Unifying Gravity and EM by Analogies to EM

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Sponsor: John A. Fries

Short Description:

Investigate an old hypothesis, that gravity is similar to EM. Clone a Lagrange density for gravity from EM. A problem has been the distance dependence. Impress friends by deriving Newton's law of gravity using perturbations of a normalized potential. The mundane chain rule may eliminate the need for dark matter and energy. The subtle underlying idea will be discussed while Grand Marnier chocolate truffles are served.

Skeptics welcome.

Day 1: Table of Contents

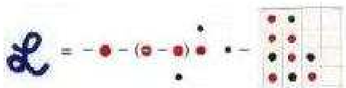
Unifying Gravity and EM by Analogies to EM

Outline for Lectures

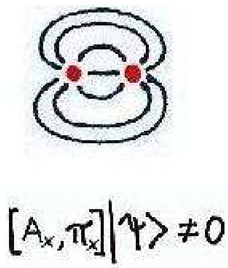
Day 1: Lagrange densities.

Day 2: Fields and quantum mechanics.

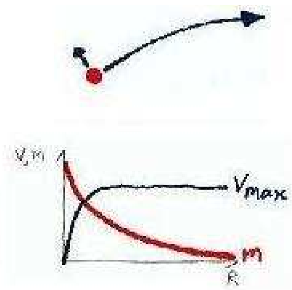
Day 3: Forces, metrics, and new physics.



Day 1



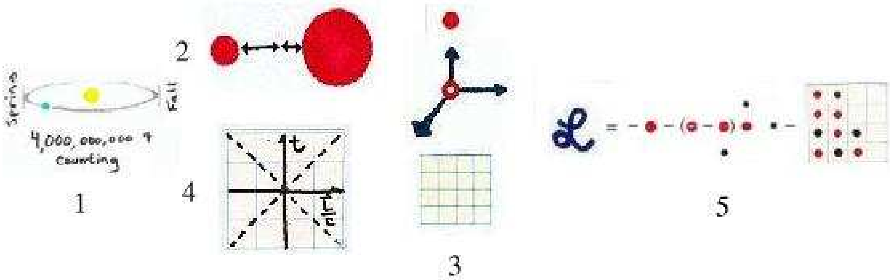
Day 2



Day 3

Outline for Day 1

1. The big picture.
2. Must do physics.
3. Tensors.
4. Units.
5. Lagrange Densities.



Required Skills

- Algebra.
- AP-level calculus.
- Ability to learn *fast*.

Helpful knowledge:

Lagrangians, calculus of variations, complex analysis, dimensional analysis, the Maxwell equations, general relativity, quantum mechanics, perturbation theory, group theory, astrophysics.

$$\frac{\partial}{\partial t}\vec{\nabla}\times\vec{A} - \nabla\times\frac{\partial \vec{A}}{\partial t} - \nabla\times\nabla\phi = 0 \checkmark$$

(Title) Information Structure

(Preamble) Definition or explanation.

- (Example 1) Slides.

- (Example 2) Slide summary.
- (Example 3) Hardcopy from web at quaternions.com.

1. (Start) Outline or math derivation.
2. (End) Interdependent task completed.

Comment, such as trying to make less than 7 info chunks/slide.

Warning: Visual information may be imprecise!

My apartment looks different.

Slide 57 count to end + random remarks.

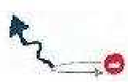
The Big Picture: A 4D Slinky

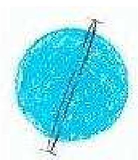
Gravity and light form a slinky in four dimensions (three for space, one for time).

- A slinky wobbles.
- The Earth has wobbled around the Sun 4 billion times.
- Light is created by electrons wobbling.

Want a description of all the interactions in a volume (called a Lagrange density) that can be used to create 4 differential equations (a 4D wave equation). The solutions to those equations must then be linked to the simple harmonic oscillations displayed by gravitational and electronic systems.

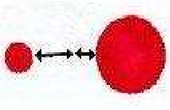
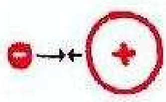
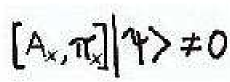
Thought experiment: slow neutrinos could wobble through the Earth act as a SHO, cycling to the other side of the Earth and back every 88 minutes. This is a longitudinal wave, because the acceleration is in the direction of the velocity.

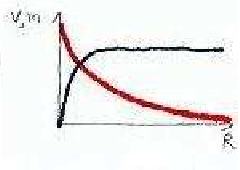




Must Do Physics

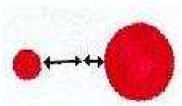
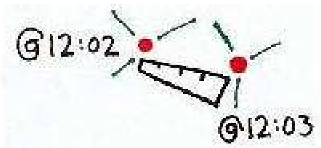
- Gravity.
- EM.
- Quantum mechanics.
- Experimental tests.



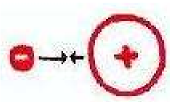
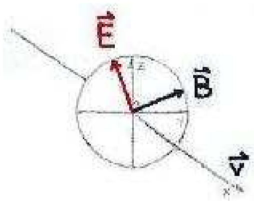
Must Do: Gravity

1. $F_g = -Gm\psi\hat{R}$ Like charges attract.
2. $+m$ One charge.
3. $\rho = \nabla^2\phi$ Newton's gravitational field equation.
4. $m\frac{d^2\vec{R}}{dt^2} = -\frac{GMm}{R^2}\hat{R}$ Newton's law of gravity under classical conditions.
5. $d\tau^2 = (1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2)\,dt^2 - (1 + 2\frac{GM}{c^2R})\,\frac{dR^2}{c^2}$
   Consistent with the Schwarzschild metric.

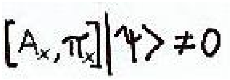
Must Do: Electrodynamics

1. $F_{\text{EM}} = q\vec{E}$ Like charges repel.
2. $\pm q$ Two distinct charges.
3. $\rho = \vec{\nabla}\cdot\vec{E} \quad \vec{J} = -\frac{\partial\vec{E}}{c\partial t} + \vec{\nabla}\times\vec{B}$ Maxwell source equations.
4. $0 = \vec{\nabla}\cdot\vec{B} \quad \vec{0} = \frac{\partial\vec{B}}{c\partial t} + \vec{\nabla}\times\vec{E}$ Maxwell homogeneous equations.
5. $F^\mu = q\frac{U_\nu}{c}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)$ Lorentz force.

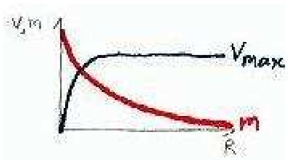
Must Do: Quantum Mechanics

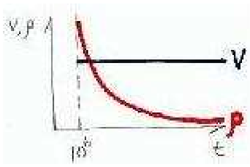
1. Unified field emission modes can be quantized.
2. Works with the standard model.
3. Indicates origin of mass.



Must Do: Experimental Tests

1. LIGO (gravity wave polarization).
2. Rotation profiles of spiral galaxies.
3. Big Bang constant velocity distribution.





Will Not Be Doing

- Review of previous efforts to unify gravity and EM.
- Regenerate Einstein's field equations, $G^{\mu\nu} = 8\pi T^{\mu\nu}$.

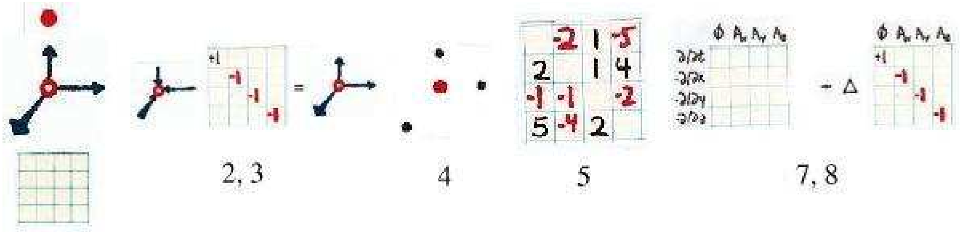
Don't bet against Einstein. Einstein viewed general relativity as an intermediate step. The last half of his life was devoted to two tasks: unifying gravity with EM, and understanding why quantum mechanics is the way it is, the logical reason driving it. He was willing to reconstruct physics from the ground up so long as guiding principles were respected. These lectures are devoted to unification. Another lecture series would be required to understand the logic of quantum mechanics, and I do think I know where the answer to that riddle lives.

Tensors

Scalars, vectors, and matrices are tensors.

Some new words are needed to generalize their properties.

1. Simple tensors
2. Covariant versus contravariant.
3. Going from covariant to contravariant.
4. Einstein's summation convention.
5. Symmetric versus antisymmetric tensors.
6. Derivatives in flat spacetime.
7. Covariant derivatives in curved spacetime.



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Simple Tensors

Useful no matter the coordinate system or dimension.

The simple tensors:

- Rank-0 tensor, a scalar.
- Rank-1 tensor, a vector.
- Rank-2 tensor, a matrix.

For these lectures, only 4-vectors and 4x4 matrices are used.

Covariant versus Contravariant

Subscript versus superscript.

- $A_\mu = (A_0, -A_1, -A_2, -A_3) = (\phi, -\vec{A})$ **Co**variant potential vector.
- $A^\mu = (A_0, A_1, A_2, A_3) = (\phi, \vec{A})$ **Contra**variant potential vector.
- μ, ν, ϖ Greek indices go from 0, 1, 2, 3.
- u, v Roman indices go from 1, 2, 3.

Memory aid: co is a commie, commies are low, negative; contras are proud, positive, up against a wall.

Going from Covariant to Contravariant

Use the rank-2 metric tensor, $g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ [flat spacetime].

- $g_{\mu\nu} A^\nu = A_\mu$ Lower an index.
- $g^{\mu\varpi} g^{\nu\sigma} A_{\varpi\sigma} = A^{\mu\nu}$ Raise two indices.

Memory aid: If the metric g has two indices raised up to the sky, it will be raising an index.

Einstein's Summation Convention

Contract same co- contra- index,

No $\sum$ needed.

- $A^\mu A_\mu = \phi^2 - \vec{A} \cdot \vec{A}$ Rank-0 tensor result.
- $A^\mu A^\nu g_{\mu\nu} = A^\mu A_\mu$ Metric contracts two contravariant vectors.

- $A^\mu A_\mu = A^\nu A_\nu$ Dummy variable names.

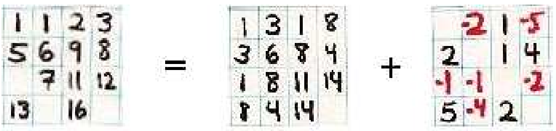
Symmetric versus Antisymmetric Tensors

Swap indices, see if sign does/does not flip for all.

- $A^{\mu\nu} = A^{\nu\mu}$ Symmetric, all keep sign.
- $A^{\mu\nu} = -A^{\nu\mu}$ Antisymmetric, all flip signs.
- $A^{\mu\nu} \neq A^{\nu\mu}$ Asymmetric, no pattern.

Any asymmetric tensor can be represented by a symmetric tensor (averaged values of 2 indices) and an antisymmetric tensor (+ and - deviations from average).

$A^{\mu\nu} = \frac{1}{2}(A^{\mu\nu} + A^{\nu\mu}) + \frac{1}{2}(A^{\mu\nu} - A^{\nu\mu})$



Average Joe & The Deviants

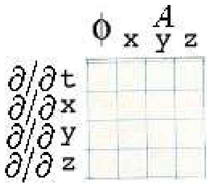
Derivatives in Flat, Euclidean Spacetime

4-derivatives: time and 3-space derivatives in a rank-1 tensor.

Signs of covariant and contravariant derivatives flip (arg!).

- $\partial_\mu = (\frac{\partial}{\partial t}, \vec{\nabla})$ Covariant derivative.
- $\partial^\mu = (\frac{\partial}{\partial t}, -\vec{\nabla})$ Contravariant derivative.
- $\partial^\mu A^\nu = A^{\nu,\mu}$ The comma convention.
- $\partial_\mu \partial^\mu = \Box^2 = (\frac{\partial^2}{\partial t^2} - \nabla^2)$ The D'Alembertian operator.

Alternate representation: The asymmetric rank-2 tensor that results from taking the 4-derivative of a 4-vector can be represented by the symmetric average amount of change tensor plus the antisymmetric deviation of change tensor.



Covariant Derivatives in Curved Spacetime

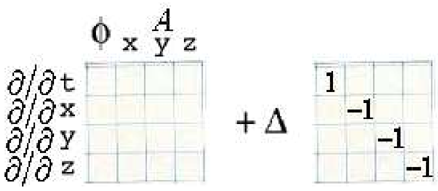
Covariant derivative = normal derivative $\pm$ derivative of the metric.

The connection or Christoffel symbol ($\Gamma_{\varpi}{}^{\mu\nu}$) handles the derivatives of the metric.

";" the semicolon convention for covariant derivatives.

- $A^{\mu;\nu} = A^{\mu,\nu} - \Gamma_{\varpi}{}^{\mu\nu} A^{\varpi}$ Derivative = normal - change in metric.
- $A^{\mu}{}_{;\nu} = A^{\mu}{}_{,\nu} + \Gamma^{\mu}{}_{\nu\varpi} A^{\varpi}$ Covariant derivative of a contravariant vector.
- $A^{\mu;\nu} - A^{\nu;\mu} = A^{\mu,\nu} - A^{\nu,\mu}$ Independent of the metric because $\Gamma^{\varpi}{}_{\mu\nu} = \Gamma^{\varpi}{}_{\nu\mu}$.
- $A^{\mu;\nu} + A^{\nu;\mu} = A^{\mu,\nu} + A^{\nu,\mu} - 2\Gamma_{\varpi}{}^{\mu\nu} A^{\varpi}$.

Omission: The details of Christoffel symbol are not discussed here.

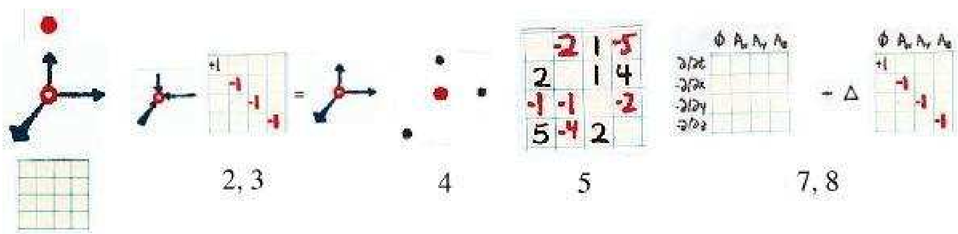


Summary: Tensors

Math:

$A^{\mu\nu} = \frac{1}{2}(A^{\mu\nu} + A^{\nu\mu}) + \frac{1}{2}(A^{\mu\nu} - A^{\nu\mu})$

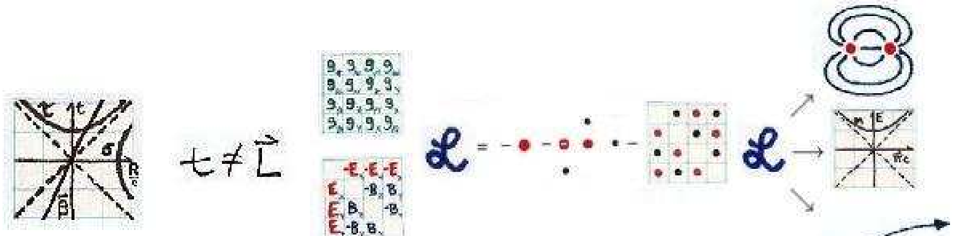
Pictures:



1 2, 3 4 5 7, 8

Units

1. Basic units.
2. Units for conversion factors.
3. Units for spacetime.
4. Units for potentials, fields, & charges.
5. Units in action:
    a) Lagrange densities.
    b) Euler-Lagrange equations (fields).
    c) Momentum.
    d) Force.



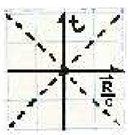
1, 3 2 4 5a 5b, c, d

Basic Units

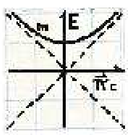
- t Time.
- L Length.
- m Mass.

For EM, Gaussian units will be used. Units of electric charge are found from Coulomb's law:

$F = \frac{qq'}{R^2} \rightsquigarrow \frac{mL}{t^2}$ so $q \rightsquigarrow \frac{\sqrt{mL^3}}{t}$ where " $\rightsquigarrow$ " means "has units of".



Spacetime

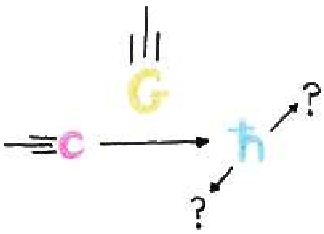


Energy/Momentum

Units for Conversion Factors

For gravity, spacetime, & quantum mechanics.

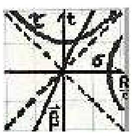
- $G \rightsquigarrow \frac{L^3}{mt^2}$ Gravitational constant.
- $c \rightsquigarrow \frac{L}{t}$ Speed of light.
- $h \rightsquigarrow \frac{mL^2}{t}$ Planck's constant.



Units for Spacetime

Where all events of gravity, EM, and quantum mechanics take place.

- $V \rightsquigarrow L^3$ Volume.
- $\tau^2 = t^2 - \frac{\vec{R}\cdot\vec{R}}{c^2} \rightsquigarrow t^2$ Interval squared.
- $\sigma^2 = \vec{R}\cdot\vec{R} - c^2t^2 = -c^2\tau^2 \rightsquigarrow L^2$ 4D-distance squared.
- $\gamma = \frac{1}{\sqrt{1-\frac{\vec{v}\cdot\vec{v}}{c^2}}} = \frac{\partial t}{\partial \tau} \rightsquigarrow -$ Stretch factor.
- $\vec{\beta} = \frac{\vec{v}}{c} \rightsquigarrow -$ Relativistic 3-velocity
- $U^\mu = (c\frac{\partial t}{\partial \tau}, \frac{\partial \vec{R}}{\partial \tau}) = (c\gamma, c\gamma\vec{\beta}) = (\frac{E}{mc}, \frac{\vec{\pi}}{mc}) \rightsquigarrow \frac{L}{t}$ Velocity vector.



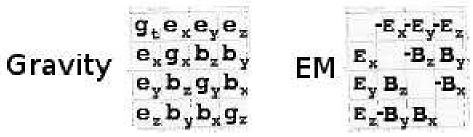
Spacetime

Units for Potentials, Fields, & Charges

The way to describe where stuff is everywhere, everywhen.

- $A^\mu = (\phi, \vec{A}) \rightsquigarrow \frac{\sqrt{m}}{\sqrt{L}}$ Potential vector.
- $A^{\mu;\nu} \rightsquigarrow \vec{g} \rightsquigarrow \vec{E} \rightsquigarrow \vec{B} \rightsquigarrow \frac{\sqrt{m}}{t\sqrt{L}}$ Derivatives of potential vectors (fields!).

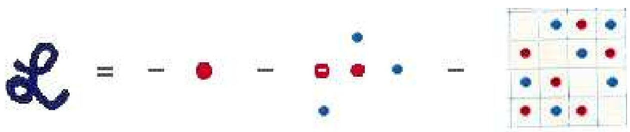
- $q \rightsquigarrow \frac{\sqrt{m L^3}}{t} \rightsquigarrow \sqrt{G}\, m \,[\frac{\sqrt{L^3}}{t\sqrt{m}} m] \rightsquigarrow \sqrt{h\,c}\,[\frac{L\sqrt{m}}{\sqrt{t}}\frac{\sqrt{L}}{\sqrt{t}}]$ Charge.
- $J^\mu = \frac{q}{V}\frac{U^\mu}{\gamma c} = (\frac{q}{V}, \frac{q}{V}\vec{\beta}) \rightsquigarrow \frac{\sqrt{m}}{t\sqrt{L^3}} \rightsquigarrow \frac{\sqrt{G}\, m}{V}\,[\frac{\sqrt{L^3}}{t\sqrt{m}} m \frac{1}{L^3}] \rightsquigarrow \frac{\sqrt{h\,c}}{V}\,[\frac{L\sqrt{m}}{\sqrt{t}}\frac{\sqrt{L}}{\sqrt{t}}\frac{1}{L^3}]$
  Current density vector.



Units in Action: Lagrange Density

Lagrange Density, where all mass, energy, and interactions are in a volume.

- $\mathfrak{L} \rightsquigarrow \frac{m}{L^3}$ Mass density.
- $\mathfrak{L} \rightsquigarrow \frac{m}{\gamma V}\,[\frac{m}{L^3}] \rightsquigarrow \frac{1}{c^2}\frac{q}{V}\frac{U^\mu}{\gamma} A_\mu [\frac{t^2}{L^2}\frac{\sqrt{m\,L^3}}{t}\frac{1}{L^3}\frac{L}{t}\frac{\sqrt{m}}{\sqrt{L}}] \rightsquigarrow$
  $\frac{\sqrt{G}}{c^2}\frac{m}{V}\frac{U^\mu}{\gamma} A_\mu [\frac{\sqrt{L^3}}{t\sqrt{m}}\frac{t^2}{L^2} m \frac{1}{L^3}\frac{L}{t}\frac{\sqrt{m}}{\sqrt{L}}] \rightsquigarrow \frac{1}{c^2} A^{\mu;\nu} A_{\mu;\nu} [\frac{t^2}{L^2}\frac{\sqrt{m}}{t\sqrt{L}}\frac{\sqrt{m}}{t\sqrt{L}}]$
  Equivalent units.

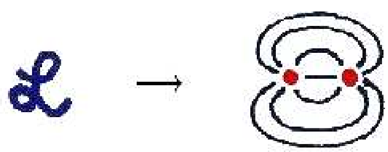


Lagrangian of E M

Units in Action: Euler-Lagrange Equations

Euler-Lagrange equations, generates field equations given a Lagrange density.

- $c\,\frac{\partial \mathfrak{L}}{\partial \phi} = c\,\partial_\mu \frac{\partial \mathfrak{L}}{\partial(\partial_\mu \phi)}$ From principle of least action.
- $c\frac{\partial \mathfrak{L}}{\partial \phi}\,[\frac{L}{t}\frac{m}{L^3}\frac{\sqrt{L}}{\sqrt{m}}] \rightsquigarrow \frac{q}{V}\,[\frac{\sqrt{m\,L^3}}{t}\frac{1}{L^3}] \rightsquigarrow \nabla A^{\mu;\nu}\,[\frac{1}{L}\frac{\sqrt{m}}{t\sqrt{L}}]$
  $\rightsquigarrow \nabla \vec{g}\,[\frac{1}{L}\frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow \nabla \vec{E}\,[\frac{1}{L}\frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow \nabla \vec{B}\,[\frac{1}{L}\frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow J^\mu [\frac{\sqrt{m}}{t\sqrt{L^3}}]$ Equivalent units.



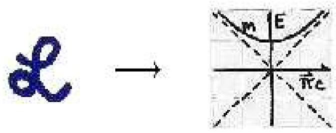
Lagrangian to fields

Units in Action: Momentum

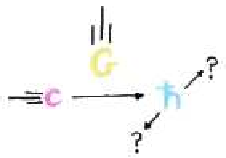
Energy and 3-momentum from a derivative of a Lagrange density.

- $\pi^{\mu} = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\,\partial t}} \rightsquigarrow \frac{m\,L^2}{t^2}$ Derivative of the Lagrange density.
- $\pi^{\mu}\,[\frac{m\,L^2}{t^2}] \rightsquigarrow h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\,\partial t}}\,[\frac{m\,L^2}{t}\,\frac{\sqrt{L^3}}{t\sqrt{m}}\,\frac{L}{t}\,\frac{t\sqrt{L}}{\sqrt{m}}\,\frac{m}{L^3}]$ Equivalent units.

Note: units suggest relativistic (c), quantum (h) gravity (G).



Lagrangian to Energy/Mo

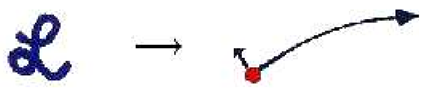


Units of Relativistic (c), quantum (h), Gravity (G)

Units in Action: Relativistic Force

- $F^{\mu} = \text{cause} = \frac{\partial m\,U^{\mu}}{\partial \tau}$ Force is a cause which has an effect on momentum.
- $F^{\mu}\,[\frac{m\,L}{t^2}] \rightsquigarrow \frac{1}{c}\,qU_{\nu}\nabla^{\mu}A^{\nu}\,[\frac{t}{L}\frac{\sqrt{m\,L^3}}{t}\frac{L}{t}\frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow \frac{\sqrt{h}}{\sqrt{c}}\,nU_{\nu}\nabla^{\mu}A^{\nu}\,[\frac{L\sqrt{m}}{\sqrt{t}}\frac{\sqrt{t}}{\sqrt{L}}\frac{L}{t}\frac{\sqrt{m}}{t\sqrt{L}}]$

  $\rightsquigarrow \frac{\sqrt{G}}{c}\,mU_{\nu}\nabla^{\mu}A^{\nu}[\frac{\sqrt{L^3}}{t\sqrt{m}}\,\frac{t}{L}\,m\,\frac{L}{t}\,\frac{\sqrt{m}}{t\sqrt{L}}]$ Equivalent units.



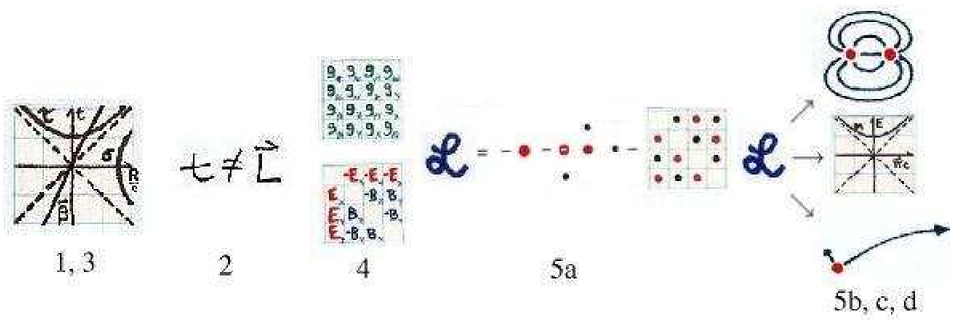
Lagrangian to Force

Summary: Units

Math:

$q \rightsquigarrow \frac{\sqrt{m\,L^3}}{t} \rightsquigarrow \sqrt{G}\,m\,[\frac{\sqrt{L^3}}{t\sqrt{m}}\,m] \rightsquigarrow \sqrt{h\,c}\,[\frac{L\sqrt{m}}{\sqrt{t}}\,\frac{\sqrt{L}}{\sqrt{t}}]$

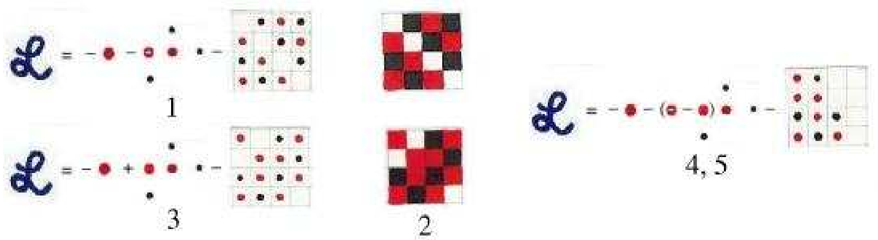
Pictures:



Lagrange Densities

Where all mass, energy, and interactions are in a volume.

1. EM Lagrange density.
2. EM to gravity by analogy.
3. Gravity Lagrange density hypothesis.
4. GEM Lagrange density.
5. GEM Lagrange density in detail.



EM Lagrange Density

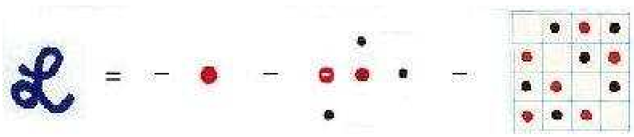
Where all EM energy is in a volume, no gravity.

$$\mathfrak{L}_{\mathrm{EM}} = -\frac{1}{\gamma}\rho_m - \frac{1}{c}J_\mu A^\mu - \frac{1}{4c^2}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$$

- $-\frac{1}{\gamma}\rho_m$ Energy density of mass in motion.
- $-\frac{1}{c}J_\mu A^\mu$ Energy density of electric charge in motion.
- $-\frac{1}{4c^2}(\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$

  Energy density of antisymmetric change in the potential.

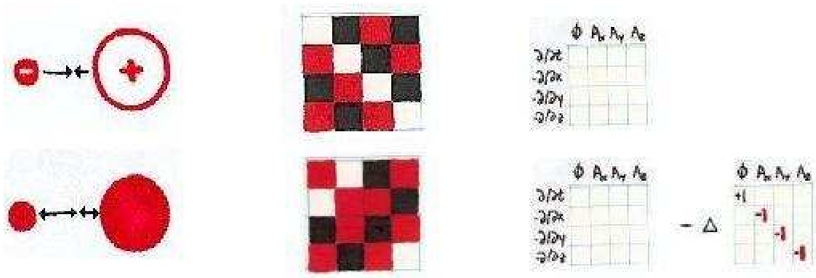
The pattern: rank-0, rank-1 contraction, and rank-2 contraction.



EM to Gravity Analogy

- $-\frac{1}{\gamma}\rho_m$ — No change for mass in motion rank-0 term.
- $-\,q \longrightarrow +\sqrt{G}\,m$ — Electric charge to mass charge.
- Change field strength tensor's symmetry.
  1. $\partial \longrightarrow \nabla$ — Derivatives to contravariant derivatives.
  2. $A - A \longrightarrow A + A$ — Anti-symmetric to symmetric tensor.

There are two sign changes, both are minus to plus. The first from -q to +m makes the law attractive. The second in the tensor changes the symmetry.



Gravity Lagrange Density Hypothesis

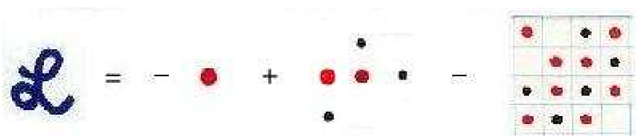
Where all gravitational energy is in a volume, no EM.

$$\mathfrak{L}_G = -\frac{1}{\gamma}\,\rho_m + \frac{1}{c}\,J^\mu_m A_\mu - \frac{1}{4c^2}(\nabla^\mu A^\nu + \nabla^\nu A^\mu)(\nabla_\mu A_\nu + \nabla_\nu A_\mu)$$

- $-\frac{1}{\gamma}\,\rho_m$ — Energy density of mass in motion.
- $+\frac{1}{c}\,J^\mu_m A_\mu$ — Energy density of mass charge in motion.
- $-\frac{1}{4c^2}\,(\nabla^\mu A^\nu + \nabla^\nu A^\mu)(\nabla_\mu A_\nu + \nabla_\nu A_\mu)$

  Energy density of symmetric change in the potential.

Only the rank-1 and rank-2 contraction terms have been changed by the analogy.



Unified Lagrange Density

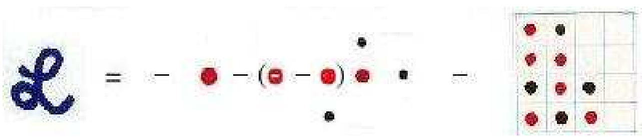
$\mathfrak{L}_{\text{GEM}}$ is the union of $\mathfrak{L}_G$ and $\mathfrak{L}_{\text{EM}}$.

- Mass in motion term is a union, not a sum.
- Sum charges in motion terms.
- Sum and simplify field strength tensor terms:

  1. $\partial \longrightarrow \nabla$ Derivatives to contravariant derivatives.
  2. $\nabla^\mu A^\nu \nabla_\nu A_\mu - \nabla^\mu A^\nu \nabla_\nu A_\mu$ Cross terms drop.
  3. $\nabla^\mu A^\nu \nabla_\mu A_\nu = \nabla^\nu A^\mu \nabla_\nu A_\mu$ Contractions are equal.

$$\mathfrak{L}_{\text{GEM}} = -\frac{1}{\gamma}\rho_m - \frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla^\mu A^\nu \nabla_\mu A_\nu$$

Note: Every property of this proposal is dictated by $\mathfrak{L}_{\text{GEM}}$!



GEM Lagrange Density in Detail

Goal: Get to individual terms, no indices.

Method: Expand, contract, and repeat.

1. Start with the GEM Lagrange density which has $1+4+16$ final terms:

   $$\mathfrak{L}_{\text{GEM}} = -\frac{1}{\gamma}\rho_m - \frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla^\mu A^\nu \nabla_\mu A_\nu$$

2. Expand J^μ , A_μ. Apply the definition of a contravariant derivative to a contravariant vector ($\nabla^\mu A^\nu = \partial^\mu A^\nu - \Gamma_\sigma{}^{\mu\nu} A^\sigma$):

   $$\mathfrak{L} = -\frac{1}{\gamma}\rho_m - (\rho_q - \rho_m)(c, -v^u)(\phi, A^u)$$

   $$-\frac{1}{2c^2}(\partial^\mu A^\nu - \Gamma_\sigma{}^{\mu\nu} A^\sigma)(\partial_\mu A_\nu - \Gamma^\sigma{}_{\mu\nu} A_\sigma)$$

3. Contract U_μ with A^μ. Multiply out final term:

$$\mathfrak{L} = -\frac{1}{\gamma}\,\rho_m - (\rho_q - \rho_m)(c\phi - v^u A^u)$$

$$-\frac{1}{2c^2}(\partial^\mu A^\nu \partial_\mu A_\nu - 2\,\Gamma_\sigma{}^{\mu\nu} A^\sigma \partial_\mu A_\nu + \Gamma_\sigma{}^{\mu\nu} A^\sigma \Gamma^\sigma{}_{\mu\nu} A_\sigma)$$

4. Expand $\partial_\mu A^\nu$ and $\partial^\mu A_\nu$. Work in local covariant coordinates where $\Gamma = 0$:

   $$\mathfrak{L} = -\frac{1}{\gamma}\,\rho_m - (\rho_q - \rho_m)(c\phi - v^u A^u) - \frac{1}{2c^2}(\frac{\partial}{\partial t}, -\nabla^v)(\phi, A^u)(\frac{\partial}{\partial t}, \nabla^v)(\phi, -A^u)$$

5. Contract:

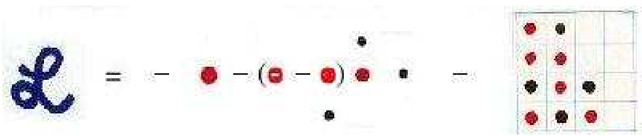
   $$\mathfrak{L} = -\frac{1}{\gamma}\,\rho_m - (\rho_q - \rho_m)\,(c\phi - v^u A^u) - \frac{1}{2c^2}((\frac{\partial\phi}{\partial t})^2 - (\nabla\phi)^{v2} - (\frac{\partial A}{\partial t})^{u2} + (\nabla A)^{uv2})$$

6. Write it ALL out:

   $$\mathfrak{L} = -\rho_m(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2} - (\rho_q - \rho_m)(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

   $$-\frac{1}{2}((\frac{\partial\phi}{c\partial t})^2 - (\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2 - (\frac{\partial\phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

   $$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2)$$

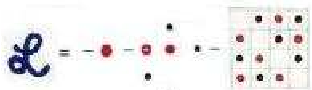


Summary: Lagrange Densities

Math:

$$\mathfrak{L}_{\mathrm{GEM}} = -\frac{1}{\gamma}\,\rho_m - \frac{1}{c}\,(J^\mu_q - J^\mu_m)A_\mu - \frac{1}{2c^2}\,\nabla^\mu A^\nu\,\nabla_\mu A_\nu$$

Pictures:



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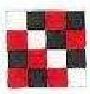

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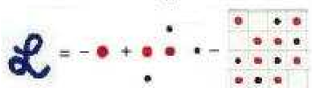

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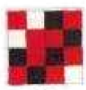


2

Summary Equations

1. Schwarzschild metric:

$$d\tau^2 = (1 - 2\frac{GM}{c^2R} + 2(\frac{GM}{c^2R})^2)\, dt^2 - (1 + 2\frac{GM}{c^2R})\, \frac{dR^2}{c^2}$$

2. Maxwell source equations:

$$\rho = \vec{\nabla} \cdot \vec{E} \quad \vec{J} = -\frac{\partial \vec{E}}{c\partial t} + \vec{\nabla} \times \vec{B}$$

3. Maxwell homogeneous equations:

$$0 = \vec{\nabla} \cdot \vec{B} \quad \vec{0} = \frac{\partial \vec{B}}{c\partial t} + \vec{\nabla} \times \vec{E}$$

4. Lorentz force:

$$F^\mu = q\, \frac{U_\nu}{c} (\nabla^\mu A^\nu - \nabla^\nu A^\mu)$$

5. Any tensor is the sum of a symmetric and antisymmetric tensor.

$$A^{\mu\nu} = \frac{1}{2}\,(A^{\mu\nu} + A^{\nu\mu}) + \frac{1}{2}\,(A^{\mu\nu} - A^{\nu\mu})$$

6. Equivalent units of charge:

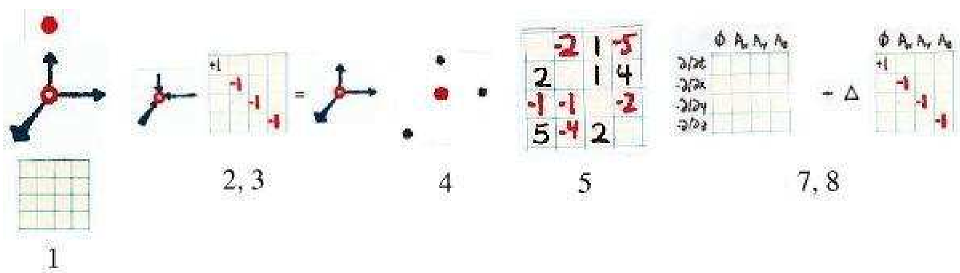
$$q \rightsquigarrow \frac{\sqrt{m\, L^3}}{t} \rightsquigarrow \sqrt{G}\, m\, [\tfrac{\sqrt{L^3}}{t\sqrt{m}}\, m] \rightsquigarrow \sqrt{h\, c}\, [\tfrac{L\sqrt{m}}{\sqrt{t}}\, \tfrac{\sqrt{L}}{\sqrt{t}}]$$

7. GEM Lagrange density

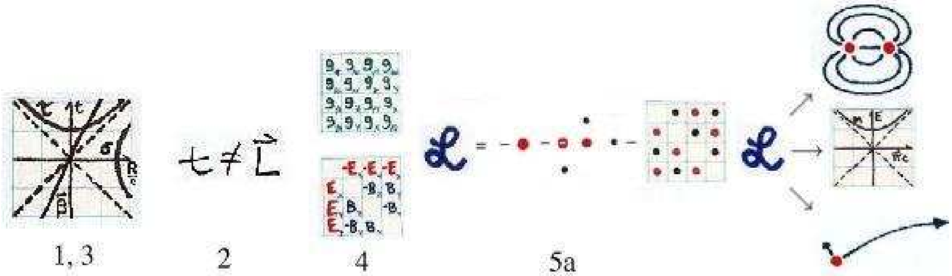
$$\mathfrak{L}_{\mathrm{GEM}} = -\,\frac{1}{\gamma}\, \rho_m - \frac{1}{c}\,(J_q^\nu - J_m^\nu) A_\nu - \frac{1}{2c^2}\, \nabla^\mu A^\nu \nabla_\mu A_\nu$$

Summary Pictures

1. Tensors:

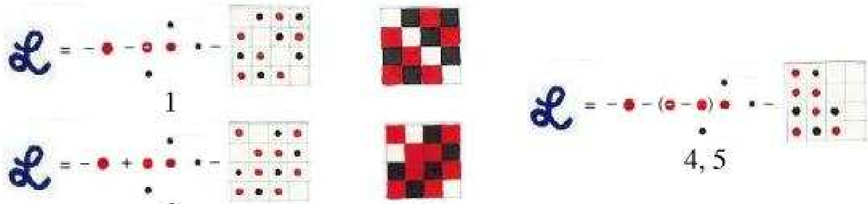


2. Units:



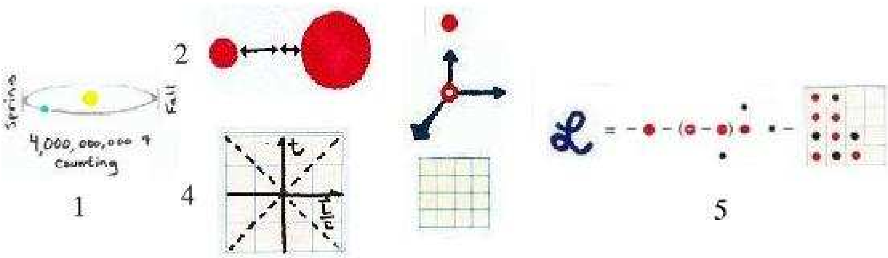
1, 3 2 4 5a 5b, c, d

3. Lagrange Densities:



1 3 2 4, 5

4. Day 1:



1 2 4 3 5