Unifying gravity and electromagnetism using analogies based on electromagnetism for gravity

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Abstract. Gravity and electromagnetism are unified at the level of a 4-potential by re-examining the most obvious choice at the Lagrangian level:

$$\mathcal{L}_{GEM} = -\frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla_\mu A^\nu \nabla^\mu A_\nu.$$

The GEM Lagrangian has mass and electric current densities coupled to the 4-potential with different signs: like charges attract for gravity and repel for electromagnetism. A four-dimensional wave equation results by varying the action with respect to the potential:

$$J_q^\mu - J_m^\mu = \nabla^\nu \nabla_\nu A^\mu.$$

If the mass current density J_m^μ is zero, one sees the Maxwell equations. If the electric charge density ρ_q is zero but the mass density ρ_m is not, in the static case, Newton's field equation for gravity results. When a constant potential is chosen, for a metric compatible, torsion-free connection, the exponential metric tensor solves the field equations:

$$g_{\mu\nu} = \begin{pmatrix} exp(-2\frac{GM}{c^2R}) & 0 & 0 & 0 \\ 0 & -exp(2\frac{GM}{c^2R}) & 0 & 0 \\ 0 & 0 & -exp(2\frac{GM}{c^2R}) & 0 \\ 0 & 0 & 0 & -exp(2\frac{GM}{c^2R}) \end{pmatrix}.$$

The exponential metric equation has the same ten parameterized post-Newtonian (PPN) coefficients as the Schwarzschild metric, so it will pass all the same tests of the equivalence principle and the metric. The second-order PPN coefficients for the two metrics differ. Light will bend around the Sun 0.7 microarcseconds more for this proposal than for the Schwarzschild metric, beyond our means to measure today. Energy loss by gravitational waves should be consistent with the proposal. When a flat metric is chosen, a normalized, linear perturbation potential function is found whose derivative has the correct inverse squared distance dependence needed for a classical force.

Analysis of the gravitational Lorentz force indicates both the standard mass times acceleration effect and a new classical gravitational effect, velocity times the change in passive inertial mass with respect to space. This may lead to new explanations for the rotation profile of thin galaxies without requiring dark matter or a modification of Newtonian mechanics (MOND).

Quantization will be very similar to the Gupta-Bleuler method of fixing the Lorenz gauge. The key difference is that the GEM field equations have two spin fields, one even, the other odd. The polarization of gravity waves are different than the transverse waves predicted by general relativity, setting the stage for a future test of the proposal.

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1. Introduction

The goal of this paper is to create one mathematical structure for gravity and electromagnetism that can be quantized. The difference between gravity and electromagnetism is one of the oldest problems facing physics, going back to studies of electromagnetism in the eighteenth century. Gravity was the first inverse square law, discovered by Isaac Newton. After twenty years of effort, he was able to show that inside a hollow, massive shell, the gravitational field would be zero. Ben Franklin, in his studies of electricity, demonstrated a similar property for an electrically charged ball inside an insulated, conducting can. Joseph Priestly realized this meant that the electrostatic force was governed by an inverse square law like gravity. Due to his experimental work, Coulomb got the credit for the electrostatic force law modeled on Newton's law of gravity.[5]

Over a hundred years later, Einstein started from the tensor formalism of electromagnetism on the road to general relativity. Instead of the antisymmetric electromagnetic field strength tensor, Einstein realized a change in the symmetric metric tensor would be governed by symmetric tensors. There is a precedence for transforming mathematical structures between gravity and electromagnetism.

Previous efforts to use 4-potentials for gravity report failures due in part to considering analogies that were too close to electromagnetism. Although free to choose the sign of a mass current, researchers chose the same sign used for electromagnetism where like charges repel.[4, 13] Other efforts created an attractive gravitational force, but forgot Einstein's insight that a symmetric tensor was required.[11, Exercise 7.2] Perhaps the greatest barrier however was the inverse squared potential that solved the field equations suggest a non-physical, inverse-cubed force law.

In this paper, analogies to electromagnetism are used for gravity, respecting differences. The mass current always has a sign opposite to electric current. The second-rank gravitational field strength tensor is symmetric. An inverse distance potential is physically relevant, so a normalized, perturbation of a 4-potential near that classical result will be the focus. By staying close to electromagnetism, particularly by working with a linear field theory, the difficulty of quantizing gravity may dissolve.

2. Lagrange densities

The classic electromagnetic Lagrange density has two terms: one for electric charge density coupling to the potential and another for the antisymmetric second-rank field strength tensor $F^{\mu\nu}$:

$$\mathcal{L}_{EM} = -\frac{1}{c} J_q^\mu A_\mu - \frac{1}{4c^2}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu). \tag{1}$$

An analogous Lagrangian for gravity should contain these components, but changes are required. Gravity would couple the potential to the mass current density, not the electric current density or a second-rank energy density tensor. Mass does not have the same units as electric charge, so mass will have to be multiplied by the square root of Newton's gravitational constant G to keep the units identical. Where there is a negative electric current density, a positive mass current density will be substituted. A change in sign is required so that like mass currents attract for gravity. Because gravity effects metrics which are symmetric, the field strength tensor for gravity must also be symmetric. In order that the symmetric object transforms like a tensor, the exterior derivative must be replaced by a covariant derivative:

$$\mathcal{L}_G = +\frac{1}{c} J_m^\mu A_\mu - \frac{1}{4c^2}(\nabla_\mu A^\nu + \nabla_\nu A^\mu)(\nabla^\mu A_\nu + \nabla^\nu A_\mu). \tag{2}$$

A mixed derivative is used for the field strength tensor so that a scalar field can be defined by taking the trace. The trace will be zero for massless particles like a graviton or photon, but nonzero for massive particles. The unified Lagrangian will be the sum of these two, $\mathcal{L}_{EM}$ and $\mathcal{L}_G$, which separately only apply if the other charge is zero and the corresponding vacuum field strength tensor is zero. Without loss of generality, the exterior derivatives in the electromagnetic Lagrangian (1) can be written as covariant derivatives. This leads to the gravity and electromagnetism (GEM) Lagrangian:

$$\mathcal{L}_{GEM} = -\frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla_\mu A^\nu \nabla^\mu A_\nu. \tag{3}$$

The Fermi Lagrangian of electromagnetism is a subset of 3. This establishes a link to electromagnetism. A covariant derivative contains the connection. For a metric compatible, torsion-free connection, the Christoffel symbol is a function of derivatives of the metric. As such, the derivatives of the metric may be constrained by the GEM Lagrangian, unlike the case with the exterior derivative of the classical electromagnetic Lagrangian. The possibility to do both gravity and electromagnetism is here.

With the Hilbert action of general relativity, one considers the second-rank metric tensor to be a field. By varying the metric, non-linear second-rank field equations result that dictate how the metric changes. In the GEM Lagrangian, a dynamic metric is possible due to a diffeomorphism symmetry in the Lagrangian introduced when changing from an exterior derivative used in the electromagnetic field strength tensor to a covariant derivative in the unified field strength tensor. Any change in the unified field strength tensor could be due to a change in the potential, or due to a change in the metric via the connection. One can choose a metric as the gauge, and then calculate the potential for a given field strength tensor, or choose a fix the potential, and calculate the appropriate connection.

3. Unified field equations

Write out the action based on the GEM Lagrange density (3):

$$S = \int (-\frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla_\mu A^\nu \nabla^\mu A_\nu)\sqrt{-det(g)}d^4x. \tag{4}$$

Varying the action by the 4-potential field generates the field equations:

$$J_q^\mu - J_m^\mu = \nabla^\nu \nabla_\nu A^\mu. \tag{5}$$

This is a four-dimensional wave equation with two sources, one for electricity, the other for gravity. The Maxwell equations are apparent in the physical situation where the mass current density J_m^μ is effectively zero (all charged particles have a non-zero mass, but the charge mass charge is more than thirteen orders of magnitude smaller than the electric charge). The gauge is not fixed in the Lagrangian, so there is more freedom, a necessity if the equation is also going to describe gravity. Newton's field equation for gravity is the first static field equation when there is no electric current density ρ_q. The field equations are covariant under a Lorentz transformation, behaving like a 4-vector. One of the justifications for general relativity - to make Newton's field equation for gravity covariant under a Lorentz transformation - is no longer compelling because 5 is manifestly covariant.[9]

Because the metric was not varied, that indicates that it must be fixed. With the Maxwell equations, there are no constraints on the metric, so a metric must be supplied as part of the background structure for the theory. For a connection that is metric compatible and torsion-free, second-order derivatives of the metric are part of the field equations. The process of finding solutions to the field equations will necessitate constraints on the dynamics of the metric. The rest of the background structure for the Maxwell equations - things like the 4-dimensional structure, the differential manifold - will still be necessary for the GEM field equations.

The classical fields can be expressed in terms of covariant derivatives of the potential. The symmetric and antisymmetric field strength tensors are very similar, differing only in the sign of the tensor $\bigtriangledown_\nu A^\mu$. The electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ together represent all the information in the antisymmetric field strength tensor. Symmetric analogs $\boldsymbol{e}$ and $\boldsymbol{b}$ are defined to do a similar job for the symmetric field strength tensor. In addition, there is a four component field for the terms along the diagonal which will be noted as g^μ. To make a connection to the classical fields of gravity and electromagnetism, use the following mappings:

$$E^i = -\partial_0 A^i + \partial_i A^0 \tag{6}$$

$$e^i = \partial_0 A^i + \partial_i A^0 + 2\Gamma_{\nu 0}{}^i A^\nu \tag{7}$$

$$\epsilon^{ijk} B^k = -\partial_i A^j + \partial_j A^i = -Curl(\boldsymbol{A}) \tag{8}$$

$$b^k = -\partial_i A^j - \partial_j A^i - 2\Gamma_{\nu i}{}^j A^\nu \qquad i \neq j \neq k \tag{9}$$

$$g^\mu = (\partial_\mu A^\mu + \Gamma_{\nu\mu}{}^\mu A^\nu)^\mu. \tag{10}$$

The set of five classical fields will transform like a second-rank asymmetric tensor. Under a Lorentz transformation, the three gravity fields will mix, as will the two electromagnetic fields. There will be no mixing between gravity and electromagnetic fields since the fields are in different, irreducible tensors.

The first row and column of the asymmetric field strength tensor is the sum of the electric field $\boldsymbol{E}$ and the symmetric analog $\boldsymbol{e}$. The rest of the off-diagonal terms are the sum of the magnetic field $\boldsymbol{B}$ and its symmetric counterpart $\boldsymbol{b}$. The diagonal of the field strength tensor is g^{μ}. The sum of the components of g^{μ} is the Lorentz invariant trace of the asymmetric field strength tensor. The trace contains information about both the electromagnetic gauge and the Christoffel symbols. The ability to choose an arbitrary electromagnetic gauge is equivalent to ignoring constraints imposed by gravity, which is common for all practical problems with electromagnetism.

Substitute the classical fields (6-10) into the field equations (5), starting with the first field equation:

$$\begin{aligned}\rho_q - \rho_m &= c\nabla^{\nu}\nabla_{\nu}\phi \\ &= \frac{c}{2}(\bigtriangledown \cdot \boldsymbol{E} + \bigtriangledown \cdot \boldsymbol{e}) + \frac{\partial g^0}{\partial t}.\end{aligned} \tag{11}$$

This has Gauss' law and analogous equation for gravity. One must note that the divergence with a contravariant derivative introduces another minus sign. The Newtonian gravitational field equation is embedded here under the following physical conditions:

$$\begin{aligned}-\rho_m =& \ -c\nabla^2\phi = c \bigtriangledown \cdot \boldsymbol{e} \\ iff \qquad & \rho_q = 0, \ \bigtriangledown \cdot \boldsymbol{E} = 0, \ and \ \frac{\partial g^0}{\partial t} = 0.\end{aligned} \tag{12}$$

If the time derivative of g^0 is not zero, then the field equations for gravity incorporate time directly. This is significant because one justification for general relativity is to make the form of Newton's field equations dynamic.[11, chapter 7]

With a particular choice of reference frame such that all symmetric derivatives of the potential vanish, the relativistic form of 12 concerns changes in the Christoffel symbol exclusively:

$$\rho_m = 2\partial_{\mu}\Gamma^{0\mu}_{\nu}A^{\nu}. \tag{13}$$

The second-order change in the metric is determined by the mass charge density. It is significant that the Riemann curvature tensor is not being used here because this unification model is fundamentally distinct from general relativity. One can find a metric which solves 13. For a static, non-rotating, uncharged mass where the change in the potential is zero, the exponential metric is a solution:

$$g_{\mu\nu} = \begin{pmatrix} exp(-2\frac{GM}{c^2R}) & 0 & 0 & 0 \\ 0 & -exp(2\frac{GM}{c^2R}) & 0 & 0 \\ 0 & 0 & -exp(2\frac{GM}{c^2R}) & 0 \\ 0 & 0 & 0 & -exp(2\frac{GM}{c^2R}) \end{pmatrix}. \tag{14}$$

As the exponent goes to zero, the metric becomes the Minkowski metric for flat spacetime in the limit. The exponential metric equation has been studied previously.[12, 17, 6, 15] The metric is consistent with weak field tests of gravity to first-order parameterized post-Newtonian (PPN) accuracy. At second-order PPN accuracy, light will bend around the Sun an additional 11.5 microarcseconds more than first order. General relativity predicts a second-order contribution of 10.8 microarcseconds.[2] At the current time, we can only measure light bending to 100 microarcseconds, and there are no experiments planned to determine the second-order PPN coefficients (Clifford Will, personal communication). In the future, the 0.7 microarcseconds difference in light bending represents a means to either confirm or reject this proposal.

Strong gravitational field tests such as the energy loss by binary pulsars due to gravity waves has ruled out Rosen's bimetric proposal. There can be a dipole mode of gravity wave emission due to the additional metric field. For the GEM model, there is no additional field to store energy or momentum, so for an isolated source, the lowest mode of emission is a quadrapole, consistent with the data from pulsars.

Gauss' law can be isolated under different physical conditions:

$$\begin{aligned}\rho_q &= \frac{1}{c}\frac{\partial^2 \phi}{\partial t^2} - c\nabla^2\phi = c \bigtriangledown \cdot \boldsymbol{E} \\ iff \quad & \rho_q = 0, \quad \bigtriangledown \cdot \boldsymbol{e} = 0 \; and \; \frac{\partial g^0}{\partial t} = 0.\end{aligned} \tag{15}$$

To make the discussion of unification more concrete, imagine a proton at the center of a 1 cm sphere. The electric charge density would be $3.82 \times 10^{-14} C/m^3$. The mass charge density would be $3.25 \times 10^{-27} C/m^3$, thirteen orders of magnitude smaller than the electric charge density. This model does not address why there is such a significant difference between the charges. The unified charge density for a proton is slightly less than the electric charge density, although not measurably so since the electric charge is not known to thirteen significant digits.

Repeat the exercise for the 3-vector field equation.

$$\begin{aligned}\boldsymbol{J}_q - \boldsymbol{J}_m &= \frac{1}{c}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - c \bigtriangledown^2 \boldsymbol{A} \\ &= \frac{1}{2}\left(-\frac{\partial \boldsymbol{E}}{\partial t} + c \bigtriangledown \times \boldsymbol{B} + \frac{\partial \boldsymbol{e}}{\partial t} - c \bigtriangledown \boxtimes \boldsymbol{b}\right) + c \bigtriangledown g^u \\ where \quad & (\boldsymbol{a} \boxtimes \boldsymbol{b})^k = a^i b^j + a^j b^i \quad i \neq j \neq k.\end{aligned} \tag{16}$$

This has Ampere's law and a symmetric analog for gravity.

The model for classical gravitational and electromagnetic field equations is expressed with 4-vectors, tensors of rank one. Einstein's field equations use tensors of rank two. Therefore the two approaches are fundamentally different. Although the field equations are rank one, the field strength tensor is second rank, consistent with arguments that a symmetric second-rank field strength tensor is required to characterize a dynamic metric. Second-order derivatives of the metric arise from the divergence of the Christoffel symbols as seen in 13.

The homogeneous Maxwell equations are vector identities, unaffected by unification.

4. GEM field equation solutions

The task is to find a physically relevant solution to the GEM field equations, 5. The Poisson field equation of classical Newtonian gravity can be solved by an inverse distance potential, $1/R$. The potential has a point singularity where $R = 0$. The GEM field equations are relativistic, so time must be incorporated. An inverse distance potential does not solve the field equations in four dimensions. The potential $A^\mu = (\frac{1}{\sigma^2}, \overrightarrow{0})$ solves the field equations, where σ^2 is the Lorentz invariant distance squared, the negative of the Lorentz invariant interval squared, $-(c\tau)^2$. Distance is used instead of the interval because classical gravity depends on distance, not time. The potential has as a singularity that is the entire lightcone, where $\sigma^2 = 0$. The potential is not relevant in the classical domain since its derivative will not be an inverse square as required for a classical gravitational force.

Gravity is a weak effect. It is common in quantum mechanics to normalize a potential and study linear perturbations of weak fields, an approach that will be followed here. Assume spherical symmetry. Form a normalized potential with a linear perturbation:

$$A^\mu = (\frac{\sqrt{G}h}{c^2\sigma^2}, \mathbf{0}) \rightarrow (\frac{c/\sqrt{G}}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}R)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2}, \mathbf{0}). \tag{17}$$

Take the covariant derivative with respect to t and R, keeping only terms to first order in the spring constant k:

$$\begin{aligned} \frac{\partial\phi}{\partial t} &= \frac{c^2}{\sqrt{G}}\frac{k}{\sigma^2} + O(k^2) \\ \nabla\phi &= -\frac{c^2}{\sqrt{G}}\frac{k}{\sigma^2} + O(k^2). \end{aligned} \tag{18}$$

The change in the potential is a function of a spring constant k over sigma squared. The classical Newtonian dependence on distance is an inverse square, so this is promising. A potential that applies exclusively to gravity is sought, yet the non-zero gradient of ϕ indicates an electric field. The sign of the spring constant k does not effect the solution to the four dimensional wave field equations but does change the derivative of the potential. A potential that only has derivatives along the diagonal of the field strength tensor $\nabla_\mu A^\nu$ can be constructed from two potentials that differ by spring constants that either constructively interfere to create non-zero derivatives, or destructively interfere to eliminate derivatives. With this in mind, construct a potential that will have no electric field:

$$A^\mu = \frac{c}{\sqrt{G}}\Big(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}t)^2}$$
$$+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}t)^2},$$
$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}t)^2}$$
$$+\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}t)^2},$$
$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}t)^2} \tag{19}$$
$$+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}t)^2},$$
$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}t)^2}$$
$$+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}x)^2+(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}y)^2+(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}z)^2-(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}t)^2}\Big).$$

Take the covariant derivative of this potential, keeping only the terms to first order in the spring constant k.

$$\nabla_\mu A^\nu = \frac{c^2}{\sqrt{G}}\frac{k}{\sigma^2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} + O(k^2). \tag{20}$$

All this work was required to get a multiple of the Minkowski matrix!

5. Relativistic gravitational force

Armed with a gravitational potential where the derivative has the correct distance dependence, it is time to examine the relativistic force. The electromagnetic Lorentz force density arises from moving charges in an electromagnetic field:

$$F^\mu_{EM} = J_{q\,\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu) = \frac{d\rho_m U^\mu}{\sqrt{G}d\tau}. \tag{21}$$

If the sign of the electric charge q where changed, this would change the sign of the 4-force, so there are two distinguishable signs for electric charge. Like electric charges repel because the force has a positive sign.

An analogous gravitational Lorentz force can be created with precisely the same substitutions as were used for the Lagrange density:

$$F_{G\,\mu} = -J_{m\,\nu}(\nabla_\mu A^\nu + \nabla_\nu A^\mu) = \frac{d\rho_m U_\mu}{\sqrt{G}d\tau}. \tag{22}$$

If the sign of the mass current were changed, then the sign of the inertial mass density in the force would also need to change, resulting in no change, so there is only one distinguishable sign for mass charge. Like mass charges attract because the force has a negative sign.

Plug the weak-field gravitational approximation (20) into the gravitational Lorentz 4-force equation (22), assuming the change in momentum is the product of mass charge density times velocity:

$$F_{G\,\mu} = \frac{\rho_m}{\sqrt{G}}(-\frac{ck}{\sigma^2}U^0, -\frac{ck}{\sigma^2}\boldsymbol{U}) = \frac{\rho_m}{\sqrt{G}}(\frac{dU^0}{d\tau}, -\frac{d\boldsymbol{U}}{d\tau}). \tag{23}$$

This is the electrically neutral, weak gravitational field Lorentz 4-force density. These are first-order differential equations. The weak equivalence principle equates inertial and passive gravitational mass densities. This principle will be assumed, but is supported by experiments.[16] So that the equations have the same variable, substitute the interval $c^2\tau^2$ for the distance $-\sigma^2$:

$$\frac{dU^0}{d\tau} - \frac{k}{c\tau^2}U^0 = 0 \tag{24}$$

$$\frac{d\boldsymbol{U}}{d\tau} + \frac{k}{c\tau^2}\boldsymbol{U} = 0. \tag{25}$$

Solve for the velocity, U^μ. The solution involves exponentials with velocity constants:

$$U^\mu = (v\ exp(-\frac{k}{\tau}), \boldsymbol{V}\ exp(\frac{k}{\tau})). \tag{26}$$

For flat spacetime, $U^\mu = (v, \overrightarrow{V})$. The constraint on relativistic velocities in flat spacetime is:

$$U^\mu U_\mu = (\frac{cdt}{d\tau})^2 - (\frac{d\boldsymbol{R}}{d\tau})^2 = v^2 - \boldsymbol{V}\cdot\boldsymbol{V} = c^2. \tag{27}$$

This will be the case for the velocity solution should the spring constant be zero or the interval infinite. Substitute the solution (26) into the constraint (27) multiplying through by $d\tau^2$.

$$d\tau^2 = exp(-2\frac{k}{\tau})dt^2 - exp(2\frac{k}{\tau})(dR/c)^2. \tag{28}$$

This metric equation represents the weak-field gravitational Lorentz 4-force solution (26) under the constraint that it yields a flat spacetime metric for a zero spring constant k or infinite interval τ. The effect of a weak gravitational force is to bend spacetime.

Assume the spring constant k is due to an active geometric mass, $k = GM/c^2$. Assume the field is static, so $\sigma^2 = R^2 - (ct)^2 \cong R'^2$. Both sigma and tau must have the same magnitude, R. To express the metric in terms of tau real, assume sigma is imaginary, so tau is real:

$$d\tau^2 = exp(-\frac{2GM}{c^2R})dt^2 - exp(\frac{2GM}{c^2R})(dR/c)^2. \tag{29}$$

The exponential metric derived from the gravitational Lorentz force law is the same as the solution to the field equations (14), a test of self-consistency. On aesthetic grounds, it is nice to see that the smallest steps away from flat spacetime uses Nature's favorite function, the exponential.

The exponential metric is not a solution to the Einstein field equations. If one chooses a constant potential, the exponential metric is a singular solution to the GEM field equation that depends only on the Christoffel symbol, 13. If instead one chooses a flat Minkowski metric, the potential solution would be the singular $1/R$ potential. The two types of singular solutions with different gauge choices is a stringent test of the consistency of the model.

6. Classical gravitational force and a new effect

Minkowski spacetime is different from Newtonian space and time due to the way one measures distance, four dimensional versus three. Spacetime symmetry must be broken. The Minkowski interval τ is a consequence of the relationship between time t and space $\boldsymbol{R}$. For classical physics, the functional relationship between time and space must be severed. In the static field approximation, there is a scalar distance R which is the same magnitude as the interval τ. If the interval τ is replaced by the scalar distance R in the relativistic 4-velocity, then that will sever the functional relationship between time and space:

$$(\frac{dt}{d\tau}, \frac{d\boldsymbol{R}}{cd\tau}) \rightarrow (c\frac{dt}{d\mid R\mid}, \frac{d\boldsymbol{R}}{d\mid R\mid}) = (0, \hat{R}). \tag{30}$$

Substitute into weak-field gravitational Lorentz 4-force density equation (23) to create a classical 3-force equation:

$$\boldsymbol{F} = \frac{\sqrt{G}M\rho_m}{R^2}\hat{R} = -\frac{d\rho_m\boldsymbol{U}}{\sqrt{G}d\tau}. \tag{31}$$

This is not quite Newton's gravitational force density law. The reason is that one must now consider the right-hand side of the force equation carefully. According to the chain rule:

$$-\frac{d\rho_m\boldsymbol{U}}{\sqrt{G}d\tau} = -\frac{\rho_m}{\sqrt{G}}\frac{d\boldsymbol{U}}{d\tau} - \frac{\boldsymbol{U}}{\sqrt{G}}\frac{d\rho_m}{d\tau}. \tag{32}$$

An open question is how should spacetime symmetry be broken for the derivatives with respect to the interval τ? An interval is composed of both changes in time and space. For the acceleration term, $\frac{\partial \boldsymbol{U}}{\partial \tau}$, if the interval is only about time, then one gets back Newtonian acceleration, a second derivative of time. One might be tempted to use time in the mass distribution in spacetime term, $\frac{\partial \rho_m}{\partial \tau}$. However, the system is presumed to be static, so this would be zero by presumption. If this derivative is to have any chance at being non-zero, it would have to change with respect to the scalar distance R as has been done earlier in the derivation. Note that this new term will not point in a radial direction. Instead, the change in mass with respect to space points in the direction of the velocity of that mass. The classical 3-force law would look like so:

$$\frac{\sqrt{G}M\rho_m}{R^2}(\hat{R}+\hat{V}) = -\frac{\rho_m}{\sqrt{G}}\frac{d^2\boldsymbol{R}}{dt^2} - \frac{c\boldsymbol{V}}{\sqrt{G}}\frac{d\rho_m}{d\mid R\mid}. \tag{33}$$

For a point source, the change in mass distribution term, $d\rho_m/d \mid R \mid$, will not make a contribution, and one gets Newton's law of gravity. It is only if the inertial mass is distributed over space as is the case for galaxies will the new effect term come into play. If the velocity is constant, then the acceleration is zero. The equation describes the distribution in space of the inertial mass density ρ_m that contributes to the total gravitational source mass M. The solution to 33 when there is no acceleration has the inertial mass distribution that decays exponentially. There is a problem with the rotation profile of thin disk galaxies.[7, 8] Once the maximum velocity is reached, the velocity stays constant while the mass density declines exponentially. It has been shown that galaxies should not be stable at all.[14] The new effect term promises a stable exponential decay of the mass distribution for large radii with constant velocity, which sounds like a fit. The new effect has an inverse distance dependence for small accelerations due to the factor of c required to get the units right, which matches the MOND proposal that has been successfully applied to explain the velocity profile of thin disk galaxies.[10]

7. Quantization

The classical electromagnetic Lagrangian cannot be quantized. One way to realize this is to calculate the canonical conjugate of the fields, the generalized 4-momentum:

$$\pi^\mu_{EM} = h\sqrt{G}\frac{\partial \mathcal{L}_{EM}}{\partial(\frac{\partial A^\mu}{c\partial t})} = -F^{0\mu}. \tag{34}$$

Unfortunately, the energy component of the momentum operator is zero. The commutator of the complementary variables of the potential and energy, $[A^0, \pi^0]$, will equal zero, and cannot be quantized. The momentum for the GEM Lagrangian does not suffer from this problem:

$$\pi^\mu_{GEM} = h\sqrt{G}\frac{\partial A^\mu}{c\partial t} = h\sqrt{G}(g_0 + (\boldsymbol{e} - \boldsymbol{E})/2). \tag{35}$$

When expressed with operators, the commutator $[A^0, \pi^0]$ will not be zero, so the field may be quantized. If the gravitational fields are zero, the generalized 4-momentum is the same as for standard electromagnetic field theory, thus passing another consistency check. The field equations (5) are similar to the classical electromagnetic Lagrangian with the choice of the Lorenz gauge, the difference being an additional mass current density source. Gupta and Bleuler have quantized the modes of radiation for a four dimensional wave equation.[1, 3] They determined that there were four modes of transmission: two transverse, one scalar, and one longitudinal mode. The scalar polarization mode represents a significant technical problem since its norm may be negative. They introduce a supplemental condition to ensure the scalar and longitudinal modes are virtual.

The field in the GEM model must represent both gravity and electromagnetism. The two transverse modes are spin-1 photons that do all the work of electromagnetism. The symmetric second-rank field strength tensor cannot be represented by a photon because photons transform differently than a symmetric tensor. For a long-range force where like charges attract, the spin must be an even integer. Whatever particle does the work must travel at the speed of light like the transverse modes. Since the symmetric tensor is rank two, that can be represented by a spin-2 graviton.

The scalar field formed from the trace of the asymmetric GEM field strength tensor may be able to play a role similar to the scalar Higgs field. Since the scalar field is entirely a function of the gravity field g^{μ}, the symmetry of the electromagnetic field is not affected. That is a key requirement for any mechanism to introduce mass into the standard model. For the graviton, the trace of the asymmetric tensor must be equal to zero so that the graviton travels at the speed of light. Particles with mass and with or without electric charge will have a non-zero trace.

There are efforts underway to detect the transverse gravitational waves predicted by general relativity. This model predicts the polarization of a gravitational wave will be either scalar or longitudinal, with the transverse modes reserved for light. The detection of the first gravitational wave polarization will mark either success or failure of the GEM field theory.

8. Conclusion

Using a nineteenth century approach, an effort to unify physics from the twentieth century has been attempted. A dynamic metric equation is found but it uses a simpler set of field equations than general relativity, a rank one tensor instead of two.

This theory makes several predictions. If weak gravitation effects are measured to second-order parameterized post-Newtonian accuracy, light will bend around the Sun 0.7 microarcseconds more than the Schwarzschild metric predicts. It remains to be seen if the complete 3-force equation matches all the data for a thin spiral galaxy. The polarization of gravitational waves will be scalar or longitudinal, not transverse as predicted by general relativity. A linear field theory for gravity can work elegantly beside the intellectual superstructure of electromagnetism, using analogy to complete the story of forces in Nature.

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