: Let $\mathcal{N}$ be the class of all lattices which are embeddable into the lattice of normal subgroups of some group; let $\mathcal{C}$ be the class of all complemented modular lattices; $(\mathcal{N}\cup\mathcal{C})^*$ is the variety generated by the union of these classes. Let $\mathcal{V}$ be the class of all lattices which are embeddable into subspace lattices of 5-dimensional vector spaces over some field of characteristic zero. We are going to prove:
THEOREM. *No class $\mathcal{L}$ of lattices with $\mathcal{V}\subset\mathcal{L}\subset(\mathcal{N}\cup\mathcal{C})^*$ is elementary, i.e. definable by finitely many first order axioms.*

[Hik2024] Tatsuyuki Hikita, *A proof of the Stanley–Stembridge conjecture* (2024), arXiv 2410.12758, primary class math.CP, DOI 10.48550/arXiv.2410.12758, available at `https://arxiv.org/abs/2410.12758`. GS 15839076723379590630

Abstract: We give a probability theoretic interpretation of the coefficients of the elementary symmetric function expansion of chromatic quasisymmetric function for any unit interval graph. As a corollary, we prove the Stanley–Stembridge conjecture.

[HinHas2015a] Wataru Hino and Ichiro Hasuo, *Varieties, Quasivarieties and Prevarieties: Completing the Picture*, 2015, available at `https://coalg.org/calco15/ei/hino.pdf`. GS 10266213490302299544.

The presentation form is [HinHas2015b].

Abstract: Variety and *quasivariety* of algebras are classic notions in universal algebra (see e.g. [3]). By definition, a variety is a full subcategory of $\mathbf{Alg}\Sigma$ specified by a set of equations; a quasivariety is specified by a set of *implications* $\forall\vec{x}((\bigwedge_{i=0}^{n} s_i = t_i) \to s = t)$. Then the famous Birkhoff theorem characterizes varieties as those which are closed under *homomorphic images, subobjects* and (arbitrary) *products* (H, S, P in Table 1). A similar characterization is possible for quasivarieties (see [3]): see Table 1, where FC means closure under *filtered colimits*.
These classic results are significantly extended through the development of categorical model theory (see e.g. [3]). This accounts for the rows of Table 1 other than the first and third rows.

[HinHas2015b] ______, *Varieties, Quasivarieties and Prevarieties: Completing the Picture*, 2015, available at `https://group-mmm.org/~wataru/sort-of-varieties.pdf`. presentation of [HinHas2015a].

Abstract: Outline: (A) Review 1: variety and quasivariety, (B) Review 2: orthogonality and prevariety, (C) New notion: sort-of-variety.

[HivScott2024] Florent Hivert and Jeanne Scott, *Diagram model for the Okada algebra and monoid*, Proceedings of the 36th International Conference on Formal Power Series and Algebraic Combinatorics (Ruhr-Universität Bochum, Germany, 2024), 2024, pp. art. 91B.25, available at `https://www.mat.univie.ac.at/~slc/wpapers/FPSAC2024/25.pdf`. GS 14698800556631969636.

Abstract: It is well known that the Young lattice is the Bratelli diagram of the symmetric groups expressing how irreducible representations restrict from $\mathfrak{S}_N$ to $\mathfrak{S}_{N-1}$. In 1988, Stanley discovered a similar lattice called the Young–Fibonacci lattice which was realized as the Bratelli diagram of a family of algebras by Okada in 1994.
In this paper, we realize the Okada algebra and its associated monoid using a labeled version of Temperley–Lieb arc-diagrams. We prove in full generality that the dimension of the Okada algebra is $n!$. In particular, we interpret a natural bijection between permutations and labeled arc-diagrams as an instance of Fomin's Robinson–Schensted correspondence for the Young–Fibonacci lattice. We prove that the Okada monoid is aperiodic and describe its Green relations. Lifting those results to the algebra allows us to construct a cellular basis of the Okada algebra.

[Ho2023] Aaron Ho, *The Stone-Čech compactification*, 2023, available at `https://math.uchicago.edu/~may/REU2023/REUPapers/Ho,Aaron.pdf`.

Abstract: The Stone-Čech Compactification serves as one of the major theorems in Point-Set Topology dealing with the construction of the largest compactification, denoted $\beta(X)$, of a completely regular topological space X. This compactification is characterized in the following: given any continuous (and bounded) map $f : X \to C$ from X to C, a compact Hausdorff space, f extends uniquely to another continuous map $g : \beta(X) \to C$ that equals f on X. In this paper, we will reiterate and define relevant definitions and concepts to build up to this theorem, utilizing and proving several other important theorems on the way, such as the Tychonoff Theorem, Urysohn's Lemma, and Urysohn's Metrization Theorem.

[Hoff2001] Michael E. Hoffman, *An analogue of covering space theory for ranked posets*, Electron. J. Comb. **8** (2001), R32, DOI 10.37236/1576, available at `https://www.combinatorics.org/ojs/index.php/eljc/article/view/v8i1r32`. GS 4398742423178623417.

Abstract: Suppose P is a partially ordered set that is locally finite, has a least element, and admits a rank function. We call P a weighted-relation poset if all the covering relations of P are assigned a positive integer weight. We develop a theory of covering maps for weighted-relation posets, and in particular show that any weighted-relationposet P has a universal cover $\tilde{P} \to P$, unique up to isomorphism, so that

(1) $\tilde{P} \to P$ factors through any other covering map $P' \to P$;
(2) every principal order ideal of $\tilde{P}$ is a chain; and
(3) the weight assigned to each covering relation of $\tilde{P}$ is 1.

If P is a poset of "natural" combinatorial objects, the elements of its universal cover $\tilde{P}$ often have a simple description as well. For example, if P is the poset of partitions ordered by inclusion of their Young diagrams, then the universal cover $\tilde{P}$ is the poset of standard Young tableaux; if P is the poset of rooted trees ordered by inclusion, then $\tilde{P}$ consists of permutations. We discuss several other examples, including the posets of necklaces, bracket arrangements, and compositions.

[Hoff2004] ———, *Updown categories* (2004), arXiv 0402450, primary class math.CO, DOI 10.48550/arXiv.math/0402450, available at `https://arxiv.org/abs/math/0402450`. GS 4651653832258076262

Abstract: A poset can be regarded as a category in which there is at most one morphism between objects, and such that at most one of $\mathrm{Hom}(c, c')$ and $\mathrm{Hom}(c', c)$ is nonempty for $c \neq c'$. If we keep in place the latter axiom but allow for more than one morphism between objects, we can have a sort of generalized poset in which there are multiplicities attached to the covering relations, and possibly nontrivial automorphism groups. We call such a category an "updown category." In this paper we give a precise definition of such categories and develop a theory for them, which incorporates earlier notions of differential posets and weighted-relation posets. We also give a detailed account of ten examples, including the updown categories of integer partitions, integer compositions, planar rooted trees, and rooted trees.

[Hop2025] Sam Hopkins, *RSK via local transformations* (2025), arXiv 2510.22082, primary class math.CO, DOI 10.48550/arXiv.2510.22082, available at `https://arxiv.org/abs/2510.22082`. GS 2819358328795655820

Abstract: We explain how to define the Robinson–Schensted–Knuth (RSK) correspondence in terms of local transformations called "toggles." (This note, which is not intended for publication and which is based on presentations of Alex Postnikov, was written in 2014 and has been circulating since then. We are finally posting it to the arXiv for preservation purposes.)

[HuhKimJKrattOk2023] Jisun Huh, Jang Soo Kim, Christian Krattenthaler, and Soichi Okada, *Bounded Littlewood identities for cylindric Schur functions*, posted on 2023, DOI 10.48550/arXiv.2301.13117, available at https://arxiv.org/abs/2301.13117. GS 5168736310574237708.

Abstract: The identities which are in the literature often called "bounded Littlewood identities" are determinantal formulas for the sum of Schur functions indexed by partitions with bounded height. They have interesting combinatorial consequences such as connections between standard Young tableaux of bounded height, lattice walks in a Weyl chamber, and noncrossing matchings. In this paper we prove affine analogs of the bounded Littlewood identities. These are determinantal formulas for sums of cylindric Schur functions. We also study combinatorial aspects of these identities. As a consequence we obtain an unexpected connection between cylindric standard Young tableaux and r-noncrossing and s-nonnesting matchings.

[Hutch1989] George Hutchinson, *Lattices and Categories of Modules*, 1989, available at https://www2.mathematik.tu-darmstadt.de/~herrmann/hutchinson/.

Abstract: This book is about rings and modules, but it is not primarily concerned with the elements, operations, and structure of particular rings or modules. Most of the results are expressed in terms of the following theories:
(1) modular lattices,
(2) abelian categories,
(3) additive relation categories,
(4) endomorphism algebras of additive relation categories, without constants,
(5) endomorphism algebras of additive relation categories, with constants.

These five diverse theories have a common overlap, which will be displayed and developed here as a unified theory.

[Jagg2005] Aaron D. Jaggard, *Subsequence containment by involutions*, Electron. J. Comb. **12** (2005), R14, DOI 10.37236/1911, available at `https://www.combinatorics.org/ojs/index.php/eljc/article/view/v12i1r14`. GS 4575520411946440393.

Counts the number of involutions of S_n which contain a given permutation $\tau \in S_k$ as a subsequence. Also provides a bijective proof of a formula counting the standard Young tableaux of size n which contain a fixed tableau of size k as a subtableau.

[Jam1978] Gordon Douglas James. *The Representation Theory of the Symmetric Groups.* Lecture Notes in Math. (LNM), vol. 682. Springer-Verlag, Berlin and New York, 1978. GS 3515895933197314846.

U. Representation theory of S_n that attempts to follow Young's outline. Includes a portrait of Alfred Young (Rev., FRS).

Abstract: The purpose of this book is to provide an account of both the ordinary and the modular representation theory of the symmetric groups. The range of applications of this theory is vast, varying from theoretical physics through combinatorics to the study of polynomial identity algebras, and new uses are still being found. So diverse are the questions which arise that we feel justified in hoping the reader might find that some part of our text inspires him to undertake research of his own into one of the many unsolved problems in this elegant branch of mathematics.
There are several different ways of approaching symmetric group representations, and while we have tried to illuminate parts of the theory by giving more than one description of it, we have made no effort to cover every view of the subject.

[Jiang2024] Yuhan Jiang, *Geometric RSK, from Discrete to Continuous*, 2024-04-25, Harvard Univ., Cambridge, Mass., US, Harvard Trivial Notions Seminar

Abstract: The Robinson-Schensted-Knuth correspondence plays a fundamental role in the theory of Young tableaux, symmetric functions, and representation theory. By a version of Greene's theorem, it can be defined as a piecewise linear map over the tropical semiring on matrices with non-negative integer entries. Berenstein and Kirillov extended it to the usual $(+, \times)$ algebra for real values matrices, which is called the geometric RSK. As RSK can be used to study Bernoulli walks, gRSK can be used to study Brownian motions and directed polymers. As RSK can be used to prove Cauchy–Littlewood identity for Schur functions, gRSK can be used to prove analogous identity for $GL(N, \mathbb{R})$-Whittaker functions.

[Jóns1953a] Bjarni Jónsson, *Modular lattices and normal subgroups. Preliminary report.*, AMS meeting (New York, April 1953), 1953, pp. 343, available at https://www.ams.org/journals/bull/1953-59-04/S0002-9904-1953-09720-8/S0002-9904-1953-09720-8.pdf. GS 17121198026752292349.

[Jóns1953b] states "The results of Section 2 were presented to the American Mathematical Society in April 1953. Cf. Jónsson [3]" and "3. B. Jónsson, *Modular lattices and normal subgroups*, To appear in the Bulletin of the American Mathematical Society." but only the abstract seems to have been published:

Abstract: The following proposition holds in every lattice of normal subgroups of a group and, more generally, in every lattice of commuting equivalence relations: If a_0, a_1, a_2, x are any elements, $y = (a_1 + x)(a_2 + x)$, $b_0 = (a_0 + y)(a_1 + a_2)$ and cyclically, $c_0 = (a_1 + a_2) \cdot (b_1 + b_2)$ and cyclically, then $c_0 + c_1 + c_2 = c_1 + c_2$. Applied to the lattice of a projective space this condition implies a special case of Desargues Theorem, and is known to fail in certain projective planes. This solves Problem 27 of G. Birkhoff's *Lattice theory* (rev. ed., 1948). It also shows that a free modular lattice with four generators is not isomorphic to a lattice of normal subgroups or of commuting equivalence relations. (Received March 10, 1953.)

[Jóns1953b] ———, *On the representation of lattices*, Math. Scand. **1** (1953), 193–206, available at https://www.mscand.dk/article/view/10377. GS 5679706934102218841.

Abstract: The concept of a modular lattice arose from the study of normal subgroups of a group, and it has been shown that most known theorems on lattices of normal subgroups are actually valid for arbitrary modular lattices. It is therefore natural to ask whether every modular lattice is isomorphic to a lattice of normal subgroups of some group. As will be shown in Section 2 below, the answer to this question is negative. In Section 3 we obtain a different kind of representation applicable to arbitrary modular lattices. Modifying slightly the methods developed there we prove in Section 4 a somewhat stronger form of the fundamental representation theorem for arbitrary lattices.

[Jóns1954a] ———, *Modular lattices and Desargues' theorem*, Math. Scand. **2** (1954), 205–314, available at https://www.jstor.org/stable/24489042. GS 14025529922627479984.

Abstract: It has been proved that every complemented modular lattice is isomorphic to a sublattice of the lattice of all subspaces of a (possibly degenerate) projective space, but very little is known about the connection between the given lattice and the corresponding space. The representation theorem shows that every complemented modular lattice B can be embedded in a complete and atomistic modular lattice A, satisfying certain additional conditions. Our main results consist in describing up to isomorphism the connection between B and A, and in showing that every identity which holds in B is also valid in A. Since it is known the Desargues' Theorem is equivalent to a lattice-theoretic identity, we are thus able to state under what conditions on B the corresponding space is Arguesian. Inasmuch as every lattice of commuting equivalence relations satisfies the identity in question, we infer that, for a complemented modular lattice, the existence of a representation by commuting equivalence relations is equivalent to the existence of a representation by subspaces of an Arguesian projective plane. For arbitrary modular lattices this is no longer true. We construct a five dimensional modular lattice which is isomorphic to a lattice of commuting equivalence relations, but not to a lattice of normal subgroups of a group.

[Jóns1954b] ———, *Representation of lattices. I.*, AMS meeting (New York, October 1953), 1954, pp. 24, available at https://www.ams.org/journals/bull/1954-60-01/S0002-9904-1954-09753-7/S0002-9904-1954-09753-7.pdf.

Abstract: Whitman showed that every lattice L is isomorphic to a lattice L' of equivalence relations. If we regard binary relations as sets of ordered pairs, then multiplication and inclusion in the lattice L' coincide with set-theoretic intersection and inclusion while the lattice sum of two relations R and S in L' is the set-theoretic union of the nondecreasing infinite sequence of relative products $R;S,\ R;S;R,\ R;S;R;S,\ \cdots$. If for every R and S in L' this sequence is constant from the nth term on, then we speak of a representation of L of type n. *Results:* Every lattice L has a representation of type 3. In order for L to have a representation of type 2, it is necessary and sufficient that L be modular. Previously we have shown that there exist modular lattices which do not have a representation of type 1 (a representation by commuting equivalence relations), cf. [Jóns1953a].

[Jóns1954c] ———, *Representations of lattices. II. Preliminary report.*, AMS meeting (New York, October 1953), 1954, pp. 24, available at https://www.ams.org/journals/bull/1954-60-01/S0002-9904-1954-09753-7/S0002-9904-1954-09753-7.pdf.

Abstract: Consider the four classes (i) $\mathcal{E}$, (ii) $\mathcal{N}$, (iii) $\mathcal{A}$, (iv) $\mathcal{L}$, consisting, respectively, of all modular lattices which can be represented isomorphically by means of (i) commuting equivalence relations, (ii) normal subgroups of a group, (iii) subgroups of an Abelian group, (iv) subspaces of a (possibly degenerate) Desarguesian projective space. Let $\mathcal{C}$ be the class of all complemented modular lattices and $\mathcal{D}_n$ the class of all n-dimensional modular lattices. *Results:* 1. $\mathcal{E}\cap\mathcal{C} = \mathcal{N}\cap\mathcal{C} = \mathcal{A}\cap\mathcal{C} = \mathcal{L}\cap\mathcal{C}$. 2. $\mathcal{E}\cap\mathcal{D}_n = \mathcal{N}\cap\mathcal{D}_n = \mathcal{A}\cap\mathcal{D}_n = \mathcal{L}\cap\mathcal{D}_n$ for $n \leq 4$. 3. $\mathcal{E}\cap\mathcal{D}_5 \neq \mathcal{N}\cap\mathcal{D}_5$. 4. If $L \in \mathcal{E}$, then L satisfies the following condition (α): For any $a_0, a_1, a_2, b_0, b_1, b_2 \in L$ if $y = (a_0 + a1)\cdot(b_0 + b_1)[(a_0 + a_2)(b_0 + b_2) + (a_1 + a_2)(b_1 + b_2)]$, then $(a_0+b_0)(a_1+b_1)(a_2+b_2) \leq a_0(a_1+y)+b_0(b_1+y)$. 5. If $L \in \mathcal{C}$ or $L \in \mathcal{D}_n$ with $n \leq 4$, and if the condition (α) holds, then $L \in \mathcal{E}$. 6. Suppose $L \in \mathcal{C}$ is of order $k \geq 3$ (in the sense of von Neumann). Then L is isomorphic to the lattice of all principal right ideals of a regular ring if and only if (α) holds.

[Jóns1959a] ———, *Representation of modular lattices and of relation algebras*, Trans. Am. Math. Soc. **92** (1959), 449–464, DOI 10.1090/S0002-9947-1959-0108459-5, available at https://www.ams.org/journals/tran/1959-092-03/S0002-9947-1959-0108459-5/. GS 15760874009066419029.

Abstract: This paper gives an axiomatic characterization of the class of all those (modular) lattices which are isomorphic to lattices of commuting equivalence relations. As might be expected, this problem turns out to be closely related to the representation problem for relation algebras, and we are able to borrow some basic ideas from the work of R. Lyndon. It turns out to be convenient to consider first the class of all those algebras $\mathfrak{A} = (A, ;, \cdot, \breve{}, 1')$ which can be represented isomorphically by means of binary relations over some set U in such a way that the operations ;, $\cdot$, $\breve{}$ correspond to relation-theoretic multiplication, set-theoretic multiplication and relation-theoretic conversion [reverse], respectively, and $1'$ corresponds to the identity relation over U. This class of algebras is characterized in Theorem 1. In Theorem 2 this result is applied to lattices. The key observation here is that a lattice $\mathfrak{A} = (A, +, \cdot)$ with a zero element 0 is isomorphic to a lattice of commuting equivalence relations if and only if the algebra $\mathfrak{A} = (A, +, \cdot, \breve{}, 0)$, where $\breve{x} = x$ for every $x \in A$, can be represented isomorphically by means of binary relations in the manner discussed above with + and 0 taking the place of ; and $1'$.
We can give a more direct, although more involved, proof of Theorem 2. The added complications are due mostly to the fact that we are then unable to take advantage of the metamathematical results of Henkin and Tarski. The trouble is that we see no direct way of proving that the class of lattices under consideration coincides with the class of all subalgebras of algebras from an arithmetic class in the wider sense. The method employed here also has the advantage that it brings out the close connection with the representation problem for relation algebras, and in this context Theorem 1 is of independent interest. Theorems 3 and 4 concern the existence of what we call weak representations for relation algebras, and the paper concludes with a brief discussion of some open questions related to our work.

[Jóns1959b] ———, *Arguesian lattices of dimension* $n \leq 4$, Math. Scand. **7** (1959), 133–145, available at `https://www.jstor.org/stable/24489014`. GS 8601356801190697074.

Abstract: It is known ([Jóns1953b]) that every lattice of commuting equivalence relations is Arguesian, but it is still an open question whether, conversely, every Arguesian lattice is isomorphic to a lattice of commuting equivalent relations. The principal purpose of this note is to establish the converse statement for lattices of dimension $n \leq 4$. Actually we prove a stronger statement, namely that every Arguesian lattice of dimension $n \leq 4$ is isomorphic to a sublattice of the lattice of all subspaces of an Arguesian projective geometry of dimension $n-1$. Thus it follows that a lattice of dimension $n \leq 4$ is isomorphic to a lattice of commuting equivalence relations if and only if it is isomorphic to a sublattice of all subspaces of an Arguesian projective geometry. The corresponding statement for $n = 5$ is false; in fact it is known (cf. [Jóns1954a]) that there exists a five dimensional lattice which is isomorphic to a lattice of commuting equivalence relations but cannot be embedded in a complemented modular lattice.

[Jóns1959c] ———, *Lattice-theoretic approach to projective and affine geometry*, 1959, Berkeley, Calif., U.S., DOI 10.1016/S0049-237X(09)70028-X, available at `https://www.sciencedirect.com/science/article/abs/pii/S0049237X0970028X`. available at `https://archive.org/details/axiomaticmethodw0000inte` GS 2547232161835439821

The results that we are going to discuss are due to several authors. The earliest work along these lines was done by Menger in the late twenties. He was joined a few years later by von Neumann and Birkhoff. A large number of more recent contributions can be found in the papers listed in bibliography; we shall in particular make use of results due to Frink and Schüzenberger on projective geometry, and by Croisot, Maeda, Sasaki and Wilcox on affine geometry and its generalizations. The bibliography includes a number of papers that are not concerned directly with geometry, but in which at least some of the ideas and methods were suggested by the investigations of geometric lattices.

[Jóns1962] ______, *Algebraic extensions of relational systems*, Math. Scand. **11** (1962), 179–205, available at https://www.mscand.dk/article/download/10665/8686. https://www.jstor.org/stable/24489338 GS 4186087394924111335.

This paper is concerned with a class K of relational systems, subject to two conditions of a rather general nature. In particular, these conditions are satisfied if K is the class of all (commutative) fields. The notion of an algebraic extension of a system in K is introduced, as well as several other related notions, and a series of results are obtained that generalize many of the basic theorems concerning algebraic field extensions.

[Jóns1967] ______, *Algebras whose congruence lattices are distributive*, Math. Scand. **21** (1967), 110–121, available at https://www.mscand.dk/article/download/10850/8871. GS 16143520069002349616. Zbl 0167.28401

This note is concerned with equational classes of algebras subject to the condition that for all algebras in the class, the lattice of all congruence relations over the algebra is distributive. In Section 2 necessary and sufficient conditions are obtained in order for this condition to hold. This result is inspired by a theorem in Malcev which gives necessary and sufficient conditions in order for all the algebras in a class to have permutable congruence relations.

[Jóns1972] ______, *The class of Arguesian lattices is self-dual*, Algebra Univers. **2** (1972), 396, DOI 10.1007/BF02945054, available at https://link.springer.com/article/10.1007/BF02945054. GS 13761165232906172754.

Proves the theorem in the title.

[Józ1989] Tadeusz Józefiak, *Characters of projective represntaitons of symmetric groups*, Expo. Math. **7** (1989), 193–247. GS 16996539617204846252. MR1007885 Zbl 0693.20009

Abstract: Issai Schur was the first who observed that the study of relations between representations of a group and representations of a factor group leads to projective representations. At the beginning of the century he laid foundations of a general theory of projective representations of finite groups. His crowning achievement in this field is a long paper published in 1991 [Schur1911] in which he determined the characters of all irreducible projective representations of the symmetric and the alternating groups using the so-called Q-functions. . . . The central object of the article is the so-called representation group $\tilde{S}_n$ whose equivalence classes of negative linear representations are in 1–1 correspondence with equivalence classes of projective representations of the symmetric group S_n for $n > 3$.

[Kac1966] Mark Kac, *Can One Hear the Shape of a Drum?*, Am. Math. Mon. **73** (1966), 1–23, DOI 10.1080/00029890.1966.11970915, available at https://www.tandfonline.com/doi/pdf/10.1080/00029890.1966.11970915. GS 838428721989267130. Zbl 0139.05603

This article poses the interesting question whether the spectrum of overtones of an idealized drum determine uniquely the shape of the drum, that is, whether the spectrum of eigenvalues of the Laplacian on a bounded region of the plane determine the shape of the region. In addition, it discusses how this problem connects with many areas of mathematical physics. Also, see https://en.wikipedia.org/wiki/Hearing_the_shape_of_a_drum for the further history and solution of this question.

[KacRotSchwartz1996] Mark Kac, Gian-Carlo Rota, and Jacob T. Schwartz. *Discrete Thoughts: Essays on Mathematics, Science, and Philosophy*. Edited by Peter Renz. Springer, Boston, 2nd ed., 1992, ISBN 9780817647759. GS 12755158262443375681.

Abstract: Sometime, in a future that is knocking at our door, we shall have to retrain ourselves or our children to properly tell the truth. The exercise will be particularly painful in mathematics. The enrapturing discoveries of our field systematically conceal, like footprints erased in the sand, the analogical train of thought that is the authentic life of mathematics. Shocking as it may be to a conservative logician, the day will come when currently vague concepts such as motivation and purpose will be made formal and accepted as constituents of a revamped logic, where they will at last be allotted the equal status they deserve, side- by-side with axioms and theorems. Until that day, however, the truths of mathematics will make only fleeting appearances, like shameful confessions whispered to a priest, to a psychiatrist, or to a wife.

[Kar2024] Eddie Karat, *The algebra of subgraph counting functions.*

Abstract: In a loose analogy to physics, where we can introduce an auxiliary field to simplify the equations of motion, we introduce an additional edge color to graphs, as well as require the analogue of a Feynman rule or Skein relation to remove it. As such, we consider graphs with an edge coloring and define a counting function of those graphs. These counting functions are related to the number of induced subgraphs isomorphic to a test graph, but our counting functions have a simpler decomposition of disconnected graphs. The resulting algebra allows us to determine the number of independent counting functions and relate those that are not independent. An explicit construction of these counting functions allows us to calculate the coefficients of those relationships directly, rather than having to sample random graphs. By working with counting functions instead of entire graphs, calculations in Ramsey Theory can require less computation and can eliminate whole sets of graphs at once rather than iterating over every single subgraph. Perhaps more importantly, we only need to know certain properties of subgraphs, rather than the structure of every possible subgraph in order to eliminate candidates for graphs in Ramsey Theory.

[Kar2024a-] ———, *The algebra of subgraph counting functions*, https://docs.google.com/document/d/1jdjvKRh2FoqIgu3JY8lH0GO5L6d_rzoMFNeCo5vEHnc. Accessed July 10, 2024.

[Kearn2023] Keith A. Kearnes, *The projective structure of a non-distributive modular lattice [answer]*, https://mathoverflow.net/questions/455850/the-projective-structure-of-a-non-distributive-modular-lattice. Accessed October 15, 2023.

Abstract: **Theorem.** If $P = [0; s, t, u; 1]$ is a projective root in the modular lattice L with $\dim(s/0) = k > 1$, then L contains projective roots $P_0 = [0; s_0, t_0, u_0; \Omega]$ with $\dim(s_0/0) = 1$ and $P^+ + = [\Omega; s^+, t^+, u^+; 1]$ with $\dim(s^+/\Omega) = k - 1$.
This produces a first prime factor P_0 of P, and you may apply the theorem to P^+ to obtain further prime factors.

[KearnNat2008] Keith A. Kearnes and James Bryant Nation, *Axiomatizable and Nonaxiomatizable Congruence Prevarieties*, Algebra Univers. **59** (2008), 323–335, DOI 10.1007/s00012-008-2068-y, available at `https://math.colorado.edu/~kearnes/Papers/nonaxfin.pdf`. GS 12246567188800420965. Zbl 1165.08002

Abstract: If $\mathcal{V}$ is a variety of algebras, let $L(\mathcal{V})$ denote the prevariety of all lattices embeddable in congruence lattices of algebras in $\mathcal{V}$. We give some criteria for the first-order axiomatizability or nonaxiomatizability of $L(\mathcal{V})$. One corollary to our results is a nonconstructive proof that every congruence n-permutable variety satisfies a nontrivial congruence identity.

[KhukhMaz2024] E. I. Khukhro and V. D. Mazurov, *Unsolved Problems in Group Theory. The Kourovka Notebook*, arXiv 1401.0300, primary class math.GR, DOI 10.48550/arXiv.1401.0300, available at `https://arxiv.org/abs/1401.0300`. GS 6120418083779391558

This is a collection of open problems in group theory proposed by hundreds of mathematicians from all over the world. It has been published every 2–4 years since 1965. This is the 20th edition, which contains 126 new problems and a number of comments on problems from the previous editions.

[KingElShar1983] Ronald C. King and Nahid G. I. El-Sharkaway, *Standard Young tableaux and weight multiplicities of the classical Lie groups*, J. Phys. A **16** (1983), 3153–3177, DOI 10.1088/0305-4470/16/14/012, available at `https://iopscience.iop.org/article/10.1088/0305-4470/16/14/012/meta`. GS 12754101850548728238. Zbl 0522.22015

Abstract: By examining the branching rules for all irreducible representations of the classical groups U(k), SU(k), SO(2k+1), Sp($2k$) and SO($2k$) on restriction to $U(1) * U(1) * U(1)$, standard Young tableaux are specified for each of these groups. It is shown that these tableaux determine the corresponding characters of the irreducible representations. The rules for constructing these tableaux are derived and in this way the determination of weight multiplicities is reduced to a simple combinatorial exercise. General formula for such weight multiplicities are given encompassing the most difficult case: namely that of SO($2k$). Illustrative examples are provided, including some yielding the explicit k-dependence of weight multiplicities.

[KissPál1998] E. W. Kiss and Péter P. Pálfy, *A lattice of normal subgroups that is not embeddable into the subgroup lattice of an Abelian group*, Math. Scand. **83** (1998), 169–176, available at `https://www.jstor.org/stable/24493130?seq=1`. GS 9489003440967396754.

Abstract: In this paper we give a negative solution to the following problem of Bjarni Jónsson:
Is the lattice of normal subgroups of every group embeddable into the subgroup lattice of an abelian group?

[Knuth1970] Donald E. Knuth, *Permutations, matrices, and generalized Young tableaux*, Pacific J. Math. **34** (1970), 709–727, DOI 10.2140/PJM.1970.34.709, available at `https://msp.org/pjm/1970/34-3/pjm-v34-n3-p09-s.pdf`. GS 12777966864690181605. MR42 #7535

U. Topics: insertion algorithm, Knuth equivalence, s_λ. Introduces Knuth equivalence.

[Knuth1986] ______. *The TEXbook*. Addison–Wesley, Reading, Mass., U.S., 1986.

Abstract: Gentle Reader: This is a handbook about TEX, a new typesetting system intended for the creation of beautiful books—and especially for books that contain a lot of mathematics. By preparing a manuscript in TEXformat, you will be telling a computer exactly how the manuscript is to be transformed into pages whose typographic quality is comparable to that of the world's finest printers; yet you won't need to do much more work than would be involved

if you were simply typing the manuscript on an ordinary typewriter. In fact, your total work will probably be significantly less, if you consider the time it ordinarily takes to revise a typewritten manuscript, since computer text files are so easy to change and to reprocess. (If such claims sound too good to be true, keep in mind that they were made by TEX's designer, on a day when TEXhappened to be working, so the statements may be biased; but read on anyway.)
This manual is intended for people who have never used TEXbefore, as well as for experienced TEXhackers. In other words, it's supposed to be a panacea that satisfies everybody, at the risk of satisfying nobody. Everything you need to know about TEXis explained here somewhere, and so are a lot of things that most users don't care about.

[Knuth1998] ———. *The Art of Computer Programming, Sorting and Searching*, Vol. 3. Addison-Wesley, Upper Saddle River, NJ, 2nd ed., 1998. original edition 1973.

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[KosMur1993] Masashi Kosuda and Jun Murakami, *Centralizer algebras of the mixed tensor representations of quantum group* $U_q(gl(n,\mathbb{C}))^*$, Osaka J. Math. **30** (1993), 475–507, available at `https://projecteuclid.org/journals/osaka-journal-of-mathematics/volume-30/issue-3/Centralizer-algebras-of-the-mixed-tensor-representations-of-quantum-group/ojm/1200784541.full`. GS 1794168069097758359. Zbl 0806.17012

Abstract: In the present paper we introduce a generalization $H^n_{N,M}(q)$ of the Iwahori Hecke algebra of type A, which is defined by generators and relations. Our main result says that the algebra $H^n_{N,M}(q)$ is isomorphic to the centralizer algebra $C_n^{(N,M)}(q)$ of the natural representation of $U_q(gl(n,\mathbb{C}))$ on $V_n^{(N,M)}$, if $n \geq N + M$ and q is generic.

[KungRotYan2009] Joseph P. S. Kung, Gian-Carlo Rota, and Catherine Huafei Yan. *Combinatorics the Rota way.* Cambridge Univ. Press, Cambridge, 2009. errata at https://people.tamu.edu/ huafei-yan/Files/ERRATA.pdf GS 4621067828615552296.

Abstract: It is difficult to convey the experience of a Rota lecture. Rota once said that the secret to successful teaching is to reveal the material so that at the end, the idea — and there should be only one per lecture — is obvious, ready for the audience to "take home." We must confess that we have failed to pull this off in this book. The immediacy of a lecture cannot (and should not) be frozen in the textuality of a book. Instead, we have tried to convey the method behind Rota's research. Although he would object to it being stated in such stark simplistic terms, mathematical research is not about *solving* problems; it is about *finding* the right problems. One way of finding the right problems is to look for ideas common to subjects, ranging from, say, category theory to statistics. What is shared may be the implicit algebraic structures that hide behind the technicalities, in which case finding the structure is part of "applied universal algebra." The famous paper *Foundations I*, which revealed the role of partially ordered sets in combinatorics, is a product of this point of view. To convey Rota's thinking, which involves all of mathematics, one must go against an *ideé re cue* of textbook writing: the prerequisites for this book are, in a sense, all mathematics. However, it is the ideas, not the technical details, that matter. Thus, in a different sense, there are no prerequisites to this book: we intend that a minimum of technical knowledge is needed to seriously appreciate the text of this book. Those parts where special technical knowledge is needed, usually in the exercises, can be skimmed over.

[KungYan2003] Joseph P. S. Kung and Catherine Huafei Yan, *Six problems of Gian-Carlo Rota in lattice theory and universal algebra*, Algebra Univers. **49** (2003), 113–127, DOI 10.1007/s00012-003-1810-8, available at `https://people.tamu.edu/~huafei-yan/Files/AU01.pdf`. GS 9474364366211746008.

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[Lam2008] Thomas F. Lam, *Signed differential posets and sign-imbalance*, J. Comb. Theory **Ser. A 115** (2008), 466–484, DOI 10.1016/j.jcta.2007.07.003, available at `https://www.sciencedirect.com/science/article/pii/S0097316507000957`. GS 7312628064545996659.

An example of a generalized differential poset where the weights on the edges of G_2 are ± 1, and thus cannot be represented by multiple edges.

[Lamp1994] William A. Lampe, *A perspective on algebraic representations of lattices*, Algebra Univers. **31** (1994), 337–364, DOI 10.1007/BF01221791, available at https://link.springer.com/article/10.1007/BF01221791. GS 10030817407850163469.

Abstract: We survey results on algebraic representations of lattices and related topics. This survey does not attempt to be completely comprehensive. Topics emphasized are often ones Bjarni J6nsson had influence on.
Throughout the paper we will mention some open problems. Most are fairly well known and mainly posed by others.

[LasSchüt1978] A. Lascoux and Marcel-Paul Schützenberger, *Monoïde plaxique [The plactic monoid]*, Colloque de Naples 1978, Quaderni de la Ricerca, vol. 109, 1978, available at http://igm.univ-mlv.fr/~berstel/Mps/Travaux/A/1981-1PlaxiqueNaples.pdf.

check up on the dates, etc. ***

[Lee1994] Jaejin Lee, *A Schensted algorithm for shifted rim hook tableaux*, J. Korean Math. Soc. **31** (1994), 179–203, available at https://jkms.kms.or.kr/journal/view.html?spage=179&volume=31&number=2.
GS 4153503936892197281. Zbl 0803.05055

Abstract: In this paper we give the Schensted algorithm for shifted rim hook tableaux. If k is a fixed odd positive integer it shows a one-to-one correspondence between all pairs (P, Q), where P is a shifted (first tail circled) k-rim hook tableau of shape λ and content k^m and Q is a circled shifted k-rim hook tableau of the same shape λ and content k^m, and all circled hook permutations of content k^m. In particular, if all the rim hooks of P were of size one and $|\omega| = 1$ then this algorithm reduces to the Schensted algorithm for ordinary shifted tableaux given by Sagan [Sag1987].

[Lee2016] ______, *Generalization of the Schensted algorithm for rim hook tableaux*, Korean J. Math. **24** (2016), 469–487, DOI 10.11568/kjm.2016.24.3.469, available at https://kkms.org/index.php/kjm/article/view/452/0. GS 15998726899040970813. Zbl 1432.05121

Abstract: In [Schen1961] Schensted constructed the Schensted algorithm, which gives a bijection between permutations and pairs of standard tableaux of the same shape. Stanton and White [StantWhite1985] gave analog of the Schensted algorithm for rim hook tableaux. In this paper we give a generalization of Stanton and White's Schensted algorithm for rim hook tableaux. If k is a fixed positive integer, it shows a one-to-one correspondence between all generalized hook permutations H of size k and all pairs (P, Q), where P and Q are semistandard k-rim hook tableaux and k-rim hook tableaux of the same shape, respectively.

[Len201a] Christian Lenart, *Combinatorial representation theory of Lie algebras. Richard Stanley's work and the way it was continued* (2014), arXiv 1406.0352, primary class math.CO, DOI 10.48550/arXiv.1406.0352, available at `https://arxiv.org/abs/1406.0352`. GS 1551376052959506527

Abstract: Richard Stanley played a crucial role, through his work and his students, in the development of the relatively new area known as combinatorial representation theory. In the early stages, he has the merit to have pointed out to combinatorialists the potential that representation theory has for applications of combinatorial methods. Throughout his distinguished career, he wrote significant articles which touch upon various combinatorial aspects related to representation theory (of Lie algebras, the symmetric group, etc.). I describe some of Richard's contributions involving Lie algebras, as well as recent developments inspired by them (including some open problems), which attest the lasting impact of his work.

[Lent2025] John Lentfer, *Diagonal Supersymmetry for Coinvariant Rings* (2025), arXiv 2505.14885, primary class math.CO, DOI 10.48550/arXiv.2505.14885, available at `https://arxiv.org/abs/2505.14885`. GS 16901073960484310834

Abstract: For finite groups G, we show that bosonic-fermionic coinvariant rings have a natural $U(\mathfrak{gl}(k|j)) \otimes C[\mathbb{C}]$-module structure. In particular, we show that their character series are a sum of super Schur functions $s_\lambda(\mathbf{q}/\mathbf{u})$ times irreducible characters of G with universal coefficients, which do not depend on k, j. In the case where G is the symmetric group with diagonal action, this proves the "Diagonal Supersymmetry" conjecture of Bergeron (2020).

[Lew2007] Joel Lewis, *On differential posets*, 2007, available at https://bpb-us-e1.wpmucdn.com/blogs.gwu.edu/dist/5/693/files/2018/04/JBLHarvardSeniorThesis-20drube.pdf. GS 17935962314847284809.

Abstract: We study differential posets, a family of partially ordered sets discovered by Richard Stanley. In the first half of this paper we present an introduction to poset theory as relevant to differential posets and theorems on the structure and combinatorial properties of differential posets, culminating in the explicit definition of a new differential poset. In the second half we focus on Young's lattice, the most well-studied differential poset, and the RSK algorithm.

[LiangSag2024] Jinting Liang and Bruce E. Sagan, *Log-concavity and log-convexity via distributive lattices* (2024), arXiv 2408.02782, primary class math.CO, DOI 10.48550/arXiv.2408.02782, available at https://arxiv.org/abs/2408.02782. GS 11931222555046424532

Abstract: We prove a lemma, which we call the Order Ideal Lemma, that can be used to demonstrate a wide array of log-concavity and log-convexity results in a combinatorial manner using order ideals in distributive lattices. We use the Order Ideal Lemma to prove log-concavity and log-convexity of various sequences involving lattice paths (Catalan, Motzkin and large Schröder numbers), intervals in Young's lattice, order polynomials, specializations of Schur and Schur Q-functions, Lucas sequences, descent and peak polynomials of permutations, pattern avoidance, set partitions, and noncrossing partitions. We end with a section with conjectures and outlining future directions.

[Lit2005] G. L. Litvinov, *The Maslov dequantization, idempotent and tropical mathematics: A brief introduction* (2005), arXiv 0507014, primary class math.GM, DOI 10.48550/arXiv.math/0507014, available at https://arxiv.org/abs/math/0507014. GS 13464044851046173

Abstract: This paper is a brief introduction to idempotent and tropical mathematics. Tropical mathematics can be treated as a result of the so-called Maslov dequantization of the traditional mathematics over numerical fields as the Planck constant $\hbar$ tends to zero taking imaginary values.

[Lit2007] ———, *The Maslov dequantization, idempotent and tropical mathematics: A brief introduction* (2007), DOI 10.1007/s10958-007-0450-5, available at `https://link.springer.com/article/10.1007/s10958-007-0450-5`. GS 13464044851046173

Abstract: This paper is a brief introduction to idempotent and tropical mathematics. Tropical mathematics can be treated as a result of the so-called Maslov dequantization of the traditional mathematics over numerical fields as the Planck constant $\hbar$ tends to zero taking imaginary values. Bibliography: 187 titles.

[LittBoll1986] John Edensor Littlewood and Béla Bollobás. *Littlewood's miscellany.* Cambridge Univ. Press, Cambridge, 1986. GS 913929921465718680.

Abstract: In 1951 the Mathematics Faculty in Cambridge asked J. E. Littlewood to give a talk, of about forty minutes, at the first of a series of 'social evenings'. A little later, the 'Archimedeans' — the Cambridge undergraduate mathematical society — invited him to address them. These two talks are the origins of a collection of essays for the general public which were published in 1953 as *A Mathematician's Miscellany.*

[Liu2025] Jasper M. Liu, Yichen Ma, Brendon Rhoades, and Hai Zhu, *Matrix loci, orbit harmonics, and shadow play*, Proceedings of the 36th International Conference on Formal Power Series and Algebraic Combinatorics (Ruhr-Universität Bochum, Germany, 2025), 2025, pp. art. 93B.126, available at `https://www.mat.univie.ac.at/~slc/wpapers/FPSAC2025/126.html`. GS 10410263644275062518. MR4974309

Abstract: Let $\mathbf{x}_{x\times n}$ be an $n\times n$ matrix of variables and let $\mathbb{C}[\mathbf{x}_{x\times n}]$ be the polynomial ring in these variables. We consider the ideal $I_n \subset \mathbb{C}[\mathbf{x}_{x\times n}]$ generated by all row sums, column sums, and products of variables in the same row or column. We prove $R_n = \mathbb{C}[\mathbf{x}_{x\times n}]/I_n$ has standard monomial theory governed by the Viennot shadow line avatar of the Schensted correspondence and has Hilbert series given by the longest increasing subsequence distribution on permutations (up to reversal). The ring R_n coincides with the orbit harmonics quotient ring attached to the permutation matrix locus in the space $\mathrm{Mat}_{n\times n}(\mathbb{C})$ of $n\times n$ complex matrices. With R_n as motivation, we prove results on orbit harmonics quotients for other matrix loci.

[Lob1825] Nikolai I. Lobachevsky, *Analytic and algebraic topology of locally Euclidean metrizations of infinitely differentiable Riemannian manifolds*, 1825.

Bozhe moy! This I know from nothing.

[LokWor2014] Daniel Lokshtanov and Dale R. Worley, *Does a universal index exist?*, `https://cstheory.stackexchange.com/questions/25118/does-a-universal-index-exist`. Accessed December 14, 2024.

Given a data table with k fields and N rows, there are well-known algorithms/data structures for efficiently stepping through the table when it is logically lexicographically ordered acording to a given "key", that is, a permutation of the k fields. Is there an algorithm/data structure which accommodates efficient stepping by any of the $k!$ permutations of the fields adding only a factor of $k \log k$ to the operation times? There is not.

[Luž2019] Borut Lužar, *IPE Tutorial (for version 7.2.12) [The Extensible Drawing Editor by Otfried Cheong]*, 2019, Košice, Slovakia, DOI 10.13140/RG.2.2.15238.93762, available at `https://www.researchgate.net/publication/335813672_IPE_Tutorial_for_version_7212_The_Extensible_Drawing_Editor_by_Otfried_Cheong`

A tutorial for the IPE drawing editor.

[Macdon1979] I. G. Macdonald. *Symmetric Functions and Hall Polynomials.* Oxford, 2nd ed., 1979, 1995. GS 14803784844471672227.

Reference on symmetric functions and their relationship to tableaux. A detailed review and summary of the 1979 edition is in [Stan1981].

[MacHal1983] Desmond MacHale, *Any Questions?*, Am. Math. Mon. **90** (1983), 42–43, DOI 10.2307/2975693, available at `https://www.jstor.org/stable/2975693`.

The classic reference of questions to ask at the end of a seminar when the speaker has totally lost everybody in the audience.

[MacHal1993] ______. *Comic Sections; The Book of Mathematical Jokes, Humour, Wit and Wisdom*. Boole Press, Dublin, 1983. GS 1041552240539297260.

The big book of mathematics humor, including "Any Questions?" [MacHal1983].

[Macl1978] Saunders Mac Lane. *Categories for the working mathematician*. Graduate Texts in Mathematics, vol. 5. Springer, New York, 2nd ed., 1978. GS 1012099057611865473.

Abstract: Category theory has developed rapidly. This book aims to present those ideas and methods that can now be effectively used by mathematicians working in a variety of other fields of mathematical research. This occurs at several levels.

[Malk1959] Willem V. R. Malkus, *Turbulent transport*, International Oceanographic Cong. (New York, 1959), Oceanography; Invited lectures presented at the International Oceanographic Congress, Vol. 67, AAAS, Washington, 1961.

Abstract: At the Woods Hole Oceanographic Institution I seek to isolate the simplest experimentally realizable types of turbulent transport which can be studied theoretically. In these controlled experiments one can hope to establish the range of validity of quantitative hypotheses concerning the field of motion. In this paper I shall (1) explore the possibility of isolating those turbulent processes in the sea which one might hope to treat theoretically, (2) explore some of the continuity relations which must exist between the separated turbulent processes, and (3) show that the more detailed mechanistic studies of the last decade support the inferred relations. In particular, I wish to paint an idealized picture of the North Atlantic circulation suggesting the various turbulent mechanisms which control the flow and to predict certain integrals of mass, heat, and momentum transport which one can compare both with observation and other theories.

[Malk1979] ______, *The amplitude of convection*, 1979, Falmouth, Mass., U.S.

Abstract: As the ubiquitous source of motion, both astrophysical and geophysical, convection has attracted theoretical attention since the last century. In the ocean, many different scales are called convection; from the deep circulation due to seasonal production of Arctic Bottom Water to the mixing by salt fingers. In the atmosphere, convection dominates the flow from sub-cloud layers to Hadley "cells". It is proposed that convection in the earth's core powers the geomagnetic field. The non-periodic reversals of that field, captured in the rock, define the evolution of the ocean basin. recognition that this latter process is caused by convection in the mantle has produced a new geophysics.
In the past, understanding the central features of convection has come from the isolation of "simplest" mechanistic examples. Although large scale geophysical convection never quite provides the idealized simplest problem, these examples (e.g., Lord Rayleigh's study of the Benard cells) have generated much of the formal language of inquiry used in the field. In the past it has served the dynamic oceanographer to follow developments in understanding of these bits of the overall geophysics.

[Mark1973] George Markowsky, *Some combinatorial aspects of lattice theory*, Proc. Univ. of Houston Lattice Theory Conf. (Houston, 1973), available at https://www.math.uh.edu/~hjm/1973_Lattice/p00036-p00068.pdf. GS 10997219563151152692.

After a few technical preliminaries we will discuss a basic representation theorem for lattices and give some applications of it, including a new characterization of distributive lattices and some combinatorial results having to do with the representation of lattices and posets by subsets of the power set of some given set. In Part II, we introduce the poset of join-irreducible and meet-irreducible elements of a lattice, a construction which bears the same relationship to the given lattice, as the poset of join-irreducible elements bears to the corresponding finite distributive lattice. After describing the properties of the poset of join-irreducible and

meet-irreducible elements, we will give some applications of this construction, including the extension of the work of Crapo and Rota 17] on the factorization of relatively complemented lattices of finite length to all lattices of finite length. We will then discuss the enumeration of the elements of the free distributive lattice on n generators, a problem first proposed by Dedekind [8] in 1897.

[Mark2001] ———, *An overview of the poset of irreducibles*, Combinatorial and Computational Mathematics, Present and Future (Pohang, Rep. of Korea, 2000), World Scientific, Singapore, 2001. `https://www.worldscientific.com/worldscibooks/10.1142/4749`, pp. 163–176, DOI 10.1142/4749, available at `http://aturing.umcs.maine.edu/~markov/posetirreducibles.pdf`.
GS 5344078877505210969.

An interesting fact that is of great practical importance is that finite lattices have an associate poset, called the poset of irreducibles that acts much like the basis of a vector space. The poset of irreducibles of a finite lattice provides a compact representation of the lattice from which many of the properties of the lattice can be deduced easily. This paper is dedicated to explaining the poset of irreducibles and providing some examples of its usefulness.

[McDon1995] John McDonald, *Fiber polytopes and fractional power series*, J. Pure Appl. Algebra **104** (1995), 213–233, DOI 10.1016/0022-4049(94)00129-5, available at `https://www.sciencedirect.com/science/article/pii/0022404994001295`.
GS 12956251035289821910. Zbl 0842.52009

Abstract: This paper explores the power series expansions of polynomial equations in N variables. Expansions considered have exponents lying in some convex conical region in $\mathbb{R}^N$. An N-variable analog of the Newton polygon construction for polynomials in two variables is used to construct such series expansions. The structure of these series is related to the theory of fiber polytopes as introduced by Billera and Sturmfels in [2], and this relationship is used to draw conclusions about certain ramification loci.

[McElStan1993] Robert J. McElice and Richard P. Stanley, *The general theory of convolutional codes*, 1993, available at https://math.mit.edu/~rstan/pubs/pubfiles/97.pdf. GS 11314049963758753688.

Abstract: In this paper, we present a self-contained introduction to the algebraic theory of convolutional codes, which is partly tutorial, but at the some time contains a number of new results which we believe will prove useful for designers of advanced telecommunication systems. Among the new concepts we introduce here are the Hilbert series for a convolutional code, and the class of compact codes.

[McLar1986] Timothy J. McLarnan, *Tableau Recursions and Symmetric Schensted Correspondences for Ordinary, Shifted and Oscillating Tableaux*, Ph.D. thesis, U. C. San Diego, 1986. GS 6563465974933598796.

U, *S*, *UD*. Topics: Generic insertion algorithms. Derives insertion algorithms based on recursive enumeration formulas for tableaux. Anticipates [Fom1995a]. Partly reprinted in [GarMcLar1987].

[MerSmir2015] Grigory Merzon and Evgeny Smirnov, *Determinantal identities for flagged Schur and Schubert polynomials*, Eur. J. Math. **2** (2015), 227–245, DOI 10.1007/s40879-015-0078-9, available at https://link.springer.com/article/10.1007/s40879-015-0078-9. GS 3630810764908776375. Zbl 1331.05223

Abstract: We prove new determinantal identities for a family of flagged Schur polynomials. As a corollary of these identities we obtain determinantal expressions of Schubert polynomials for certain vexillary permutations.

[MilStash2022] John W. Milnor and James D. Stasheff. *Characteristic classes.* Ann. Math. Studies, vol. 76. TEXromancers, 2022. GS 3270252869678688816.

Abstract: The theory of characteristic classes provides a meeting ground for the various disciplines of differential topology, differential and algebraic geometry, cohomology, and fiber bundle theory. As such, it is a fundamental and an essential tool in the study of differentiable manifolds.

In this volume, the authors provide a thorough introduction to characteristic classes, with detailed studies of Stiefel-Whitney classes, Chern classes, Pontrjagin classes, and the Euler class. Three appendices cover the basics of cohomology theory and the differential forms approach to characteristic classes, and provide an account of Bernoulli numbers.
Based on lecture notes of John Milnor, which first appeared at Princeton University in 1957 and have been widely studied by graduate students of topology ever since, this published version has been completely revised and corrected.

[Miy1966] Yōichi Miyashita, *Quasi-projective modules, perfect modules, and a theorem for modular lattices*, Hokkaido Math. J. **19** (1966), 86–110, available at `https://eprints.lib.hokudai.ac.jp/dspace/bitstream/2115/56072/1/JFSHIU_19_N2_086-110.pdf`. GS 4148440594279391123. Zbl 0142.27904

[Mont2014] Marie-José Montpetit, *A short network coding primer*, 2014

[Mooers1959] Calvin N. Mooers, *The Next Twenty Years in Information Retrieval: Some Goals and Predictions*, Proceedings of the Western Joint Computer Conference (San Francisco, 1959), pp. 81–86, DOI 10.1145/1457838.1457853, available at `https://dl.acm.org/doi/pdf/10.1145/1457838.1457853`. GS 6065334782042092209.

Classic paper on the general problem of information retrieval. Generally considered to predict the development of the Web, but that is secondary in the paper to the problem of helping users find the particular information they need in massive archives of unstructured information.

[Moor2017] G. Eric Moorhouse, *Projective planes of small order*, `https://ericmoorhouse.org/pub/planes/`. Accessed January 23, 2024.

Abstract: This site is intended to provide a current list of known projective planes of small order. ... The completeness of this list is known only for planes of order n at most 10 [C.W.H. Lam, G. Kolesova and L. Thiel (1988); C.W.H. Lam, L. Thiel and S. Swiercz (1988)]. There is also a substantial literature classifying (or showing nonexistence of) planes of certain small orders (such as 11, 12, 15) admitting automorphisms of certain orders, or containing certain embedded configurations.

[Mor1962] Alun Owen Morris, *On Q-functions*, J. Lond. Math. Soc. **s1-37** (1962), 445–455, DOI 10.1112/jlms/s1-37.1.445, available at https://academic.oup.com/jlms/article-abstract/s1-37/1/445/805303. GS 17650943999131566166. Zbl 0112.02302

Abstract: In this paper, a certain class of symmetric functions, which we call Q-functions, which were first introduced by Schur [Schur1911] are discussed. A relation exists between Q-functions and the spin characters of the spin group Γ_n of the symmetric group $\mathfrak{S}_n$ similar to that between S-functions and the ordinary characters of $\mathfrak{S}_n$ [1, 2, 3].
We prove a few properties of Q-functions. In the first place, a method for the multiplication of Q-functions is given, which is similar to the method given by Murnaghan [5] for the multiplication of S-functions. In [2] a recurrence relation giving the spin characters of the classes containing a cycle of order s in the spin group Γ_{n+s} provided the spin characters of Γ_n are known was stated. This result is proved here. Finally, a method is given for writing Q-functions as a sum of S-functions.

[MoraPakPan2019] Alejandro H. Morales, Igor Pak, and Greta Panova, *Asymptotics of principal evaluations of Schubert polynomials for layered permutations*, Proc. Am. Math. Soc. **147** (2019), 1377–1389, DOI 10.1090/proc/14369, available at https://www.ams.org/journals/proc/2019-147-04/S0002-9939-2019-14369-0/. GS 16730424172716953585. Zbl 1405.05003

Abstract: Denote by $u(n)$ the largest principal specialization of the Schubert polynomial $u(n) := \max_{w\in\mathfrak{S}_w}(1,\ldots,1)$. Stanley conjectured that there is a limit $\lim_{n\to\infty}\frac{1}{n^2}\log u(n)$ and asked for a limiting description of permutations achieving the maximum $u(n)$. Merzon and Smirnov conjectured in [MerSmir2015] that this maximum is achieved on layered permutations. We resolve both of Stanley's problems restricted to layered permutations.

[MoraPanPet2025] Alejandro H. Morales, Greta Panova, Leonid Petrov, and Damir Yeliussizov, *Grothendieck Shenanigans: Permutons from pipe dreams via integrable probability* (2025), arXiv 2407.21653, primary class math.PR, DOI 10.48550/arXiv.2407.21653, available at https://arxiv.org/abs/2407.21653. GS 3813051572599441845

Abstract: We study random permutations arising from reduced pipe dreams. Our main model is motivated by Grothendieck polynomials with parameter $\beta = 1$ arising in K-theory of the flag variety. The probability weight of a permutation is proportional to the principal specialization (setting all variables to 1) of the corresponding Grothendieck polynomial. By mapping this random permutation to a version of TASEP (Totally Asymmetric Simple Exclusion Process), we describe the limiting permuton and fluctuations around it as the order n of the permutation grows to infinity. The fluctuations are of order $n^{\frac{1}{3}}$ and have the Tracy–Widom GUE distribution, which places this algebraic (K-theoretic) model into the Kardar–Parisi–Zhang universality class. We also investigate non-reduced pipe dreams and make progress on a recent open problem on the asymptotic number of inversions of the resulting permutation. Inspired by Stanley's question for the maximal value of principal specializations of Schubert polynomials, we resolve the analogous question for $\beta = 1$ Grothendieck polynomials, and provide bounds for general β.

[MorJon2003] Alun Owen Morris and Huw I. Jones, *Projective Representations of Generalized Symmetric Groups*, Séminaire Lotharingien de Combinatoire **50** (2003), art. B50b, available at `https://www.emis.de/journals/SLC/wpapers/s50morris.html`. GS 15343604290019433082. Zbl 1068.20015

Abstract: The representation theory of generalized symmetric groups has been of interest over a long period dating back to the classical work of W. Specht [28,29] and M.Osima - an exposition of this work and other references may be found in [12]. Furthermore, the projective representations of these groups have been considered by a number of authors, much of the this work was not published or was published in journals not readily accessible in the western world. The first comprehensive work on the projective representations of the generalized symmetric groups was due to E. W. Read [24] which was followed later by an improvement in the work of M. Saeed-ul-Islam, see, for example, [26]. Of equal interest has been the representation theory of the hyperoctahedral groups, which are a special case of the generalized symmetric groups. The projective representations of these groups was

considered by M. Munir in his thesis [20] which elaborated on the earlier work of E. W. Read and M. Saeed-ul-Islam and also by J. Stembridge [31] who gave an independent development which was more complete and satisfactory in many respects. This approach later influenced that used by H. I. Jones in his thesis [13] where the use of Clifford algebras was emphasized.
More recently, the generalized symmetric groups have become far more predominant in the context of complex reflection groups and the corresponding cyclotomic Hecke algebras where they and their subgroups form the infinite family $G(m, p, n)$, see for example [3], [4] and [5]. In view of this interest, it was thought worthwhile to present this work which is based on the earlier work of H. I. Jones which has not been published. As this article is also meant to be partially expository, a great deal of the background material is also presented.

[Most1969] Andrzej Mostowski, *Review of A. I. Mal'cév. Modél'nyé [sic] sootvéctviá (Model correspondences)*, J. Symb. Log. **34** (1969), 299–300, DOI 10.2307/2271119, available at https://www.cambridge.org/core/journals/journal-of-symbolic-logic/article/abs/i-malcev-modelnye-sootvectvia-model-correspondences-izvestia-akademii-nauk-sssr-seria-matematiceskaa-vol-23-1959-pp-313336/61FFCB7C58624B7058DFC922D1B6F756. GS 5586338310459870669.

Terse introduction to Mal'cev's concept of axiomatic correspondence.

[MosWy1955] Leo Moser and Max Wyman, *On solutions of $x^d = 1$ in symmetric groups*, Can. J. Math. **7** (1955), 159–168, DOI 10.4153/CJM-1955-021-8, available at https://www.cambridge.org/core/journals/canadian-journal-of-mathematics/article/on-solutions-of-xd-1-in-symmetric-groups/8F90642D9472FA7326164E54BE3BE57B. GS 10909011387343360213.

Analysis of the number of solutions of $x^d = 1$ in S_n, including asymptotic analysis.

[Munk2014] James Munkres. *Topology*. Pearson, Harlow, Essex, U. K., 2, Pearson New International, 2014. GS 14116254435604492157.

[Nach1965] Leopoldo Nachbin. *The Haar integral*. Translated by Lulu Bechtolsheim. Van Nostrand, Princeton, N.J., U.S., 1965.

Abstract: The present volume contains the material of an introductory course on the Haar integral which the author had the opportunity of giving successively, during the second semester of 1959, at the *Faculdade Nacional de Filosofia* of the University of Brazil (Rio de Janeiro, Guanabara) and at the *Instituto de Fisica e Matemática* of the University of Recife (Recife, Pernambuco),
This text is written with the main concern of making it reasonably self-sufficient: it presupposes of the reader only a rudimentary knowledge of the Stieltjes and Lebesgue integrals, of algebra, and of general topology.
In order to acquaint the reader who is interested in this chapter of functional analysis with the material to be considered, the text will follow the shortest line between the elementary prerequisites mentioned and the existence and uniqueness theorem of the Haar integral.

[Nat1994] James Bryant Nation, *Jónsson's contributions to lattice theory*, Algebra Univers. **31** (1994), 430–445, DOI 10.1007/BF01221797, available at `https://link.springer.com/article/10.1007/BF01221797`.
GS 8251348821874564111.

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[Nat2017] ______. *Notes on Lattice Theory*, 2017. GS 2514237753731863273 Links to individual chapters are in `https://math.hawaii.edu/~jb/books.html`.

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[Nat2018] ______, *Tribute to Bjarni Jónsson*, Algebra Univers. **79** (2018), art. 57, DOI 10.1007/s00012-018-0542-8, available at `https://math.hawaii.edu/~jb/bjarni_jonsson_3.pdf`. GS 4641080885284972445 `https://link.springer.com/article/10.1007/s00012-018-0542-8`.

Biography of Jónsson, including bibliography of Jónsson's work.

[Nath2017] Rishi Nath, *Advances in the Theory of Cores and Simultaneous Core Partitions*, Am. Math. Mon. **124** (2017), 844–861, DOI 10.4169/amer.math.monthly.124.9.844, available at `https://www.tandfonline.com/doi/abs/10.4169/amer.math.monthly.124.9.844`. GS 11715475618298208530. Zbl 1391.05042

Describes a number of abacus constructions.

Abstract: The theory of s-core partitions, integer partitions whose hook sets avoid hooks of length s, lies at the intersection of a surprising number of fields, including number theory, combinatorics, and representation theory. A more recent trend has been to study partitions whose hook sets avoid multiple lengths, known as simultaneous core partitions. This paper, divided into five sections, is a review of five recent papers in this area by undergraduates ([3], [4], [5], [18], [67]. All of the authors surveyed conducted their research while participating in the University of Minnesota Duluth REU.
In the first section, we introduce partitions, the abacus, s-core partitions, and their connections to several fields. In the second section, we turn to self-conjugate s-core partitions and discuss several theorems of L. Alpoge on their asymptotic behavior and their connection, for small s, with points on curves. In the third section, we discuss simultaneous (s, t)-core partitions and the work of A. Aggarwal and V. Wang on the Armstrong conjecture. The fourth section highlights results of A. Aggarwal, A. Berger, and V. Wang on the enumeration, weight, and containment properties of simultaneous (s, t, u)-core partitions. In the final section, we mention some areas of ongoing research connected to the work discussed here.
The techniques used across these papers, ranging from generating functions and modular forms to more combinatorial tools such as abaci, posets, and lattice paths, give a flavor of the richness of the subject. We provide illustrative examples when full proofs are too lengthy.

[Nath2025] ———, *Introduction to core and quotient partitions*, 2025, CCNY, New York, New York Combinatorics Day 2025

Describes a number of abacus constructions.

Abstract: t-core partitions for positive integers t first arose in the study of the representation theory of the symmetric group. Since then, researchers have found connections between certain core partitions and modular forms, Dyck paths and parking functions. Here we survey important results in this field and highlight new and open problems.

[NatPick1987] James Bryant Nation and Douglas A. Pickering, *Arguesian lattices whose skeleton is a chain*, Algebra Univers. **24** (1987), 91–100, DOI 10.1007/BF01188386, available at `https://link.springer.com/article/10.1007/BF01188386`. GS 8996748697701020082.

Attacks the problem of finding "nice" classes of arguesian lattice, all of whose members have type 1 representations. Christian Herrmann devised a way to decompose a finite dimensional modular lattice into complemented blocks, called the lattice's skeleton. The main result of the paper is that if L is a finite dimensional arguesian lattice and its skeleton is a chain, then L has a type 1 representation.

[NguyenVulWood2025] Son Nguyen, Joseph Vulakh, and Dora Woodruff, *A generalization of RSK to d-complete posets* (2025), arXiv 2508.13988, primary class math.CO, DOI 10.48550/arXiv.2508.13988, available at `https://arxiv.org/abs/2508.13988`. GS 5041213084471026261

Abstract: The hook length formula for d-complete posets expresses the number of linear extensions of a d-complete poset P in terms of hooks of P. It generalizes the usual hook length formula for standard Young tableaux, as well as hook length formulas for shifted Young tableaux and trees. We give a new proof of the hook length formula for d-complete posets which is elementary and purely combinatorial. Our approach is to define a generalization of the Robinson–Schensted–Knuth bijection for d-complete posets, which may be of independent interest.

[NLabNat] *natural transformation*, `https://ncatlab.org/nlab/show/natural+transformation`. Accessed November 9, 2024.

nLab entry "natural transformation".

[NLabPoint] *pointed endofunctor*, https://ncatlab.org/nlab/show/pointed+endofunctor. Accessed November 9, 2024.

nLab entry "pointed endofunctor".

[Nzeut2007] Janvier Nzeutchap, *On the Young-Fibonacci insertion algorithm* (2007), arXiv 0704.1969, primary class math.CO, DOI 10.48550/arXiv.0704.1969, available at https://arxiv.org/abs/0704.1969. GS 13265785212974760874

Abstract: This work is concerned with some properties of the Young-Fibonacci insertion algorithm and its relation with Fomin's growth diagrams. It also investigates a relation between the combinatorics of Young-Fibonacci tableaux and the study of Okada's algebra associated to the Young-Fibonacci lattice. The original algorithm was introduced by Roby and we redefine it in such a way that both the insertion and recording tableaux of any permutation are conveniently interpreted as chains in the Young-Fibonacci lattice. A property of Killpatrick's evacuation is given a simpler proof, but this evacuation is no longer needed in making Roby's and Fomin's constructions coincide. We provide the set of Young-Fibonacci tableaux of size n with a structure of graded poset, induced by the weak order on permutations of the symmetric group, and realized by transitive closure of elementary transformations on tableaux. We show that this poset gives a combinatorial interpretation of the coefficients in the transition matrix from the analogue of complete symmetric functions to analogue of the Schur functions in Okada's algebra. We end with a quite similar observation for four posets on Young-tableaux studied by Taskin.

[OBrienQuin2024] Cian O'Brien and Rachel Quinlan, *Groups of singular alternating sign matrices* (2024), arXiv 2405.13611, primary class math.RA, DOI 10.48550/arXiv.2405.13611, available at https://arxiv.org/abs/2405.13611. GS 8847928710170183986

Abstract: We investigate multiplicative groups consisting entirely of singular alternating sign matrices (ASMs), and present several constructions of such groups. It is shown that every finite group is isomorphic to a group of singular ASMs, with a singular idempotent ASM as its identity element. The relationship between the size, the rank, and the possible multiplicative orders of singular ASMs is explored.

[Ok1990] Soichi Okada, *Wreath products by the symmetric groups and product posets of Young's lattices*, J. Comb. Theory **Ser. A 55** (1990), 14–32, DOI 10.1016/0097-3165(90)90044-W, available at https://www.sciencedirect.com/science/article/pii/009731659090044W. GS 5577402980031524002. Zbl 0707.05062

Abstract: In this note we study the connections between the wreath products $\Gamma \wr \mathfrak{S}_n$ of a finite group Γ by the symmetric groups $\mathfrak{S}_n$, and the product poset Y^r of Young's lattices Y. We construct a generalized Robinson–Schensted correspondence for $\Gamma \wr \mathfrak{S}_n$. And we give a complete set of orthogonal eigenvectors for the linear transformation $\mathrm{Ind}_{\Gamma\wr\mathfrak{S}_{n-1}}^{\Gamma\wr\mathfrak{S}_n} \circ \mathrm{Res}_{\Gamma\wr\mathfrak{S}_{n-1}}^{\Gamma\wr\mathfrak{S}_n}$ of the vector space of class functions on $\Gamma \wr \mathfrak{S}_n$.

[Ok1994] ———, *Algebras associated to the Young-Fibonacci lattice*, Trans. Am. Math. Soc. **346** (1994), 549–568, available at https://www.ams.org/journals/tran/1994-346-02/S0002-9947-1994-1273538-7/. GS 8840185111387790989.

Abstract: The algebra $\mathcal{F}_n$ generated by $E_1, \ldots, E_n$ subject to the defining relations $E_i^2 = x_i E_i$ $(i = 1, \ldots, n-1)$, $E_{i+1}E_iE_{i+1} = y_iE_{i+1}$ $(i = 1, \ldots, n-2)$, $E_iE_j = E_jE_i$ $(|i-j| \geq 2)$ is shown to be a semisimple algebra of dimension $n!$ if the parameters $x_1, \ldots, x_{n-1}, 1, \ldots, y_{n-2}$ are generic. We also prove that the Bratteli diagram of the tower $(\mathcal{F}_n)_{n>0}$ of these algebras is the Hasse diagram of the Young-Fibonacci lattice, which is an interesting example, as well as Young's lattice, of a differential poset introduced by R. Stanley. A Young-Fibonacci analogue of the ring of symmetric functions is given and studied.

[Ok2024] ———, *Algebraic structures associated with the Young–Fibonacci lattice*, 2024-06-07, Harvard Univ., Cambridge, Mass., US. video at https://www.youtube.com/watch?v=b8Bw_ODX79g

[Okam2001] Yoshio Okamoto, *Several Aspects of Antimatroids and Convex Geometries*, Masters Thesis, Univ. of Tokyo, 2001, https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=843350518c9a92f1f72dff57a90410296466da80. GS 9457105020841686187.

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[Ol1987] Jørn B. Olsson, *Frobenius symbols for partitions and degrees of spin characters*, Math. Scand. **61** (1987), 223–247, DOI 10.7146/math.scand.a-12201, available at https://www.mscand.dk/article/view/12201. GS 5148054076614415698. Zbl 0658.20012

Abstract: In the first section of this paper we describe a theory of cuts in partition sequences which associates to a given partition λ an infinite sequence of symbols which include all the β-sets for λ as well as Frobenius "characteristic" for λ. Also we study the relation between an arbitrary Frobenius symbol for λ and the hook structure of λ. The author discovered the Frobenius symbols during a study of the "bar-structure" of 2-regular partitions of n. These partitions index the spin characters of a covering groups $\hat{S}_n$ of S_n. It was first realized by Morris that "bars" play a similar role for spin characters as hooks for ordinary characters of symmetric groups. The results of section 1 may be used to give a perhaps more transparent description of the p-bar quotient of a 2-regular partition (see [12]) which is essential for further work. Theorem (2.3) is particularly important. It leads to an explicit description of the power of an odd prime dividing the degree of a spin character. This is done in section 3. In section 4 we study then the power of 2 dividing the degree of a spin character. Again the results of section 1 are essential. Whereas the distribution of spin characters into p-blocks is known for p odd (see [5],[14]) the same question is open for $p = 2$. We apply the results of section 4 to determine those spin characters, which are contained in 2-blocks of small defect, and finish with a conjecture.

[Ol1994] ______. *Combinatorics and Representations of Finite Groups*. Vorlesungen aus dem Fachbereich Mathematik der Universität Essen, vol. 20. Fachbereich Mathematik, Universität Essen, *, 1994. GS 14020511370963206679.

Abstract: These notes contain an expanded version of some lectures given during my stay in Essen in the first half of 1993 The main purpose is to give a description of the interplay between combinatorics (especially the study of partitions and related object) and irreducible representations of some classes of finite groups (especially symmetric groups and their coverings and some linear groups). The origin was the study of (numerical) properties of the representations of symmetric groups including the degrees and the distribution into blocks. Of course the symmetric groups are very special in several respects, but as it has turned out some of the combinatorial analysis applied there may be modified to give results for other groups.

[Ore1942] Øystein Ore, *Theory of equivalence relations*, Duke Math. J. **9** (1942), 573–627, DOI 10.1215/S0012-7094-42-00942-6, available at `https://projecteuclid.org/journals/duke-mathematical-journal/volume-9/issue-3/Theory-of-equivalence-relations/10.1215/S0012-7094-42-00942-6.short`. GS 8845904821420179904.

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[Pak2001] Igor Pak, *Hook Length Formula and Geometric Combinatorics*, Séminaire Lotharingien de Combinatoire **46** (2001), art. B46f, available at `https://www.mat.univie.ac.at/~slc/wpapers/s46pak.html`. GS 17986425221030588085. MR1877632 Zbl 0982.05109

Abstract: Motivated by weighted Hurwitz theory and its connection to integrability, we introduce a (q,t)-tau function that deforms the classical case of hypergeometric tau functions using Macdonald polynomials, while simultaneously generalizing several important series that have already appeared in enumerative geometry, gauge theory, and integrability. We prove that this function is uniquely characterized by a family of differential equations and demonstrate a positive combinatorial expansion of these PDEs in terms of a new family of operators encoded by alternating paths. As a byproduct of our techniques, we establish a connection between path operators and the Delta conjecture.

[Pál2001] Péter P. Pálfy, *Groups and lattices*, Groups St Andrews 2001 in Oxford: Volume 2 (Oxford, 2001), Lond. Math. Soc. Lecture Note Series, vol. 305, Cambridge Univ., Cambridge, 2003, 2001, available at https://web.archive.org/web/20121022011544/https://math.hawaii.edu/~williamdemeo/latticetheory/Palfy-GroupsAndLattices-GStA-2001.pdf. GS 3870854386669262006.

Abstract: In this survey paper we discuss some topics from the theory of subgroup lattices. After giving a general overview, we investigate the local structure of subgroup lattices. A major open problem asks if every finite lattice occurs as an interval in the subgroup lattice of a finite group. Next we investigate laws that are valid in normal subgroup lattices. Then we sketch the proof that every finite distributive lattice is the normal subgroup lattice of a suitable finite solvable group. Finally, we discuss how far the subgroup lattice of a direct power of a finite group can determine the group.

[PálSzab1995] Péter P. Pálfy and Cs. Szabó, *An identity for subgroup lattices of Abelian groups*, Algebra Univers. **33** (1995), 191–195, DOI 10.1007/BF01190930, available at https://link.springer.com/article/10.1007/BF01190930. GS 3373915366505029858.

"In this note we exhibit an identity valid in the subgroup lattice of every Abelian group, and we show that this identity fails in the lattice of normal subgroups of a certain finite 2-group." Thus, there is a lattice of normal subgroups of a group that cannot be embedded in the lattice of subgroups of any Abelian group.

[PatrPyl2018] Rebecca Patrias and Pavlo Pylyavskyy, *Dual filtered graphs*, Algebr. Comb. **1** (2018), 441–500, DOI 10.5802/alco.21, available at https://alco.centre-mersenne.org/item/ALCO_2018__1_4_441_0/. GS 5875943387514643312. Zbl https://zbmath.org/1397.05202

Abstract: We define a K-theoretic analogue of Fomin's dual graded graphs, which we call dual filtered graphs. The key formula in the definition is $DU - UD = D + I$. Our major examples are K-theoretic analogues of Young's lattice, of shifted Young's lattice, and of the Young–Fibonacci lattice. We suggest notions of tableaux, insertion algorithms, and growth rules whenever such objects are not already present in the literature. (See the table below.) We also provide a large number of other examples. Most of our examples arise via two constructions, which we call the Pieri construction and the Möbius construction. The Pieri construction is closely related to the construction of dual graded graphs from a graded Hopf algebra, as described in [1, 19, 16]. The Möbius construction is more mysterious but also potentially more important, as it corresponds to natural insertion algorithms.

[Pick1988] Douglas A. Pickering, *A selfdual Arguesian inequality*, Algebra Univers. **22** (1988), 99, DOI 10.1007/BF01190740, available at https://link.springer.com/article/10.1007/BF01190740.
GS 8890993649973702965.

Abstract: It is well known that the class of Arguesian lattices is selfdual [Jóns1972]. In this note we give a selfdual inequality equivalent to the Arguesian inequality.

[PosFarb1976] Jonathan B. Postel and David J. Farber, *Graph Modeling of Computer Communication Protocols*, 1976, available at https://escholarship.org/content/qt2p87f03x/qt2p87f03x_noSplash_362cea8fc80600eddb09b3d6813ba64a.pdf.
GS 10130318056270778004.

Abstract: The design of computer-to-computer communications protocols for networks of computers is one of the challenging problems facing the computer scientist. In the last few years, a number of large scale computer networks have been discussed and several of these are in various phases of implementation. It is clear that the design of the communications protocols for these networks is a difficult problem (though the evidence for this is mainly in internal memoranda, see [CARR70], [CROC72] , [FARB73], [FRAN72l, and [HEAR70]). Many of the problems in designing these communications protocols, which involve asynchronous parallel processes, are due to the difficulty in recognizing all of the possible orderings in which the significant events can occur and the possible execution sequences which arise from these orderings.

This paper draws from experience with the development of a computer network sponsored by the Advanced Research Projects Agency (ARPA) called the ARPANET [ROBE70], and the development of the UCLA Graph Model of Computation (ESTR83), especially the advances made by Gostelow [GOST71] and Cerf [CERF72c] which gave this model of parallel processing additional analytical power. One especially attractive aspect of this modeling technique is the existence of a computer program for testing for the property of proper termination. Proper termination indicates that the process modeled by the graph is in a certain sense well behaved; for example it has no deadlocks. These developments suggest that communications protocols can be modeled by the UCLA Graph Model in a useful and revealing way.

[Post2005] Alexander Postnikov, *Affine approach to quantum Schubert calculus*, Duke Math. J. **128** (2005), 473–509, DOI 10.1215/S0012-7094-04-12832-5, available at `https://math.mit.edu/~apost/papers/affine.pdf`. GS 11731329525404378458. MR2145741 Zbl 1081.14070

Concerns cylindric and toric tableaux.

Abstract: This paper presents a formula for products of Schubert classes in the quantum cohomology ring of the Grassmannian. We introduce a generalization of Schur symmetric polynomials for shapes that are naturally embedded in a torus. Then we show that the coefficients in the expansion of these toric Schur polynomials, in terms of the regular Schur polynomials, are exactly the 3-point Gromov-Witten invariants, which are the structure constants of the quantum cohomology ring. This construction implies three symmetries of the Gromov-Witten invariants of the Grassmannian with respect to the groups S_3, $(\mathbb{Z}/n\mathbb{Z})^2$, and $\mathbb{Z}/2\mathbb{Z}$. The last symmetry is a certain *curious duality* of the quantum cohomology which inverts the quantum parameter q. Our construction gives a solution to a problem posed by Fulton and Woodward about the characterization of the powers of the quantum parameter q which occur with nonzero coefficients in the quantum product of two Schubert classes. The curious duality switches the smallest such power of q with the highest power. We also discuss the affine nil-Temperley–Lieb algebra that gives a model for the quantum cohomology.

[Priest1970] Hilary Ann Priestley, *Representation of distributive lattices by means of ordered Stone spaces*, Bull. Lond. Math. Soc. **2** (1970), 186–190, DOI 10.1112/blms/2.2.186, available at `https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=441f6bd252853e8a9774c52914150387a0c29a91`. GS 11362711773578304365. MR265242 Zbl 0201.01802

Abstract: Stone, in [Stone1938], developed for distributive lattices a representation theory generalizing that for Boolean algebras. This he achieved by topologizing the set X of prime ideals of a distributive lattice A (with a zero element) by taking as a base $\{P_a : a \in A\}$ (where P_a denotes the set of prime ideals of A not containing a), and by showing that the map $a \mapsto P_a$ is an isomorphism representing A as the lattice of all open compact subsets of its dual space X.
The topological spaces which arise as duals of Boolean algebras may be characterized as those which are compact and totally disconnected (i.e. the Stone spaces); the corresponding purely topological characterization of the duals of distributive lattices obtained by Stone is less satisfactory. In the present paper we show that a much simpler characterization in terms of ordered topological spaces is possible. The representation theorem itself, and much of the duality theory consequent on it [8, 6], becomes more natural in this new setting, and certain results not previously known can be obtained.

[Proct1982] Robert A. Proctor, *Representations of $\mathfrak{sl}(2,\mathbb{C})$ on posets and the Sperner property*, SIAM J. Algebr. Disc. Meth. **3** (1982), 275–280, DOI 10.1137/0603026, available at `https://epubs.siam.org/doi/abs/10.1137/0603026`. GS 3401363903828155882. MR655567 Zbl 0496.06004

Abstract: A ranked partially ordered set is said to be *Sperner* if it has no antichain bigger than its largest rank. A necessary and sufficient condition for a ranked partially ordered set to be rank symmetric, rank unimodal and strongly Sperner is presented. This condition involves representations of $\mathfrak{sl}(2,\mathbb{C})$. It is used to provide a new, short proof that this combination of properties is preserved under the product operation.

The sufficient part of this condition is also used to provide new, simpler proofs that certain combinatorially interesting partially ordered sets are rank symmetric, rank unimodal and strongly Sperner.

[Proct1999] ———, *Dynkin diagram classification of λ-minuscule Bruhat lattices and of d-complete posets*, J. Algebr. Comb. **9** (1999), 61–94, DOI 10.1023/A:1018615115006, available at `https://link.springer.com/article/10.1023/A:1018615115006`. GS 4067945276743999854. MR1676724 Zbl 0920.06003

Abstract: d-Complete posets are defined to be posets which satisfy certain local structural conditions. These posets play or conjecturally play several roles in algebraic combinatorics related to the notions of shapes, shifted shapes, plane partitions, and hook length posets. They also play several roles in Lie theory and algebraic geometry related to λ-minuscule elements and Bruhat distributive lattices for simply laced general Weyl or Coxeter groups, and to λ-minuscule Schubert varieties. This paper presents a classification of d-complete posets which is indexed by Dynkin diagrams.

[PudTům1980] Pavel Pudlák and Jiří Tůma, *Every finite lattice can be embedded in a finite partition lattice*, Algebra Univers. **10** (1980), 74–95, DOI 10.1007/BF02482893, available at `https://link.springer.com/article/10.1007/BF02482893`. GS 3014669489435746571.

Abstract: We give here a proof of the theorem stated in the title. The theorem was conjectured by P. M. Whitman in [Whit1946]. The proof, mostly of combinatorial character, is based on "the regraph power technique" making use of special edge-colored graphs, called regraphs, as construction schemes. We use the proof of Combinatorial Lemma 6.1 by B. Wolk and B. Sands, which is shorter and more elegant than the original one. The list of references is a random collection of papers related to Whitman's conjecture.

[Qing2008] Yulan Qing, *Differential Posets and Dual Graded Graphs*, Masters Thesis, Massachusetts Inst. of Tech., 2008, `https://dspace.mit.edu/handle/1721.1/47899`. GS 9596728992570937515.

Abstract: In this thesis I study r-differential posets and dual graded graphs. Differential posets are partially ordered sets whose elements form the basis of a vector space that satisfies $DU - UD = rI$, where U and D are certain order-raising and order-lowering operators. New results are presented related to the growth and classification of differential posets. In particular, we prove that the rank sequence of an r-differential poset is bounded above by the Fibonacci sequence and that there is a unique poset with such a maximum rank sequence. We also prove that a 1-differential lattice is either Young's lattice or the Fibonacci lattice. In the second part of the thesis, we present a series of new examples of dual graded graphs that are not isomorphic to the ones presented in Fomin's original paper.

[Rap2025] Gabriel Raposo, *Global fluctuations for standard Young tableaux* (2025), arXiv 2507.18601, primary class math.PR, DOI 10.48550/arXiv.2507.18601, available at `https://arxiv.org/abs/2507.18601`. GS 6400335075125843958

Abstract: We introduce the notion of a Young generating function for a probability measure on integer partitions. We use this object to characterize probability distributions over integer partitions satisfying a law of large numbers and those that satisfy a central limit theorem. We further establish a multilevel central limit theorem, which enables the study of random standard Young tableaux. As applications of these results, we describe the fluctuations of height functions associated with (i) the Plancherel growth process, (ii) random standard Young tableaux of fixed shape, and (iii) probability distributions induced by extreme characters of the infinite symmetric group S_∞. In all cases, we identify the limiting fluctuations as a conditioned Gaussian Free Field.

[RenDun2005] Paul Renteln and Alan Dundes, *Foolproof: A Sampling of Mathematical Folk Humor*, Not. Am. Math. Soc. **52** (2005), 24–34, available at `https://community.ams.org/journals/notices/200501/fea-dundes.pdf`. GS 829031054094557161.

In the discipline known as folkloristics [D1] (the study of folklore), a folk is defined as any group whatsoever that shares at least one common linking factor. The factor could be nationality, ethnicity, religion, or occupation. Members of a profession would also qualify as a folk group. Hence, mathematicians constitute a folk. And, like all folk groups, mathematicians have their own folk speech (slang), proverbs, limericks, and jokes, among other forms of folklore. It is precisely the folklore of a group that defines that group. So, mathematicians as a group share a common core of mathematical folklore. Some of this folklore tends to be quite esoteric and intelligible only to members of the group. Outsiders not possessing the requisite mathematical vocabulary and knowledge rarely know such esoteric material, and even if they did they would probably not understand it. But there is also exoteric mathematical folklore that is known to a limited number of outsiders, for example, physicists, chemists, and engineers. Much of this exoteric folklore consists of classic jokes contrasting members of different but related academic disciplines. We intend to offer a brief sampling of both esoteric and exoteric mathematical folklore, concentrating on humorous genres such as jokes. We are persuaded that these data not only serve as a basis for identity among mathematicians but also provide a unique window on mathematical culture in general, and even a clue as to the nature of mathematical thinking.

[RichGebSturmTheo2003] Jürgen Richter-Gebert, Bernd Sturmfels, and Thorsten Theobald, *First Steps in Tropical Geometry* (2003), arXiv 0306366, primary class math.AG, DOI 10.48550/arXiv.math/0306366, available at https://arxiv.org/abs/math/0306366. GS 12630966512609487340

Abstract: Tropical algebraic geometry is the geometry of the tropical semiring $(\mathbb{R}, \min, +)$. Its objects are polyhedral cell complexes which behave like complex algebraic varieties. We give an introduction to this theory, with an emphasis on plane curves and linear spaces. New results include a complete description of the families of quadrics through four points in the tropical projec- tive plane and a counterexample to the incidence version of Pappus' Theorem.

[Rob1938] G. de B. Robinson, *On the representations of the symmetric group.*, Am. J. Math. **60** (1938), 745–760, DOI 10.2307/2371609, available at https://www.jstor.org/stable/2371609;, Am. J. Math. **69** (1947), 286–298, DOI 10.2307/2371853, available at https://www.jstor.org/stable/2371853;, Am. J. Math.

70 (1948), 277–294, DOI 10.2307/2372326, available at https://www.jstor.org/stable/2372326. MR1507943

U. Introduction of insertion algorithm.

[Roby1991] Thomas W. Roby, *Applications and extensions of Fomin's generalization of the Robinson-Schensted correspondence to differential posets*, Ph.D. thesis, Massachusetts Inst. of Tech., 1991, https://dspace.mit.edu/handle/1721.1/13517. GS 9893116474914970883.

U U, tableauUDT. Good introduction section. Intuitive description of how the Fibonacci differential poset is constructed. Uses [Fom1994] construction in various variations to construct insertion algorithms, both previously known and new. Also applies these techniques to "sequentially differential posets" which have differing differential degree at different ranks of the poset.

[Roby1995] ______, *The connection between the Robinson-Schensted correspondence for skew oscillating tableaux and graded graphs*, Discrete Math. **139** (1995), 481–485, DOI 10.1016/0012-365X(94)00151-8, available at https://www.sciencedirect.com/science/article/pii/0012365X94001518. GS 8525652298298107321.

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[Rot1997a] Gian-Carlo Rota, *The many lives of lattice theory*, Not. Am. Math. Soc. **44** (1997), 1440–1445, available at https://www.ams.org/notices/199711/comm-rota.pdf. GS 15038564556962815673.

Brief introduction to the history of lattice theory and its connections to other sectors of mathematics.

[Rot1997b] ______. *Indiscrete Thoughts.* Modern Birkhäuser Classics. Birkhäuser, Boston, 1997. GS 14868341420392467256.

Indiscrete Thoughts gives a glimpse into a world that has seldom been described that of science and technology as seen through the eyes of a mathematician. The era covered by this book, 1950 to 1990, was surely one of the golden ages of science as well as the American university.

Cherished myths are debunked along the way as Gian-Carlo Rota takes pleasure in portraying, warts and all, some of the great scientific personalities of the period.

[Sag1979a] Bruce E. Sagan, *Partially ordered sets with hooklengths: — An algorithmic approach*, Ph.D. thesis, Massachusetts Inst. of Tech., 1979, `https://dspace.mit.edu/bitstream/handle/1721.1/154325/07117617-MIT.pdf`. GS 8802861476287111626.

Combinatorialists have developed an extensive theory of reverse plane partitions and standard Young Tableaux (a partial listing will be found in the bibliography, see also [1], [34]). These arrays are also of interest in group theory where they can be used to get information about representations of the symmetric group. The generating Function for reverse plane partitions is by definition

$$F(X) = \sum_{m=0}^{\infty} a_m X^m$$

where a_m is the number of reverse plane partitions whose parts sum to m. This generating function has the important property that it can be expressed as a product

$$F(X) = \prod_i \frac{1}{1 - X^{h_i}} \qquad (1)$$

The positive integers h_i, are called the hooklengths of the partition.
There are two other related arrays, i.e. shifted partitions and rooted trees, whose generating functions have a similar form but they have been almost totally ignored in the literature. In my thesis I show that three known algorithms for the usual arrays have analogs in the shifted and tree-like cases. The first algorithm, due originally to Hillman and Grassl, provides a combinatorial proof of the identity (1). In fact I extend their techniques to a number of other generating functions as well. Next I provide a method for picking a random tableau or tree (generalized from a paper of Greene, Nijenhuis, and Wilf). The last algorithm yields a combinatorial proof of a character formula from the work of Schur on projective representations of the symmetric group.

[Sag1979b] ———, *An analog of Schensted's algorithm for shifted Young tableaux*, J. Comb. Theory **Ser. A 27** (1979), 10–18, DOI 10.1016/B978-0-12-428780-8.50007-6, available at https://users.math.msu.edu/users/bsagan/Papers/Old/asa-pub.pdf. GS 15694654829594027728. MR80k 05029

S. Topics: insertion algorithm, Knuth equivalence. This describes the "first" shifted insertion algorithm.

[Sag1987] ———, *Shifted tableaux, Schur Q-functions, and a conjecture of R. P. Stanley*, J. Comb. Theory **Ser. A 45** (1987), 62–103, DOI 10.1016/0097-3165(87)90047-1, available at https://users.math.msu.edu/users/bsagan/Papers/Old/sts-pub.pdf. GS 17936398118720156955.

S. Topics: insertion algorithm, Greene invariant, jeu de taquin, Knuth equivalence, s_λ, P_λ, Q_λ.

[Sag1988] ———, *The ubiquitous Young tableau*, Invariant Theory and Tableaux (Dennis Stanton, ed.), IMA Volumes in Math. and Its Appls., vol. 19, Springer-Verlag, Berlin and New York, 1990, 1988, pp. 262–298, available at https://users.math.msu.edu/users/bsagan/Papers/Old/uyt.pdf. GS 3203592384563717131.

U, *S*, *UD*. Topics: insertion algorithm, jeu de taquin, Greene invariant, Knuth equivalence, s_λ, Q_λ, sp_λ, S_n, $Gl(n,\mathbb{C})$, $Sp(n,\mathbb{C})$, projective representation. Introduces all three types of tableaux, showing the similarity of the three theories.

[Sag2020] ———. *Combinatorics: The art of counting.* Grad. Stud. in Math., vol. 210. American Mathematical Society, Providence, 2020. Errata at https://users.math.msu.edu/users/bsagan/Books/Aoc/errata.pdf GS 3283103461630751988.

Abstract: Enumerative combinatorics has seen an explosive growth over the last 50 years. The purpose of this text is to give a gentle introduction to this exciting area of research. So, rather than trying to cover many different topics, I have chosen to give a more leisurely treatment of some of the highlights of the field. My goal has been to write the exposition so it could be read by a student at the advanced undergraduate or beginning graduate level, either as part of a course or for independent study. The reader will find it similar in tone to my book on the symmetric group. I have tried to keep the prerequisites to a minimum, assuming only basic courses in linear and abstract algebra as background. Certain recurring themes are emphasized, for example, the existence of sum and product rules first for sets, then for ordinary generating functions, and finally in the case of exponential generating functions. I have also included some recent material from the research literature which, to my knowledge, has not appeared in book form previously, such as the theory of quotient posets and the connection between pattern avoidance and quasisymmetric functions.

[SagStan1990] Bruce E. Sagan and Richard P. Stanley, *Robinson-Schensted Algorithms for Skew Tableaux*, J. Comb. Theory **Ser. A 55** (1990), 161–193, DOI 10.1016/0097-3165(90)90066-6, available at `https://www.sciencedirect.com/science/article/pii/0097316590900666`. GS 7820900345122408014.

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[Schen1961] Craige Schensted, *Longest increasing and decreasing sequences*, Can. J. Math. **13** (1961), 179–191, DOI 10.4153/CJM-1961-015-3, available at `https://www.cambridge.org/core/services/aop-cambridge-core/content/view/B5098D9BC8B226C575402B971852C05E/S0008414X00013146a.pdf/longest-increasing-and-decreasing-subsequences.pdf`. MR121305

U. Topics: insertion algorithm, Greene invariant.

[Schmidt1991] Elegius Tamás Schmidt, *Cover-preserving embedding*, Period. Math. Hung. **23** (1991), 17–84, DOI 10.1007/bf02260390, available at `https://akjournals.com/view/journals/10998/23/1/article-p17.xml`. GS 177133605269744681. Zbl 0753.06009

Abstract: A finite lattice K has the cover-preserving embedding property, abbreviated as CPEP with respect a variety V of lattices, if whenever K can be embedded into a finite lattice L in V, then K has a cover-preserving embedding into L, that is an embedding ϕ with the property that if a covers b in K then $\phi(a)$ covers $\phi(b)$ in L. This concept was introduced by E. Fried, G. Grätzer and H. Lakser in [2], and it was proved that a finite projective geometry P

(i.e. a simple complemented modular lattice) has the cover-preserving embedding property with respect to the variety M of all modular lattices if and only if one of the following three conditions hold:
(i) the length of P is 1;
(ii) the length of P is 2 and P is isomorphic to M_3;
(iii) the length of P is greater than 2 and either P is non-arguesian or P is arguesian and for some prime p, each interval of P of length 2 contains $p+1$ atoms.

[Schmidt2012] ———, *A structure theorem of semimodular lattices: the patchwork representation*, 2012, available at `http://math.bme.hu/~schmidt/papers/patchwork.pdf`. GS 15628230190451375212.

Abstract: In this paper we give a structure theorem of semimodular lattices, which generalize the the results given in [4] for planar semimodular lattices. The main result of [4] asserts, that every planar semimodular lattice is the patchwork of special intervals as show in Figure 1.
The "**building stones**" are special rectangular lattices (in most cases the surface of the diagram is a rectangular shape), we get these from Boolean lattices with a special construction, the *pigeonhole procedure*. The "**building tool**" is a kind of gluing, the *patchwork construction*. It is related to the Hall-Dilworth gluing, for instance we glue together two cubes (i.e. 2^3 Boole algebras) over faces, see Figure 2. As technical tool we use special *block matrices*, see Figure 21.

[Schmitt2006a] William R. Schmitt, *A concrete introduction to categories*, 2006, available at `https://web.archive.org/web/20240524060725/https://home.gwu.edu/~wschmitt/papers/cat.pdf`. GS 9853324641433241820.

A short introduction to category theory, oriented toward applications in combinatorics.

[Schmitt2006b] ———, *Notes on modules and algebras*, 2006, available at `https://web.archive.org/web/20240524060732/https://home.gwu.edu/~wschmitt/papers/notes2006.pdf`.

A short introduction to modules and algebras, oriented toward applications in combinatorics.

[Schur1911] Issai Schur, *Über die Darstellung der symmetrischen und der alternierenden Gruppe durch gebrochene lineare Substitutionen [On the representation of the symmetric and alternating groups by fractional linear substitutions]*, J. reine angew. Math. **139** (1911), 155–250, available at https://eudml.org/doc/149348. Zbl 42.0154.02

English translation is [Schur1911-en]. Schur published under both names "I. Schur" and "J. Schur".

[Schur1911-en] ______, *On the Representation of the Symmetric and Alternating Groups by Fractional Linear Substitutions*, translated by Marc-Felix Otto, Int. J. Theor. Phys. **40** (2001), 413–458, DOI 10.1023/A:1003772419522, available at https://link.springer.com/article/10.1023/A:1003772419522. GS 2561768707700493724. MR1820589 Zbl 0969.20002

English translation of [Schur1911]. Schur published under both names "I. Schur" and "J. Schur".
Abstract: In the present work, I deal with the task of determining all finite groups of fractional linear substitutions that are [homomorphic] to the symmetric or alternating group of n numbers in the first degree. This task is carried out insofar as an exact outline of the desired collineation groups is gained. In the following, I call the symmetric group of n numbers S_n, the alternating group A_n.

[Schüt1945] Marcel-Paul Schützenberger, *Sur certains axioms de la theorie des structures*, C. R. Acad. Sci. **221** (1945), 218–222, available at https://gallica.bnf.fr/ark:/12148/bpt6k3173p.item. GS 10841751010223657184.

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[Schüt1972] ______, *Promotion des morphisms d'ensembles ordonnes [Promotion of morphisms of ordered sets]*, Discrete Math. **2** (1972), 73–94, DOI 10.1016/0012-365X%2872%2990062-3, available at https://www.sciencedirect.com/science/article/pii/0012365X72900623?via%3Dihub. GS 4600998994934421107.

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[Schüt1976] ———, *La correspondance de Robinson [Robinson's correspondence]*, Combinatoire et Représentation du Groupe Symétrique [Combinatorics and representation of the symmetric group] (Strasbourg, 1976), Lecture Notes in Math. (LNM), vol. 579, Springer-Verlag, Berlin and New York, 1976, DOI 10.1007/BFb0090012, available at https://link.springer.com/chapter/10.1007/BFb0090012.

U. Greene invariant, jeu de taquin, insertion algorithm, Knuth equivalence, s_λ. Introduces jeu de taquin.

[Scott2020] Jeanne Scott, *What's the right notion of content for the Young-Fibonacci lattice?*, 2020-06-04, Inst. of Math. Sciences, Chennai, India, IMSc algebraic combinatorics seminar, available at https://www.youtube.com/watch?v=ZiOOgcqnv70

Abstract: The Young-Fibonacci lattice $\mathbb{YF}$ is a ranked lattice invented by R. Stanley as an example of a differential poset; a nice consequence of this feature is that saturated chains (which a fixed top) are counted by a generalized hook-length formula. In 1994 S. Okada showed that $\mathbb{YF}$ is also the branching poset for a tower of complex semi-simple algebras $\mathfrak{F}(n)$, each having a simple Coxeter-like presentation. The representation theory of these algebras strongly parallels the story of the symmetric groups $S(n)$ — in particular each element w of rank $\mathrm{rk}(w) = n$ in the $\mathbb{YF}$ lattice corresponds to an irreducible representation $V(w)$ of $\mathfrak{F}(n)$ whose basis is indexed by saturated chains in the $\mathbb{YF}$ lattice ending at w. Furthermore there is a theory of $\mathbb{YF}$-Schur functions obeying a Littlewood–Richardson rule whose structure coefficients coincide with the induction product multiplicities for representations of the Okada algebras.
As in any tower of semi-simple algebras with a simple braching poset, we may define the Gelfand-Tsetlin algebra $\mathrm{GT}(n)$ as the (maximal) commutative subalgebra of $\mathfrak{F}(n)$ generated by the centers $Z\mathfrak{F}(1), Z\mathfrak{F}(2), \ldots, Z\mathfrak{F}(n)$. The problem I would like to address is how to find (additive) Jucys-Murphy elements, namely an infinite sequence of elements $J(n)$ such that:
(1) each $J(n)$ resides in $\mathrm{GT}(n)$
(2) $J(1), \ldots, J(n)$ generate $\mathrm{GT}(n)$
(3) the sum $J(1) + \cdots + J(n)$ resides in $Z\mathfrak{F}(n)$

(4) each $J(k)$ acts diagonally on the irreducible representation $V(w)$ and its eigenvalue, with respect to a basis vector indexed by a saturated chain $u(0) \lhd \cdots \lhd u(n)$, depend only on the covering relation $u(k-1) \lhd u(k)$ in $\mathbb{YF}$.

This local eigenvalue $c(u \lhd v)$ is called the content of covering relation $u \lhd v$ with respect to the choice of Jucys-Murphy generators. Keep in mind that there are many different systems of elements $J(n)$ satisfying properties (1), (2), (3), and (4). However, not any assignment of covering weights $c(u \lhd v)$ can be realized as contents for such a system. Indeed a necessary condition requires that two saturated chains coincide if and only if the corresponding sequences of covering weights are equal; see recent work of S. Doty et. al.
Since the Jucys-Murphy problem is under-determined it is natural to use the tower of symmetric groups $S(n)$ together with its branching poset, the Young lattice $\mathbb{Y}$, as a guide to impose further constraints. For example, one might try determine a system of Jucys-Murphy elements by forcing the attending system of contents to satisfy a specialization formula for the $\mathbb{YF}$-Schur functions in analogy with the principal specialization of classical Schur functions. This is work in progress.

[Scott2024] ______, *Non-crossing diagrammatics for the Okada algebra*, 2024-01-24, Harvard Univ., Cambridge, Mass., U.S., Stanley Seminar in Combinatorics

Abstract: It is well known that the Young lattice is the Bratelli diagram of the symmetric groups, expressing how irreducible representations restrict from $S(n)$ to $S(n-1)$. In 1975 Stanley discovered a similar lattice called the Young-Fibonacci lattice which was later realized as the Bratelli diagram of a family of algebras by Okada in 1994.
In joint work with Florent Hivert (Université Paris-Sud) we realize the n-th Okada algebra as a diagram algebra with a multiplicative/monoid basis consisting of n-strand Temperley-Lieb diagrams, each equipped with a “height” labeling of its strands. The proof involves a diagrammatic version of Fomin’s Robinson-Schensted correspondence for the Young-Fibonacci lattice. This basis is cellular, which affords us with a novel, diagrammatic presentation of the irreducible representations of the Okada algebra (i.e. cell modules).
Outline:

1st Okada Algebras and its Representation Theory
2nd Young-Fibonacci Lattice and its combinatorics
3rd Fully Packed-Loop configurations and Okada Arc-Diagrams
4th Diagrammatic realization of the Okada Algebra

[Ser1997] Jean-Pierre Serre. *Linear Representations of Finite Groups.* GTM, vol. 42. Springer, New York, 1977. https://link.springer.com/book/10.1007/978-1-4684-9458-7 GS 2178243470076620893.

Abstract: This book consists of three parts, rather different in level and purpose:
The first part was originally written for quantum chemists. It describes the correspondence, due to Frobenius, between linear representations and characters. This is a fundamental result, of constant use in mathematics as well as in quantum chemistry or physics. I have tried to give proofs as elementary as possible, using only the definition of a group and the rudiments of linear algebra. The examples (Chapter 5) have been chosen from those useful to chemists.
The second part is a course given in 1966 to second-year students of l'Écoie Normale. It completes the first on the following points:

a degrees of representations and integrality properties of characters (Chapter 6);
b induced representations, theorems of Artin and Brauer, and applications (Chapters 7-11);
c rationality questions (Chapters 12 and 13).

The methods used are those of linear algebra (in a wider sense than in the first part): group algebras, modules, noncommutative tensor products, semisimple algebras.
The third part is an introduction to Brauer theory: passage from characteristic 0 to characteristic p (and conversely). I have freely used the language of abelian categories (projective modules, Grothendieck groups), which is well suited to this sort of question. The principal results are:

a The fact that the decomposition homomorphism is surjective: all irreducible representations in characteristic p can be lifted "virtually" (i.e., in a suitable Grothendieck group) to characteristic O.
b The Fong–Swan theorem, which allows suppression of the word "virtually" in the preceding statement, provided that the group under consideration is p-solvable.

[Set2025] Linus Setiabrata, *Newton polytopes of Schubert and Grothendieck polynomials*, 2025-11-08, Ohio State U., Columbus, Ohio, US, Algebra, Geometry and Combinatorics Day, available at https://math.mit.edu/~setia/algecom-nov-2025.pdf

Abstract: The Newton polytope $\mathrm{conv}\{a : x^a \text{ appears in } \mathfrak{S}_w\}$ of a Schubert polynomial has fascinating combinatorial and discrete-geometric properties, and the study of these polytopes in their own right tells us a lot about the Schubert polynomials themselves. I will survey some of the basic theory of these polytopes and discuss recent works, joint with Hafner–Mészáros–St. Dizier and with Chou, where we aim to extend this story to Grothendieck polynomials.

[Sham2024] Khodr Shamseddine, *On the complex Levi-Civita field: algebraic and topological structures and foundations for analysis*, Khayyam J. Math. **10** (2024), 70–89, DOI 10.22034/KJM.2023.378706.2735, available at http://www2.physics.umanitoba.ca/u/khodr/Publications/Complex-Levi-Civita-Field-2023.pdf. GS 3595051974167619362.

In this paper, we introduce the complex Levi-Civita field $\mathcal{C}$. We start by reviewing the algebraic structure of the field; in particular, $\mathcal{C}$ is the smallest non-Archimedean valued field extension of the complex numbers field $\mathcal{C}$ that is algebraically closed and complete in the valuation topology.
Two topologies on $\mathcal{C}$ will be studied in detail: the valuation topology induced by a non-Archimedean valuation on the field and another weaker topology induced by a family of seminorms, which we will call weak topology. We show that each of the two topologies results from a metric on $\mathcal{C}$ and that the valuation topology is not a vector topology, while the weak topology is. Then, we give simple characterizations of open, closed, and compact sets in both topologies.
Finally, we define continuity and differentiability for a $\mathcal{C}$-valued function at a point or on a subset of $\mathcal{C}$, we present key results for such functions, and we set the foundations for a Cauchy-like analysis theory on the field $\mathcal{C}$.

[ShamBerz2010] Khodr Shamseddine and Martin Berz, *Analysis on the Levi-Civita field, a brief overview*, Contemp. Math. **508** (2010), 215–237, DOI 10.1090/conm/508/10002, available at `http://www2.physics.umanitoba.ca/u/khodr/Publications/RS-Overview-offprints.pdf`. GS 11547142407722996351. Zbl 1198.26030

In this paper, we review the algebraic properties of various non-Archimedean ordered structures, extending them in various steps which lead naturally to the smallest non-Archimedean ordered field that is Cauchy-complete and real closed. In fact, the Levi-Civita field is small enough to allow for the calculus on the field to be implemented on a computer and used in applications such as the fast and accurate computation of the derivatives of real functions as "differential quotients" up to very high orders.
We then give an overview of recent research on the Levi-Civita field. In particular, we summarize the convergence and analytical properties of power series, showing that they have the same smoothness behavior as real power series; and we present a Lebesgue-like measure and integration theory on the field. Moreover, based on continuity and differentiability concepts that are stronger than the topological ones, we discuss solutions to one-dimensional and multi-dimensional optimization problems as well as existence and uniqueness of solutions of ordinary differential equations.

[Shen2025a] Guozhen Shen, *A choice-free proof of Mal'cev's theorem on quasivarieties* (2025), arXiv 2501.00766, primary class math.LO, DOI 10.48550/arXiv.2501.00766, available at `https://arxiv.org/abs/2501.00766`. GS 18133591108731611985

Abstract: In 1966, Mal'cev proved that a class $\mathcal{K}$ of first-order structures with a specified signature is a quasivariety if and only if $\mathcal{K}$ contains a unit and is closed under isomorphisms, substructures, and reduced products. In this article, we present a proof of this theorem in ZF (the Zermelo–Fraenkel set theory without the axiom of choice).

[Shen2025b] ———, *A choice-free proof of Mal'cev's theorem on quasivarieties*, Algebra Univers. **86** (2025), art. 25, DOI 10.1007/s00012-025-00902-x. GS 18133591108731611985.

Abstract: In 1966, Mal'cev proved that a class K of first-order structures with a specified signature is a quasivariety if and only if K contains a unit and is closed under isomorphic images, substructures, and reduced products. In this article, we present a proof of this theorem in ZF (i.e., the Zermelo–Fraenkel set theory without the axiom of choice).

[Skub2013] Benedek Skublics, *Isometrical embeddings of lattices into geometric lattices*, Order **30** (2013), 797–806, DOI 10.1007/s11083-012-9277-x, available at `https://www.researchgate.net/profile/Benedek-Skublics/publication/257635254_Isometrical_Embeddings_of_Lattices_into_Geometric_Lattices`. GS 17448337368194318945.

Abstract: A lattice is said to be *finite height generated* if it is complete and every element is the join of some elements of finite height. Extending former results by G. Grätzer and E.W. Kiss [6] on finite lattices, we prove that every *finite height generated* algebraic lattice that has a pseudorank function is isometrically embeddable into a geometric lattice.

[Stan1971] Richard P. Stanley, *Theory and application of plane partitions: Part I*, Stud. Appl. Math. **50** (1971), 167–188, DOI 10.1002/SAPM1971502167, available at `https://onlinelibrary.wiley.com/doi/epdf/10.1002/sapm1971502167`. GS 11441760227524011350. MR48 #3754

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[Stan1975] ______, *The Fibonacci lattice*, Fibonacci Quart. **13** (1975), 215–232, available at `https://www.fq.math.ca/Scanned/13-3/stanley.pdf`. GS 17005617859175324338. MR387143 Zbl 0328.06007

Abstract: Our object is to investigate a certain distributive lattice $\underline{F}_i$ closely related to the Fibonacci numbers. First we will review some basic properties of distributive lattices and discuss some general combinatorial problems associated with them. Thus this paper can be regarded as a semi-expository survey of some combinatorial aspects of distributive lattices.

[Stan1981] ———, *Review: I. G. Macdonald, Symmetric functions and Hall polynomials*, Bull. Am. Math. Soc. **4** (1981), 254–265, available at https://projecteuclid.org/journals/bulletin-of-the-american-mathematical-society-new-series/volume-4/issue-2/Review-I-G-Macdonald-Symmetric-functions-and-Hall-polynomials/bams/1183548016.full. GS 6567790723008896888.

A detailed review and summary of the 1979 edition of [Macdon1979].

[Stan1982] ———, *Some Aspects of Groups Acting on Finite Posets*, J. Comb. Theory **Ser. A 32** (1982), 132–161, DOI 10.1016/0097-3165(82)90017-6, available at https://www.sciencedirect.com/science/article/pii/0097316582900176. GS 15723791584781072219. Zbl 0496.06001

Abstract: Let P be a finite poset and G a group of automorphisms of P. The action of G on P can be used to define various linear representations of G, and we investigate how these representations are related to one another and to the structure of P. Several examples are analyzed in detail, viz., the symmetric group $\mathfrak{S}_n$ acting on a boolean algebra, $GL_n(q)$ acting on subspaces of an n-dimensional vector space over $GF(q)$, the hyperoctahedral group B_n acting on the lattice of faces of a cross-polytope, and $\mathfrak{S}_n$ acting on the lattice Π_n, of partitions of an n-set. Several results of a general nature are also proved. These include a duality theorem related to Alexander duality, a special property of geometric lattices, the behavior of barycentric subdivision, and a method for showing that certain sequences are unimodal. In particular, we give what seems to be the simplest proof to date that the q-binomial coefftcient $\begin{bmatrix} k+l \\ k \end{bmatrix}$ has unimodal coefficients.

[Stan1983] ———, *Unimodality and Lie superalgebras*, Stud. Appl. Math. **72** (1985), 263–281, DOI 10.1002/sapm1985723263, available at https://math.mit.edu/~rstan/pubs/pubfiles/62.pdf. GS 8636126962291835829. MR790132 Zbl 0614.17004

Abstract: It is well-known how the representation theory of the Lie algebra sl$(2, \mathbb{C})$ can be used to prove that certain sequences of integers are unimodal and that certain posets have the Sperner property. Here an analogous theory is developed for the Lie superalgebra $1, 2$. We obtain new classes of unimodal sequences (described in terms of cycle index polynomials) and a new class of posets (the "superanalogue" of the lattice $L(m, n)$ of Young diagrams contained in an $m \times n$ rectangle) which have the Sperner property.

[Stan1984] ———, *Problem 4 in "Problem session"*, Contemp. Math. **34** (1984). *** to appear.

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[Stan1988a] ———, *Differential Posets*, J. Am. Math. Soc. **1** (1988), 919–961, DOI 10.2307/1990995, available at `https://www.jstor.org/stable/1990995`. GS 5318386056862341375. MR941434 Zbl 0658.05006

Introduces differential posets.

[Stan1988b] ———, *Professor Eubanks in Zetaland*, Math. Intell. **10** (1988), 21–23, available at `https://math.mit.edu/~rstan/eubanks.pdf`. GS 10132780334540601063.

Cosmic horror at the frontiers of mathematics.

[Stan1990] ———, *Variations on differential posets*, Invariant Theory and Tableaux (Dennis Stanton, ed.), IMA Volumes in Math. and Its Appls., vol. 19, Springer-Verlag, Berlin and New York, 1990, 1990, pp. 145–165, available at `https://math.mit.edu/~rstan/pubs/pubfiles/78.pdf`. GS 2941535162033905939. MR1035494 Zbl 0707.06001

U. Introduces three variations on differential posets, including "sequential" differential posets (where the differential degree r varies between different ranks of the poset) and the weighting used on the lattice of shifted partitions.

[Stan1999] ———. *Enumerative Combinatorics, Volume 2.* Cambridge Studies in Advanced Mathematics, vol. 62. Cambridge University Press, Cambridge, 1999, ISBN 978-1139639484.

U. Topics: symmetric functions, s_λ, insertion algorithm, growth diagram, S_n, $Gl(n, \mathbb{C})$.

[Stan2001] ______, *On the Enumeration of Skew Young Tableaux*, J. Comb. Theory **30** (2003), 283–294, DOI 10.1016/S0196-8858(02)00537-7, available at `https://arxiv.org/pdf/math/0106115.pdf`. GS 6375444149607743197.

Provides a formula counting the standard Young tableaux of size n which contain a fixed tableau of size k as a subtableau. Studies the asymptotic behavior of these quantities.

[Stan2003a] ______, *Supplementary Exercises for Chapter 7 (symmetric functions) of Enumerative Combinatorics, vol. 2 by Richard P. Stanley, version of 28 March 2023*, 2023, available at `https://math.mit.edu/~rstan/ec/ch7supp.pdf`.

*** not read

[Stan2003b] ______, *Solutions to Supplementary Exercises for Chapter 7 (symmetric functions) of Enumerative Combinatorics, vol. 2 by Richard P. Stanley, version of 24 March 2023*, 2023, available at `https://math.mit.edu/~rstan/ec/ch7suppsol.pdf`.

*** not read

[Stan2007] ______, *Increasing and decreasing subsequences and their variants*, International Congress of Mathematicians (Madrid, 2006), Vol. 1, EMS Press, Zürich, 2007, DOI 10.4171/022-1/21, available at `https://math.mit.edu/~rstan/papers/ids.pdf`. GS 9347272877055132718. MR2334203 Zbl 1133.05002

Abstract: We survey the theory of increasing and decreasing subsequences of permutations. Enumeration problems in this area are closely related to the RSK algorithm. The asymptotic behavior of the expected value of the length $\mathrm{is}(w)$ of the longest increasing subsequence of a permutation w of $1, 2, \cdots, n$ was obtained by Vershik–Kerov and (almost) by Logan–Shepp. The entire limiting distribution of $\mathrm{is}(w)$ was then determined by Baik, Deift, and Johansson. These techniques can be applied to other classes of permutations, such as involutions, and are related to the distribution of eigenvalues of elements of the classical groups. A number of generalizations and variations of increasing/decreasing subsequences are discussed, including the theory of pattern avoidance, unimodal and alternating subsequences, and crossings and nestings of matchings and set partitions.

[Stan2012] ______. *Enumerative Combinatorics, Volume 1.* Cambridge Studies in Advanced Mathematics, vol. 49. Cambridge University Press, Cambridge, 2nd ed., 1986, 2012. first edition 1986.

First volume of the encyclopedic work on enumerative combinatorics.

[Stan2017] ______, *Some Schubert shenanigans* (2017), arXiv 1704.00851, primary class math.CO, DOI 10.48550/arXiv.1704.00851, available at `https://arxiv.org/abs/1704.00851`. GS 342754532158627046

Abstract: We give a conjectured evaluation of the determinant of a certain matrix $\tilde{D}(n,k)$. The entries of $\tilde{D}(n,k)$ are either 0 or specializations $\mathfrak{S}_w(1,\ldots,1)$ of Schubert polynomials. The conjecture implies that the weak order of the symmetric group S_n has the strong Sperner property. A number of peripheral results and problems are also discussed.

[Stant1990] Dennis Stanton (ed.) *Invariant Theory and Tableaux.* IMA Volumes in Math. and Its Appls., vol. 19. Springer-Verlag, Berlin and New York, 1990, ISBN 978-0387971704.

[StantWhite1985] Dennis Stanton and Dennis E. White, *A Schensted algorithm for rim hook tableaux*, J. Comb. Theory **Ser. A 40** (1985), 211–247, DOI 10.1016/0097-3165(85)90088-3, available at `https://www.sciencedirect.com/science/article/pii/0097316585900883`. GS 6819589354835413324. Zbl 0577.05001

Abstract: We consider hooks, hook tableaux, rim hooks, and rim hook tableaux where all hooks and rim hooks have length k. Using a rim hook version of the "jeu d'taquin" of Schützenberger and the orientation of a rim hook (its mod k distance from the main diagonal), we give a simple alternative description of the *-diagrams of Robinson. The rim hook Schensted correspondence given by one of us in a previous paper then decomposes into a k-tuple of ordinary Schensted correspondences. This decomposition is used to "lift" the important applications of the ordinary Schensted correspondence to the rim hook Schensted correspondence. These include applications to inverses, increasing and decreasing subsequences. and the Schützenberger evacuation procedure.

[SteenSeeb1978] Lynn Arthur Steen and J. Arthur Seeback. *Counterexamples in topology.* Springer, New York, 2nd ed., 1978. GS 17754130900085227545.

Abstract: The creative process of mathematics, both historically and individually, may be described as a counterpoint between theorems and examples. Although it would be hazardous to claim that the creation of significant examples is less demanding than the development of theory, we have discovered that focusing on examples is a particularly expeditious means of involving undergraduate mathematics students in actual research. Not only are examples more concrete than theorems-and thus more accessible-but they cut across individual theories and make it both appropriate and necessary for the student to explore the entire literature in journals as well as texts. Indeed, much of the content of this book was first outlined by undergraduate research teams working with the authors at Saint Olaf College during the summers of 1967 and 1968.
In compiling and editing material for this book, both the authors and their undergraduate assistants realized a substantial increment in topological insight as a direct result of chasing through details of each example. We hope our readers will have a similar experience. Each of the 143 examples in this book provides innumerable concrete illustrations of definitions, theorems, and general methods of proof. There is no better way, for instance, to learn what the definition of metacompactness really means than to try to prove that Niemytzki's tangent disc topology is not metacompact.
The search for counterexamples is as lively and creative an activity as can be found in mathematics research.

[Stem1990] John R. Stembridge, *On symmetric functions and the spin characters of S_n*, Banach Center Pub. **26** (1990), 433–453, DOI 10.4064/-26-2-433-453 , available at `https://www.impan.pl/en/publishing-house/banach-center-publications/all/26/0/`

106968/on-symmetric-functions-and-the-spin-characters-of-s-n. GS 9928861059733703261.

Abstract: The purpose of this paper is twofold. Primarily, it is to survey the interconnections between projective (or spin) characters of S_n, shifted tableaux, and the theory of symmetric functions. For proofs of most of the results, the reader will be referred to the literature. The remainder of this paper is devoted to providing new proofs of some known results in order to illustrate the use of combinatorial methods. Specifically, in §5, we provide a new proof of the Morris character recurrence. The novelty of this proof lies in its use of shifted tableaux where previous proofs have employed the machinery of Hall–Littlewood functions. In §6, we give a purely combinatorial proof that Schur's so-called Q-functions are symmetric.

[Stone1938] Marshall Harvey Stone, *Topological representations of distributive lattices and Brouwerian logics*, Čas. Mat. Fys. **67** (1938), 1–25, available at https://eudml.org/doc/27235. GS 13361446076336132270. Zbl 0018.00303

Abstract: In a series of papers, the writer has developed a theory of Boolean algebras dealing with their algebraic structure, their representation by algebras of classes, and their relations to general topology.1) It is the object of the present paper to outline an extension of the main features of this theory to the more general systems known variously as distributive lattices, C-lattices, or arithmetic structures.2)

[Sulz2009] Jay Sulzberger, *On the Second Half of Boole's Laws of Thought*, 2009, available at https://www.panix.com/~jays/Boole.four.page.summary.for.1.August.2006.Cork.conference.Sulzberger.v.5.pdf. GS 5351877968224812310.

Abstract: George Boole in his Laws of Thought [3], published in 1854, defined the following problem of estimating an unknown probability given certain data. We are given $\mathbf{B}$, a finite Boolean algebra and a subset E of the events of $\mathbf{B}$. The pair $(\mathbf{B}, E)$ is called a Venn diagram. In general E will not be a Boolean subalgebra of $\mathbf{B}$. We are also given a function $q : E \to \mathbb{R}$, that is, from E to the reals. We wish now to find a probability p on $\mathbf{B}$ which extends q.

[Sund1986] Sheila Sundaram, *On the Combinatorics of Representations of* $Sp(2n, \mathbb{C})$, Ph.D. thesis, Massachusetts Inst. of Tech., 1986, https://dspace.mit.edu/handle/1721.1/15060. GS 3851931148765577100.

UD. Topics: symmetric functions, s_λ, insertion algorithm, jeu de taquin, evacuation, representations of $Sp(n, \mathbb{C})$, sp_λ, Knuth equivalence. Introduction to tableaux, symmetric functions, insertion algorithms, representations of $Gl(n, \mathbb{C})$, representations of $Sp(n, \mathbb{C})$, sp_λ.

Abstract: In this thesis we study the finite-dimensional representations of the symplectic group $Sp(2n, \mathbb{C})$ from a combinatorial point of view. It is known that the irreducible representations of $Sp(2n, \mathbb{C})$ are indexed by partitions λ with at most n parts, and also that to each irreducible representation $\tilde{\phi}_\lambda$ corresponding to the partition λ, one can associate a finite set of semi-standard tableaux (which we call symplectic tableaux) of shape λ whose cardinality is precisely the dimension of $\tilde{\phi}_\lambda$. On passing from the representation $\tilde{\phi}_\lambda$ to its character one obtains a polynomial $sp_\lambda(x_1, x_1^{-1}, \ldots, x_n, x_n^{-1})$ (in the $2n$ variables $x_1, x_1^{-1}, \ldots, x_n, x_n^{-1}$) which can be expressed as an integral linear combination of monomials in $\{x_1, x_1^{-1}, \ldots, x_n, x_n^{-1}\}$, each monomial arising from a symplectic tableau of shape λ. From the tableau description or otherwise it follows that the characters $sp_\lambda(x_1, x_1^{-1}, \ldots, x_n, x_n^{-1})$ are invariant polynomials under the action of the hyperoctahedral group B_n, the Weyl group of $Sp(2n, \mathbb{C})$. The finite-dimensional irreducible representations of $Sp(2n, \mathbb{C})$ may therefore be studied by means of the "symplectic" Schur functions sp_λ where λ runs over all partitions with at most n parts. For $Sp(2n, \mathbb{C})$ a Schensted-type algorithm which produces symplectic tableaux was recently developed by Allan Berele [Ber1986]. Here the standard tableaux occurring as the right tableaux of the Knuth–Schensted insertion process are replaced by sequences of shapes $S_\mu^k = (\emptyset = \mu^0, \mu^1, \ldots, \mu^k = \mu)$ where any two consecutive shapes differ by exactly one box. We refer to such a sequence as an up-down tableau.

[Sund1990a] ———, *Tableaux in the Representation Theory of the Classical Lie Groups* **19** (1990), 191–225 pp., available at https://www.researchgate.net/publication/234460143_Tableaux_in_the_Representation_

Theory_of_the_Classical_Lie_Groups.
GS 5823057563823619308. Zbl 0707.22004

Abstract: Column-strict tableaux were introduced by I. Schur in his study of the characters of $Gl(n)$. Following early work of Alfred Young, (who introduced partitions and tableaux in the study of the representations of the symmetric group), and Specht, Weyl used Young symmetrisers to construct the irreducible modules for the polynomial representations of the classical Lie groups. This paper surveys the connections between the combinatorics and the algebra which arise from these classical constructions.

[Sund1990b] ———, *Orthogonal tableaux and an insertion algorithm for $SO(2n+1)$*, J. Comb. Theory **Ser. A 53** (1990), 239–256, DOI 10.1016/0097-3165(90)90059-6, available at https://www.sciencedirect.com/science/article/pii/0097316590900596.
GS 1828873941940003787. Zbl 0723.05119

Abstract: A new set of tableaux indexing the weights of the irreducible representations of $SO(2n+1)$ is presented. These tableaux are used to produce an insertion scheme which gives a combinatorial description of the decomposition of the kth tensor power of the natural action of $SO(2n+1)$ into irreducibles. In particular, the multiplicities in this decomposition are described explicitly.

[Sund1990c] ———, *The Cauchy identity for $Sp(2n)$*, J. Comb. Theory **Ser. A 53** (1990), 209–238, DOI 10.1016/0097-3165(90)90058-5, available at https://www.sciencedirect.com/science/article/pii/0097316590900585.
GS 16996139505809647818. Zbl 0707.05005

Abstract: A bijection establishing the Cauchy identity for $Sp(2n)$

$$\prod_{1\le i<j\le n}(1-t_it_j)\prod_{i,j=1}^{n}(1-t_ix_j)^{-1}(1-t_ix_j^{-1})^{-1}$$

$$=\sum_{\substack{\mu\\ \ell(\mu)\le n}} sp_\mu(x_1^{\pm1},\dots,x_n^{\pm1})s_\mu(t_1,\dots,t_n)$$

is presented, using the insertion algorithm of Berele. A key element in the bijection is a new encoding of up-down tableaux. We present this as a correspondence proving the following enumerative formula for the number of up-down tableaux of length k and shape μ:

$$\tilde{f}^{\mu}_{k} = \sum_{\substack{\beta \vdash (k-|\mu|) \\ \beta \text{ even}}} \binom{k}{|\mu|} f^{\beta} f^{\mu}$$

[Swan2019] Joshua P. Swanson, *A gentle introduction to coinvariant algebras*, 2019, available at https://www.jpswanson.org/talks/2019_gentle_coinvariants.pdf.

[Thib1997] Jean-Yves Thibon, A. Lascoux, and B. Leclerc, *The plactic monoid*, Algebraic Combinatorics on Words (M. Lothaire, ed.), Cambridge University, Cambridge, 2002, 1997, DOI 10.1007/s00233-004-0146-9, available at http://igm.univ-mlv.fr/~jyt/ARTICLES/plactic.ps.

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[Thrall1951] R. M. Thrall, *On the Projective Structure of a Modular Lattice*, Proc. Am. Math. Soc. **2** (1951), 146–152, available at https://www.ams.org/journals/proc/1951-002-01/S0002-9939-1951-0041104-4/S0002-9939-1951-0041104-4.pdf. GS 4998496641867146321.

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[Thur1994] William P. Thurston, *On proof and progress in mathematics* (1994), arXiv 9404236, primary class math.HO, DOI 10.48550/arXiv.math/9404236, available at https://arxiv.org/abs/9404236. GS 18255111716587044243

Abstract: This essay on the nature of proof and progress in mathematics was stimulated by the article of Jaffe and Quinn, "Theoretical Mathematics: Toward a cultural synthesis of mathematics and theoretical physics". Their article raises interesting issues that mathematicians should pay more attention to, but it also perpetuates some widely held beliefs and attitudes that need to be questioned and examined. . . .

Responses to the Jaffe–Quinn article have been invited from a number of mathematicians, and I expect it to receive plenty of specific analysis and criticism from others. Therefore, I will concentrate in this essay on the positive rather than on the contranegative. I will describe my view of the process of mathematics, referring only occasionally to Jaffe and Quinn by way of comparison.

[Tref2008] LLoyd Nicholas Trefethen, *Ten digit algorithms*, 2008, available at https://people.maths.ox.ac.uk/trefethen/tda05.pdf. GS 7630042518326471738.

Abstract: A view has come into focus for me of a kind of computing which I summarize by the notion of a "ten digit algorithm". I believe that ten digit algorithms can be useful for education, for communication, and for research. Many of the programs I've written in recent years are in this mode, and if you spend your time computing with numbers, I urge you too to make ten digit algorithms a part of your operating practice. A ten digit algorithm is a little gem of a program to compute something numerical. The jingle summarizes the three defining conditions. The program can be at most one page long, and it has to solve your problem to at least ten digits of accuracy on your machine in less than five seconds.

[Tru2024] Luke Trujello, *Category Theory*, https://ltrujello.github.io/category_theory/. Accessed October 9, 2024.

Abstract: These are a set of notes on category theory I worked on for my latter two years as a mathematics undergraduate. It covers many different areas of category theory, contains many examples in pure mathematics, and has hundreds of original mathematical diagrams.

[Tům1995] Jiří Tůma, *A new proof of Whitman's embedding theorem*, J. Algebra **173** (1995), 459–462, DOI 10.1006/jabr.1995.1097, available at https://core.ac.uk/download/pdf/82382814.pdf. https://www.sciencedirect.com/science/article/pii/S0021869385710976 GS 3767872871398892251.

Abstract: This paper contains a short and direct proof of the following result.

Theorem. Every lattice can be embedded in the subgroup lattice of some infinite group.

[User100478-2014] User100478, *Haar measure on locally compact semigroups*, https://mathoverflow.net/questions/180019/haar-measure-on-locally-compact-semigroups. Accessed August 4, 2024.

Abstract: I'm reading on Haar measure and we know that every locally compact group admits a Haar measure, is the same true for semigroups? if not, is there a class of semigroups that admits a Haar measure?

[VandenBerg2012a] Benno van den Berg, *Craig interpolation, Beth's definability theorem, Chang-Los-Suszko theorem*, 2012, available at https://staff.fnwi.uva.nl/b.vandenberg3/Onderwijs/Modeltheory_2012/slides_modeltheorie_lecture4.pdf.

Discusses the topics in model thory named in the title. Particularly useful for the Chang-Łoś-Suszko theorem, which says that if a Π_2 sentence is true of all the objects in a direct system, it is true of the direct limit.

[VanLeeu1995] Marc A. A. van Leeuwen, *The Robinson-Schensted and Schützenberger algorithms, an elementary approach*, Electron. J. Comb. **3** (1996), DOI 10.37236/1273, available at https://www.combinatorics.org/ojs/index.php/eljc/article/view/1273. GS 1634788482809564912.

Abstract: We discuss the Robinson–Schensted and Schützenberger algorithms, and the fundamental identities they satisfy, systematically interpreting Young tableaux as chains in the Young lattice. We also derive a Robinson–Schensted algorithm for the hyperoctahedral groups. Finally we show how the mentioned identities imply some fundamental properties of Schützenberger's glissements.

[VanLeeu1997] Marc A. A. van Leeuwen, *Edge Sequences, Ribbon Tableaux, and an Action of Affine Permutations*, Report — Modelling, Analysis and Simulation **7** (1997), 1–13, available at http://www-math.univ-poitiers.fr/~maavl/pdf/edgeseqs.pdf. GS 6918139501590261128.0918.05102

Sec. 2.1 details the correspondences between edge sequences and several other ways of describing partitions.

Abstract: An overview is provided of some of the basic facts concerning rim hook lattices and ribbon tableaux, using a representation of partitions by their edge sequences. An action is defined of the affine Coxeter group of type $\tilde{A}_{r-1}$ on the r-rim hook lattice, and thereby on the sets of standard and semistandard r-ribbon tableaux, and this action is related to the concept of chains in r-ribbon tableaux.

[Warn1989] Seth Warner. *Topological fields.* Mathematics Studies, vol. 157. North-Holland, Amsterdam, 1989. GS 9310640854402547515.

Abstract: This text brings to the frontiers of current research in topological fields (more precisely, topological rings that algebraically are fields) a reader having an acquaintance with some very basic point-set topology and algebra, which is normally presented in semester courses at the beginning graduate or even undergraduate level. Not every result of importance is included in the text, but it does give the reader more than enough background for tackling the current literature without undue additional preparation. Many results not in the text and many illustrations by example of theorems in the text are included among the exercises, sufficient hints for the solution of which are offered so that solving them does not become a major research effort for the reader. Within certain constraints, a bibliography intended to be complete is given. Expectations of a reader include some familiarity with topological, Hausdorff, metric, compact and locally compact spaces and basic properties of continuous functions, also with groups, rings, fields, vector spaces and modules, and with Zorn's Lemma. Additional topology and algebra are developed in the text as needed.

[Weig2018] Anna E. Weigandt, *Schubert polynomials, 132-patterns, and Stanley's conjecture*, Algebr. Comb. **1** (2018), 415–423, DOI 10.5802/alco.27, available at `https://www.numdam.org/item/10.5802/alco.27.pdf`. GS 4936433957059909693. Zbl 1397.05205

Abstract: Motivated by a recent conjecture of R. P. Stanley we offer a lower bound for the sum of the coefficients of a Schubert polynomial in terms of 132-pattern containment.

[Weil1974] André Weil. *Basic number theory.* Springer, Berlin, 3rd ed., 1974. GS 15357378455036434493.

Abstract: The first part of this volume is based on a course taught at Princeton University in 1961-62; at that time, an excellent set of notes was prepared by David Cantor, and it was originally my intention to make these notes available to the mathematical public with only quite minor changes. Then, among some old papers of mine, I accidentally came across a long-forgotten manuscript by Chevalley, of pre-war vintage (forgotten, that is to say, both by me and by its author) which, to my taste at least, seemed to have aged very well. It contained a brief but essentially complete account of the main features of classfield theory, both local and global; and it soon became obvious that the usefulness of the intended volume would be greatly enhanced if I included such a treatment of this topic. It had to be expanded, in accordance with my own plans, but its outline could be preserved without much change. In fact, I have adhered to it rather closely at some critical points.

[Weyl1946] Hermann Weyl. *The classical groups, their invariants and representations.* Princeton landmarks in mathematics and physics. Princeton Univ. Press, Princeton, N.J., US, 1946. GS 11975299567775909453.

Abstract: Ever since the year 1925, when I succeeded in determining the characters of the semi-simple continuous groups by a combination of E. Cartan's infinitesimal methods and I. Schur's integral procedure, I have looked toward the goal of deriving the decisive results for the most important of these groups by direct algebraic construction, in particular for the full group of all non-singular linear transformations and for the orthogonal group. Owing mainly to R. Brauer's intervention and collaboration during the last few years, it now appears that I have in my hands all the tools necessary for this purpose. The task may be characterized precisely as follows: with respect to the assigned group of linear transformations in the underlying vector space, to decompose the space of tensors of given rank into its irreducible invariant subspaces. In other words, our concern is with the various kinds of "quantities" obeying a linear transformation law, which may be prepared under the reign of each group from the material of tensors. Such is the problem which forms one of the mainstays of this book, and in accordance with the algebraic approach its solution is sought for not only in the field of real numbers on which analysis and physics fight their battles, but in an arbitrary field of characteristic zero. However, I have made no attempt to include fields of prime characteristic.

[Whit1946] Philip M. Whitman, *Lattices, equivalence relations, and subgroups*, Bull. Am. Math. Soc. **52** (1946), 507–522, available at https://community.ams.org/journals/bull/1946-52-06/S0002-9904-1946-08602-4/S0002-9904-1946-08602-4.pdf. https://www.ams.org/journals/bull/1946-52-06/S0002-9904-1946-08602-4/ GS 4422758468344119213.

Abstract: It is well known that the collection of all equivalence relations on a given set $\mathfrak{S}$ forms a lattice. Ore [4] has characterized lattices which are isomorphic to the lattice $\mathfrak{L}^*$ of all equivalence relations on some set $\mathfrak{S}$. One could go farther and ask what lattices are isomorphic to sublattices of $\mathfrak{L}^*$. Our answer to this is: *any* lattice is isomorphic to a sublattice of the lattice of all equivalence relations on some set; more concisely, any lattice is a lattice of equivalence relations.
Garrett Birkhoff has shown [Birk1935] that any lattice of equivalence relations is isomorphic to a lattice of subgroups. Therefore the result stated in the previous paragraph implies: any lattice is isomorphic to a sublattice of the lattice of all subgroups of a suitable group.

[WikiAt] https://en.wikipedia.org/wiki/Atom_(order_theory). Accessed June 26, 2024.

Wikipedia entry “Atom (order theory)”.

[WikiComp] https://en.wikipedia.org/wiki/Complex_reflection_group. Accessed April 15, 2026.

Wikipedia entry “Complex reflection group”.

[WikiCong] https://en.wikipedia.org/wiki/Congruence-permutable_algebra. Accessed April 11, 2025.

Wikipedia entry “Congruence-permutable algebra”.

[WikiDir] https://en.wikipedia.org/wiki/Direct_limit. Accessed January 15, 2024.

Wikipedia entry “Direct limit”.

[WikiDirSet] https://en.wikipedia.org/wiki/Directed_set. Accessed November 29, 2024.

Wikipedia entry "Directed set".

[WikiDualDist] https://en.wikipedia.org/wiki/Duality_theory_for_distributive_lattices. Accessed May 8, 2026.

Wikipedia entry "Duality theory for distributive lattices".

[WikiEl] https://en.wikipedia.org/wiki/Elementary_class. Accessed February 12, 2024.

Wikipedia entry "Elementary class".

[WikiElEq] https://en.wikipedia.org/wiki/Elementary_equivalence. Accessed September 9, 2024.

Wikipedia entry "Elementary equivalence", including the definition of "elementary substructure".

[WikiFin] https://en.wikipedia.org/wiki/Finite_field. Accessed January 15, 2024.

Wikipedia entry "Finite field". Section "Algebraic closure" discusses assembling $\mathbb{F}_{p^n}$ for all $n \geq 1$ into their algebraic closure $\bar{\mathbb{F}}_p$.

[WikiFunc] https://en.wikipedia.org/wiki/Functor. Accessed November 9, 2024.

Wikipedia entry "Functor".

[WikiGal] https://en.wikipedia.org/wiki/Galois_connection. Accessed April 21, 2024.

Wikipedia entry "Galois connection".

[WikiHahn] https://en.wikipedia.org/wiki/Hahn_series. Accessed July 9, 2024.

Wikipedia entry "Hahn series".

[WikiHyp] https://en.wikipedia.org/wiki/Hyperoctahedral_group. Accessed April 23, 2026.

Wikipedia entry "Hyperoctahedral group".

[WikiLev] https://en.wikipedia.org/wiki/Levi-Civita_field. Accessed July 9, 2024.

Wikipedia entry "Levi-Civita field".

[WikiNat] https://en.wikipedia.org/wiki/Natural_transformation. Accessed November 9, 2024.

Wikipedia entry "Natural transformation".

[WikiPapp] https://en.wikipedia.org/wiki/Pappus's_hexagon_theorem. Accessed October 22, 2024.

Wikipedia entry "Pappus's hexagon theorem".

[WikiPuis] https://en.wikipedia.org/wiki/Puiseux_series. Accessed July 3, 2024.

Wikipedia entry "Puiseux series".

[WikiRed] https://en.wikipedia.org/wiki/Reduced_product. Accessed July 3, 2025.

Wikipedia entry "Reduced product".

[WikiUlt] https://en.wikipedia.org/wiki/Ultraproduct. Accessed October 20, 2024.

Wikipedia entry "Ultraproduct".

[WikiVar] https://en.wikipedia.org/wiki/Variety_(universal_algebra). Accessed January 21, 2024.

Wikipedia entry "Variety (universal algebra)". Includes summary of Birkhoff's variety theorem.

[WikiVeb] https://en.wikipedia.org/wiki/Veblen-Young_theorem. Accessed October 22, 2024.

Wikipedia entry "Veblen–Young theorem".

[WikiWedd] https://en.wikipedia.org/wiki/Wedderburn%27s_little_theorem. Accessed October 22, 2024.

Wikipedia entry "Wedderburn's little theorem".

[WiktJeu] https://en.wiktionary.org/wiki/jeu_de_taquin. Accessed June 9, 2023.

Wiktionary entry "jeu de taquin".

[Wild1992] Marcel Maria Wolfgang Wild, *Modular lattices of finite length*, 1992, available at `https://www.researchgate.net/publication/259778597_Modular_lattices_of_finite_length`. GS 15293954682574294326.

Abstract: We review some well known results on *complemented* modular lattices, and indicate how they will be generalized to arbitrary modular lattices in the subsequent sections. These sections are 2) Modular lattice geometry, 3) Axiomatizing bases of lines, 4) Lower bounds for the number of join irreducibles, 5) Existence of linear representations, 6) Classification of linear representations. Sections 2, 4, 5, 6 are mainly based on [HW], which in turn was inspired by [FH], [JN], [W2]. Carrying out a suggestion of G.C. Rota, the exposition given here is more detailed (with additional introduction, figures, examples). Furthermore some corrections and extension have been incorporated. Section 3 is based on [HPR].

[Wild2022a] ———, *Modular lattices of finite length (Part A)* (2022), arXiv 2201.10815, primary class math.CO, DOI 10.48550/arXiv.2201.10815, available at `https://arxiv.org/abs/2201.10815`. GS 9231186045234909430

Abstract: This is Part A of four Parts dedicated to modular lattices of finite length. It builds on 1992 notes of the author (available on ResearchGate), and in so doing heeds a wish of the late Gian-Carlo Rota. Part A is in fairly final form and mainly features known material, exceptions being a short proof of the distributivity of congruence lattices of lattices, as well as the concept of a point-splitting (which applies to arbitrary partial linear spaces). The planned content of Parts B,C,D is given in the introduction of Part A. Suffice it to say that deep results from [this http URL] and [this http URL] will be treated in English for the first time. All of this (even Part A) is work in process and comments/contributions are welcome.

[Wild2022b] ———, *Modular lattices of finite length (Part B)* (2022), arXiv 2202.04120, primary class math.CO, DOI 10.48550/arXiv.2202.04120, available at `https://arxiv.org/abs/2202.04120`. GS 9231186045234909430

Abstract: Part B (of a project involving four Parts) is about “bases of lines”, a concept introduced by C. Herrmann and the author in the late 80’s. Bases of lines attempt to describe a given modular lattice in a geometric way akin to how projective geometries describe complemented modular lattices. This e.g. yields the result that each modular lattice L of finite length $d(L)$, and having $s(L)$ many maximal congruences, has at least $2d(L) - s(L)$ many join-irreducible elements. Furthermore, an algorithm is proposed that calculates, in a compressed way, the (full) submodule lattice $\mathrm{Sub}(W)$ of any (sufficiently known) finite R-module W.

[Wildon2018] Mark Wildon, *A generalized SXP rule proved by bijections and involutions*, Ann. Comb. **22** (2018), 885–905, DOI 10.1007/s00026-018-0409-x, available at `https://link.springer.com/article/10.1007/s00026-018-0409-x`. GS 13362973608770886694. Zbl 1407.05242

Abstract: This paper proves a combinatorial rule expressing the product $s_\tau(s_{\lambda/\mu} \circ p_r)$ of a Schur function and the plethysm of a skew Schur function with a power-sum symmetric function as an integral linear combination of Schur functions. This generalizes the SXP rule for the plethysm $s_\lambda \circ p_r$. Each step in the proof uses either an explicit bijection or a sign- reversing involution. The proof is inspired by an earlier proof of the SXP rule due to Remmel and Shimozono (Discrete Math. 193:257–266, 1998). The connections with two later combinatorial rules for special cases of this plethysm are discussed. Two open problems are raised. The paper is intended to be readable by non-experts.

[Will2025] Lauren Kiyomi Williams, *Cyclic partial orders, Parke–Taylor polytopes, and the magic number conjecture for the amplituhedron*, 2025-3-08, Univ. of Washington, Seattle, Wash., U.S., Cascade Lectures in Combinatorics, available at `https://pages.uoregon.edu/plhersh/CALICO/MagicNumberCascade.pdf`

Abstract:
(1) Partial cyclic orders and bicolored subdivisions
(2) Applications to Parke–Taylor identities and Parke–Taylor polytopes
(3) What is the amplituhedron?
(4) Magic number conjecture for the amplituhedron

(5) Proof of Magic number conjecture when m = 2

[Willi2023] Daniel Williams, *The marketplace of rationalizations*, Economics and Philosophy **39** (2023), 99–123, DOI 10.1017/S0266267121000389, available at https://www.cambridge.org/core/journals/economics-and-philosophy/article/marketplace-of-rationalizations/41FB096344BD344908C7C992D0C0C0DC. GS 5811287759851912342.

Abstract: Recent work in economics has rediscovered the importance of belief-based utility for understanding human behaviour. Belief 'choice' is subject to an important constraint, however: people can only bring themselves to believe things for which they can find rationalizations. When preferences for similar beliefs are widespread, this constraint generates rationalization markets, social structures in which agents compete to produce rationalizations in exchange for money and social rewards. I explore the nature of such markets, I draw on political media to illustrate their characteristics and behaviour, and I highlight their implications for understanding motivated cognition and misinformation.

[Woo2025] Alexander Woo, *Towards Hessenberg–Schubert calculus*, 2025-03-08, Univ. of Washington, Seattle, Wash., U.S., Cascade Lectures in Combinatorics, available at https://pages.uoregon.edu/plhersh/CALICO/CALICO-AW.pdf

Abstract:
(1) Motivation: Stanley–Stembridge conjecture
(2) Cohomology rings of Hessenberg varieties
(3) Schubert varieties and classes
(4) Calculating Hessenberg–Schubert classes
(5) The permutahedral cas

[Wor1984] Dale R. Worley, *A Theory of Shifted Young Tableaux*, Ph.D. thesis, Massachusetts Inst. of Tech., 1984, https://dspace.mit.edu/handle/1721.1/15599. GS 7106851617217394040. MR2941073

S. Greene invariant, insertion algorithm, jeu de taquin, symmetric functions.

Abstract: This thesis develops a combinatorial theory of shifted Young tableaux that parallels the existing theory of (unshifted) Young tableaux. A shifted Young tableau is similar to an unshifted Young tableau, except that each successive row is indented one position rightward from the preceding row. The structure of the theory is modeled after that given by Schützenberger in the Strasbourg conference in 1976 [Schüt1976], and contains the following elements: (1) a version of the Robinson–Schensted–Knuth correspondence which transforms sequences of numbers into pairs of shifted tableaux, (2) the Knuth relations, a set of elementary transformations on sequences of numbers which characterize the sets of sequences which produce the same first tableau under the insertion algorithm, (3) a combinatorial function similar to the one discovered by Greene that characterizes these sets of sequences, and (4) a version of Schützenberger's operation "jeu de taquin" under which a diagonal line of numbers is reduced to the first tableau of its corresponding pair. Several applications of the theory to symmetric functions and several open problems are discussed.

[Wor2023a] ______, *An extension of Birkhoff's representation theorem to infinite distributive lattices* (2023), arXiv 2303.04267, primary class math.CO, DOI 10.48550/arXiv.2303.04267, available at `https://arxiv.org/abs/2303.04267`. GS 18024783941681048958

Proof of an extension of Birkhoff's Representation Theorem to a class of infinite distributive lattices. Includes an alternative antecedent that is conditions on infinite ascending and descending chains.

[Wor2023b] ______, *Does counting extensions of standard Young tableaux determine its shape?*, `https://mathoverflow.net/questions/444983/does-counting-extensions-of-standard-young-tableaux-determines-its-shape`. Accessed June 21, 2023.

Queries the research community whether counting the extensions of a standard Young tableaux of different sizes determines its shape.

[Wor2023c] ———, *On the combinatorics of tableaux — Graphical representation of insertion algorithms* (2023), arXiv 2306.11140, primary class math.CO, DOI 10.48550/arXiv.2306.11140, available at https://arxiv.org/abs/2306.11140. GS 2318818649542884742

Introduces "insertion diagrams" for representing the R-correspondences which are the add-in to growth diagrams that define different insertion algorithms. Also a listing of many of the described insertion algorithms.

[Wor2024a] ———, *Can we extend "every finite lattice is a sublattice of partitions of a finite set" to linear and/or finitary lattices?*, https://mathoverflow.net/questions/461875/can-we-extend-every-finite-lattice-is-a-sublattice-of-partitions-of-a-finite-se. Accessed February 13, 2024.

Abstract: Pudlák and Tůma https://link.springer.com/article/10.1007/BF02482893 proved that every finite lattice can be embedded as a sublattice of the partition lattice of a finite set. Can this be generalized in either of the following ways, or preferably both together?
If the finite lattice L can be represented linearly as partitions of some set, can an embedding into the partition lattice of a finite set B always be chosen?
Relax the condition on L from finite to finitary, that is, every principal (lower) ideal is finite. Clearly L as a whole cannot be represented by partitions of any finite set B. But can we require that given any principal ideal $[\hat{0}, x]$ of L, restricting the representation to the ideal makes it "quasi-finite" in some suitable sense?

[Wor2024b] ———, *A survey of lattice properties: modular, Arguesian, linear, and distributive* (2024), arXiv 2403.19677, primary class math.HO, DOI 10.48550/arXiv.2403.19677, available at https://arxiv.org/abs/2403.19677. GS 16209358479822905797

This is a survey of characterizations and relationships between some properties of lattices, particularly the modular, Arguesian, linear, and distributive properties, but also some other related properties. The survey emphasizes finite and finitary lattices and deemphasizes complemented lattices.

A final section is a restatement of the open questions, which may prove to be a source of thesis problems.

[Wor2024c] ———, *Extending Birkhoff's representation theorem to modular lattices*, 2024-06-03, Harvard Univ., Cambridge, Mass., U.S., available at `https://theworld.com/~worley/Math/representation-modular-lattices.v1.pdf`. video at `https://youtu.be/Oa7iXILyN8U?t=1390`

Abstract: Goal: Construct a nice way to represent/classify modular lattices
Applications:
(1) Construct "tableaux" that nicely represent saturated chains in general modular lattices.
(2) Find/classify (weighted) differential structures on general modular lattices for use in RSK algorithms.

[Wor2025a] ———, *On the structure of modular lattices — Unique gluing and dissection* (2025), arXiv 2502.08934, primary class math.CO, DOI 10.48550/arXiv.2502.08934, available at `https://arxiv.org/abs/2502.08934`. GS 3646492554472739512

Abstract: This work proves that the process of gluing finite lattices to form a larger lattice is bijective, that is each lattice is the glued sum of a unique system of finite lattices, provided the class of lattices is constrained to modular, locally-finite lattices with finite covers. The results of this work are not surprising given the prior literature, but this seems to be the first proof that the processes of gluing and dissection can be made inverses, and hence that gluing is bijective.

[Wor2025b] ———, *On the structure of modular lattices — Axioms for gluing* (2025), arXiv 2504.05507, primary class math.CO, DOI 10.48550/arXiv.2504.05507, available at `https://arxiv.org/abs/2504.05507`

Abstract: This paper explores alternative statements of the axioms for lattice gluing, focusing on lattices that are modular, locally finite, and have finite covers, but may have infinite height. We give a set of "maximal" axioms that maximize what can be immediately adduced about the structure of a valid gluing. We also give a set of "minimal" axioms that minimize what needs to be adduced to prove that a system of blocks is a valid gluing. This system appears to be novel in the literature. A distinctive feature of the minimal axioms is that they involve only relationships between elements of the skeleton which are within an interval $[x \wedge y, x \vee y]$ where either x and y cover $x \wedge y$ or they are covered by $x \vee y$. That is, they have a decidedly *local* scope, despite that the resulting sum lattice, being modular, has *global* structure, such as the diamond isomorphism theorem.

[Wor2026a] ———, *On the combinatorics of tableaux — A notebook of open problems* (2026), arXiv 2509.25446, primary class math.CO, DOI 10.48550/arXiv.2509.25446, available at https://arxiv.org/abs/2509.25446

Abstract: Inspired by the the Kourovka Notebook of unsolved problems in group theory[KhukhMaz2024], this is a notebook of unsolved problems in the combinatorics of tableaux. Contributions to the notebook are invited.

[Wor2026b] ———, *On the combinatorics of tableaux — Classification of lattices underlying Schensted correspondences* (2026), arXiv 2511.07611, primary class math.CO, DOI 10.48550/arXiv.2511.07611, available at https://arxiv.org/abs/2511.07611

Abstract: The celebrated Robinson–Schensted algorithm and each of its variants that have attracted substantial attention can be constructed using Fomin's "growth diagram" construction from a modular lattice that is also a weighted-differential poset. We classify all such lattices that meet certain criteria; the main criterion is that the lattice is distributive. Intuitively, these criteria seem excessively strict, but all known Fomin lattices satisfy all of these criteria, with the sole exception of one family that is not even distributive, the Young–Fibonacci lattices. Disappointingly, we discover no new Fomin lattices.

[Wor2026c] ———, *An extension of Birkhoff's representation theorem to locally-finite distributive lattices* (2026), arXiv 2603.05841, primary class math.CO, DOI 10.48550/arXiv.2603.05841, available at https://arxiv.org/abs/2603.05841

Abstract: Birkhoff's representation theorem for finite distributive lattices states that any finite distributive lattice is isomorphic to the lattice of order ideals (lower sets) of the partial order of the join-irreducible elements of the lattice. We present a simplified version of Stone's extension of this theorem to general distributive lattices. We then apply this formulation to locally finite distributive lattices to produce a novel representation theorem: The lattice is isomorphic to the order ideals of the poset of prime filters of the lattice whose symmetric difference from a particular ideal is finite.

[Wor2026d] ———, *Representation of locally-finite distributive lattices*, 2026-04-13, Brandeis Univ., Waltham, Mass., U.S., Brandeis Combinatorics Seminar, available at `https://theworld.com/~worley/Math/locally-finite-pres.pdf`

Presentation of [Wor2026c].

[Wor2026e] ———, *Constructing/analyzing differential distributed lattices* (2026), arXiv 2603.23741, primary class math.CO, DOI 10.48550/arXiv.2603.23741, available at `https://arxiv.org/abs/2603.23741`

Abstract: We restate a process presented by Stanley as a technique to prove that there exists exactly one d-differential distributive lattice for any positive integer d. This process can be trivially extended to apply to distributive finitary lattices that have a variety of differential poset structures. It can be viewed as an algorithm for constructing such lattices. Alternatively, it can be viewed as an algorithm for analyzing and characterizing such lattices.
We show that the process can be used to prove properties of all weighted-differential lattices with positive weights. We present this with the hope that this approach can be used as the basis for a complete characterization of distributive lattices with a weighted-differential structure with positive weights.

[Yan1998] Catherine Huafei Yan, *Distributive laws for commuting equivalence relations*, Discrete Math. **181** (1998), 295–298, DOI 10.1016/S0012-365X(97)00061-7, available at `https://core.ac.uk/download/pdf/82571977.pdf`. GS 17278291977600796519.

[Ore1942] found necessary and sufficient conditions under which the modular and distributive laws hold in the lattice of equivalence relations on a set S. In the present paper, we consider commuting equivalence relations. It has been proved by [Jóns1953b] that the modular law holds in the lattice of commuting equivalence relations. We give some necessary and sufficient conditions for the distributive law and its dual to hold for commuting equivalence relations.

[Yuan2023] Qiaochu Yuan, *Name of a family of bijections among a family of sets that is closed under composition [answer]*, `https://math.stackexchange.com/questions/4785851/name-of-a-family-of-bijections-among-a-family-of-sets-that-is-closed-under-compo`. Accessed October 13, 2023.

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