Unifying Gravity and EM by Analogies to EM

Doug Sweetser, '84, sweetser@alum.mit.edu, quaternions.com

Abstract:

Photons are generated by oscillating electrons. Gravity makes the planets oscillate around the Sun. A four dimensional simple harmonic oscillator might be able to accommodate both modes of oscillation. Investigate an old hypothesis, that gravity is similar to EM. Clone a Lagrange density for gravity from EM. The field equations generated by the Euler-Lagrange equation can be written out explicitly in terms of the electric and magnetic fields, and two analogs of these fields and one other field that characterizes the diagonal of the 4-derivative of the 4-potential, $A^{\mu,\nu}$. A problem with solutions to the field equations has been the distance dependence of the derivative. The derivative of the perturbation of a potential normalized to an arbitrary point has the correct form, a spring constant over distance squared. The gravitational analog to the Lorentz force is solved for a weak field. That solution can be rearranged into a metric equation that is consistent with the Schwarzschild metric of general relativity to parameterized post-Newtonian accuracy. For higher order terms, the two metrics could be distinguished experimentally. Derive Newton's law of gravity by breaking spacetime symmetry. Repeat the derivation, but look for a constant velocity solution. That solution shows mass decays exponentially with distance. This may be a classical gravity mechanism for the inflation hypothesis of the big bang.

Table of Contents

Unifying Gravity and EM by Analogies to EM

Outline for Proposal

1. The big picture: a 4D simple harmonic oscillator with modes for gravity and EM.

2. Propose a Lagrange density for gravity and EM based on an analogy to EM.

3. Generate the fields equations using the Euler-Lagrange equation.

4. Write out classical field equations for EM and gravity: Gauss' law, Ampere's law, and a relativistic form of the Newtonian gravitational field equation (not Einstein's field equations because these are linear and rank 1).

5. The distance dependence of the potential that solves the field equations ($\frac{1}{\text{distance}^2}$) creates a problem for a physically relevant force law. Take the derivative of the normalized, perturbation solution for a weak field which results in a spring constant over distance squared ($\nabla \frac{\frac{1}{(R_0 + \frac{k}{2R_0} r)^2}}{\frac{1}{R_0^2}} \cong \frac{k}{R_0^2} = \frac{GM}{c^2 R_0^2}$).

6. Propose a force for gravity based on an analogy to EM.

7. Solve the relativistic gravitational force for a weak field to generate a metric equation.

8. Break spacetime symmetry to derive Newton's gravitational force law.

9. Repeat the derivation above, but search for a constant velocity solution caused by gravity. The solution generates a exponential change in mass density. This may be a classical gravitational mechanism for the inflation hypothesis.

Appendices:

A) Quantization.

B) The standard model.

C) Stress tensor.

D) Geodesics.

E) Units.

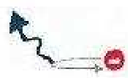
The Big Picture: A 4D Slinky

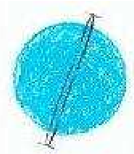
Gravity and light form a slinky in four dimensions (three for space, one for time).

- A slinky wobbles.
- The Earth has wobbled around the Sun 4 billion times.
- Light is created by electrons wobbling.

Want a description of all the interactions in a volume (called a Lagrange density) that can be used to create 4 differential equations (a 4D wave equation). The solutions to those equations for a very weak field must then be linked to the simple harmonic oscillations displayed by gravitational and electronic systems.

Thought experiment: slow neutrinos could wobble through the Earth act as a SHO, cycling to the other side of the Earth and back every 88 minutes. This is a longitudinal wave, because the acceleration is in the direction of the velocity.

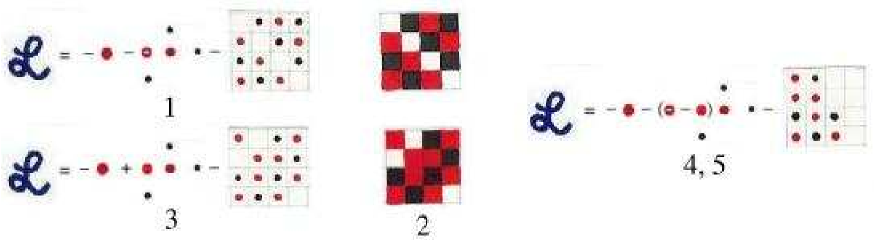




Lagrange Densities

Where all mass, energy, and interactions are in a volume.

1. EM Lagrange density.
2. EM to gravity by analogy.
3. Gravity Lagrange density hypothesis.
4. GEM Lagrange density.
5. GEM Lagrange density in detail.



EM Lagrange Density

Where all EM energy is in a volume, no gravity.

$\mathfrak{L}_{\mathrm{EM}} = -\frac{m}{\gamma V} - \frac{q}{c^2 V}\frac{U_\mu}{\gamma}A^\mu - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$

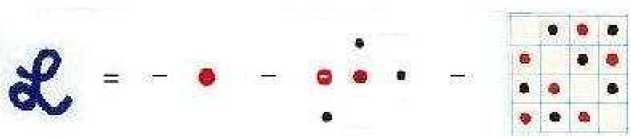
- $-\frac{m}{\gamma V}$ Energy density of mass in motion.
- $-\frac{q}{c^2 V}\frac{U_\mu}{\gamma}A^\mu$ Energy density of electric charge in motion.

  [Notation note: The electric current density is usually written as J_μ instead of its parts: charge * retivistic speed / volume, $\frac{q}{c^2 V}\frac{U_\mu}{\gamma}$. This was done because only the charge q will be changed soon.]

- $-\frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$

  Energy density of antisymmetric change in the potential.

The pattern: rank-0, rank-1 contraction, and rank-2 contraction.

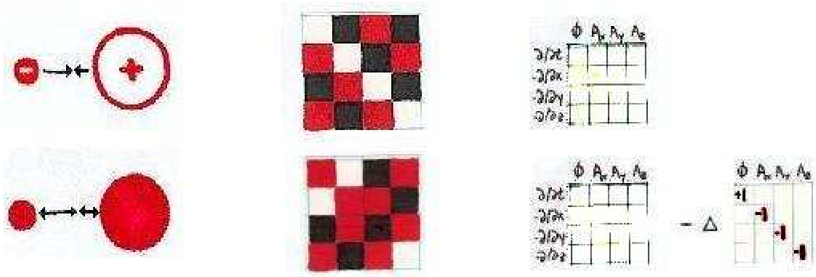


EM to Gravity Analogy

- $-\frac{m}{\gamma V}$ No change for mass in motion rank-0 term.
- $-q \longrightarrow +\sqrt{G}\,m$ Electric charge to mass charge.
- Change field strength tensor's symmetry.

  1. $, \longrightarrow ;$ Derivatives to contravariant derivatives.
  2. $A - A \longrightarrow A + A$ Anti-symmetric to symmetric tensor.

There are two sign changes, both are minus to plus. The first from -q to +m makes the law attractive. The second in the tensor changes the symmetry.



Gravity Lagrange Density Hypothesis

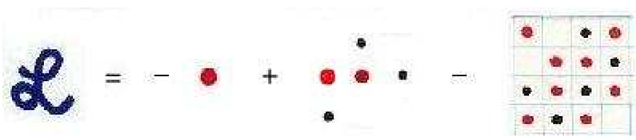
Where all gravitational energy is in a volume, no EM.

$$\mathfrak{L}_G = -\frac{m}{\gamma V} + \frac{\sqrt{G}\,m}{c^2 V}\frac{U_\mu}{\gamma}A^\mu - \frac{1}{4c^2}(A^{\mu;\nu} + A^{\nu;\mu})(A_{\mu;\nu} + A_{\nu;\mu})$$

- $-\frac{m}{\gamma V}$ Energy density of mass in motion.
- $+\frac{\sqrt{G}\,m}{c^2 V}\frac{U_\mu}{\gamma}A^\mu$ Energy density of mass charge in motion.
- $-\frac{1}{4c^2}(A^{\mu;\nu} + A^{\nu;\mu})(A_{\mu;\nu} + A_{\nu;\mu})$

  Energy density of symmetric change in the potential.

Only the rank-1 and rank-2 contraction terms have been changed by the analogy.



GEM Lagrange Density

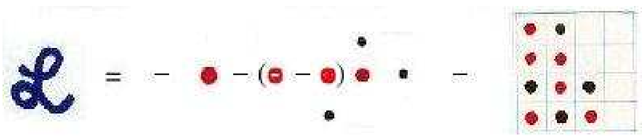
$\mathfrak{L}_{\rm GEM}$ is the union of $\mathfrak{L}_G$ and $\mathfrak{L}_{\rm EM}$.

- Mass in motion term is a union, not a sum.
- Sum charges in motion terms.
- Sum and simplify field strength tensor terms:

  1. $, \longrightarrow ;$ Derivatives to contravariant derivatives.
  2. $A^{\mu;\nu} A_{\nu;\mu} - A^{\mu;\nu} A_{\nu;\mu} = 0$ Cross terms drop.
  3. $A^{\mu;\nu} A_{\mu;\nu} = A^{\nu;\mu} A_{\nu;\mu}$ Contractions are equal.

$$\mathfrak{L}_{\rm GEM} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\, m)}{c^2 V} \frac{U_\mu}{\gamma} A^\mu - \frac{1}{2\, c^2} A^{\mu;\nu} A_{\mu;\nu}$$

Note: Every property of this proposal is dictated by $\mathfrak{L}_{\rm GEM}$!



GEM Lagrange Density in Detail

Goal: Get to individual terms, no indices.

Method: Expand, contract, and repeat.

1. Start with the GEM Lagrange density which has $1 + 4 + 16$ final terms:

$$\mathfrak{L}_{\rm GEM} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\, m)}{c^2 V} \frac{U_\mu}{\gamma} A^\mu - \frac{1}{2c^2} A^{\mu;\nu} A_{\mu;\nu}$$

2. Expand U_μ , A^μ. Apply the definition of a contravariant derivative to a contravariant vector ($A^{\mu;\nu} = A^{\mu,\nu} - \Gamma_\varpi{}^{\mu\nu} A^\varpi$) to $A^{\mu;\nu}$ and $A_{\mu;\nu}$:

$$\mathfrak{L} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\, m)}{c^2 V} (c, -v^u)(\phi, A^u) - \frac{1}{2c^2} (A^{\mu,\nu} - \Gamma_\varpi{}^{\mu\nu} A^\varpi)(A_{\mu,\nu} - \Gamma^\sigma{}_{\mu\nu} A_\sigma)$$

3. Contract U_μ with A^μ. Multiply out final term:

$$\mathfrak{L} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\, m)}{c^2 V} (c\phi - v^u A^u)$$

$$-\frac{1}{2c^2}(A^{\mu,\nu}A_{\mu,\nu} - 2\Gamma_{\varpi}{}^{\mu\nu}A^{\varpi}A_{\mu,\nu} - \Gamma_{\varpi}{}^{\mu\nu}A^{\varpi}\Gamma^{\sigma}{}_{\mu\nu}A_{\sigma})$$

4. Expand $A^{\mu,\nu}$ and $A_{\mu,\nu}$. Work in local covariant coordinates where $\Gamma = 0$:

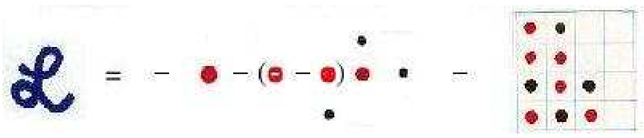
$$\mathfrak{L} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\,m)}{c^2 V}(c\phi - v^u A^u) - \frac{1}{2c^2}\left(\frac{\partial}{\partial t}, -\nabla^v\right)(\phi, A^u)\left(\frac{\partial}{\partial t}, \nabla^v\right)(\phi, -A^u)$$

5. Contract, using lines as a visual guide:

$$\mathfrak{L} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\,m)}{c^2 V}(c\phi - v^u A^u) - \frac{1}{2c^2}\left(\left(\frac{\partial \phi}{\partial t}\right)^2 - (\nabla\phi)^{v\,2} - \left(\frac{\partial A}{\partial t}\right)^{u\,2} + (\nabla A)^{uv\,2}\right)$$

6. Write it ALL out:

$$\mathfrak{L} = -\frac{m}{V}\left(\sqrt{1 - \left(\frac{\partial x}{c\partial t}\right)^2 - \left(\frac{\partial y}{c\partial t}\right)^2 - \left(\frac{\partial z}{c\partial t}\right)^2} - \frac{(q - \sqrt{G}\,m)}{c^2 V}\left(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z\right)\right.$$
$$-\frac{1}{2}\left(\left(\frac{\partial \phi}{c\partial t}\right)^2 - \left(\frac{\partial \phi}{\partial x}\right)^2 - \left(\frac{\partial \phi}{\partial y}\right)^2 - \left(\frac{\partial \phi}{\partial z}\right)^2 - \left(\frac{\partial A_x}{c\partial t}\right)^2 + \left(\frac{\partial A_x}{\partial x}\right)^2 + \left(\frac{\partial A_x}{\partial y}\right)^2 + \left(\frac{\partial A_x}{\partial z}\right)^2\right.$$
$$\left. - \left(\frac{\partial A_y}{c\partial t}\right)^2 + \left(\frac{\partial A_y}{\partial x}\right)^2 + \left(\frac{\partial A_y}{\partial y}\right)^2 + \left(\frac{\partial A_y}{\partial z}\right)^2 - \left(\frac{\partial A_z}{c\partial t}\right)^2 + \left(\frac{\partial A_z}{\partial x}\right)^2 + \left(\frac{\partial A_z}{\partial y}\right)^2 + \left(\frac{\partial A_z}{\partial z}\right)^2\right)$$

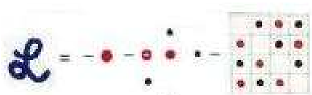


Summary: Lagrange Densities

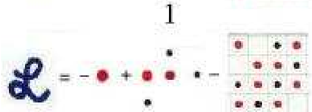
Math:

$$\mathfrak{L}_{\mathrm{GEM}} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\,m)}{c^2 V}\frac{U_\mu}{\gamma}A^\mu - \frac{1}{2c^2}A^{\mu;\nu}A_{\mu;\nu}$$

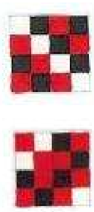
Pictures:



1



3



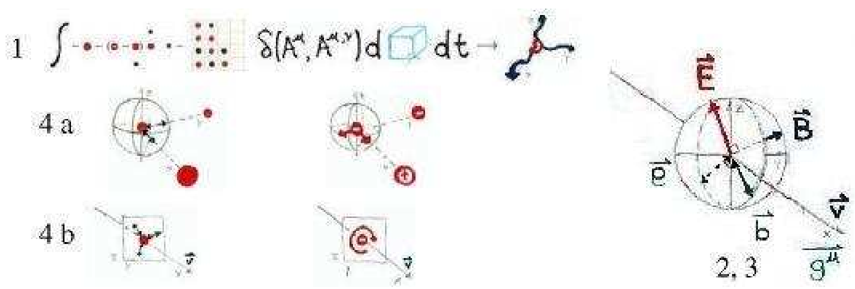
2

4, 5

Fields and Field Equations

The players.

1. Euler-Lagrange equation applied to the GEM Lagrange density.
2. Classical fields to represent $A^{\mu,\nu}$.
3. Classical fields to represent $A^{\mu,\nu}$ in detail (their components).
4. Classical field equations:
   a) Gauss' law and Newton's [relativistic] gravitational field.
   b) Ampere's law and mass current.
   c) Vector identities.



The Players

A table of the players in fields and field equations. The new analogous fields for gravity ($\vec{e}$, $\vec{b}$, and g^{μ}) will be introduced subsequently.

Rank	*Symbol*	*Name*
0	$\mathfrak{L}$	Lagrange density
1	$c\frac{\partial \mathfrak{L}}{\partial A^{\mu}} = c(\frac{\partial \mathfrak{L}}{\partial A^{\mu,\nu}})^{,\nu}$	Field equations
1	A^{μ}	Potential
2	$A^{\mu,\nu}$	Derivative of the potential
2	$\vec{E}, \vec{e}, \vec{B}, \vec{b}, g^{\mu}$	Classical fields which constitute $A^{\mu,\nu}$
1	$\frac{\partial \vec{E}}{c\partial t}, \vec{\nabla}\times\vec{E}, \frac{\partial \vec{e}}{c\partial t}, \frac{\partial \vec{B}}{c\partial t}, \vec{\nabla}\times\vec{B}, \vec{\nabla}\boxtimes\vec{b}, \nabla g^{\mu}$	Field equations (above) as classical fields

Apply Euler-Lagrange to GEM Lagrange Density

1. Start with the Euler-Lagrange equation, $\frac{\partial \mathfrak{L}}{\partial A^{\mu}} = (\frac{\partial \mathfrak{L}}{\partial A^{\mu,\nu}})^{,\nu}$, written without indices:

   $$c\frac{\partial \mathfrak{L}}{\partial \phi} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial \phi}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \phi}{\partial z})}))$$

$$c\frac{\partial \mathfrak{L}}{\partial A_x} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_x}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_x}{\partial z})}))$$

$$c\frac{\partial \mathfrak{L}}{\partial A_y} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_y}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Ay}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_y}{\partial z})}))$$

$$c\frac{\partial \mathfrak{L}}{\partial A_z} = c(\frac{\partial}{\partial t}(\frac{\partial \mathfrak{L}}{\partial(\frac{\partial A_z}{\partial t})}) - \frac{\partial}{\partial x}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial x})}) - \frac{\partial}{\partial y}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial \mathrm{Az}}{\partial y})}) - \frac{\partial}{\partial z}(\frac{\partial \mathfrak{L}}{\partial(-\frac{\partial A_z}{\partial z})}))$$

2. Write out GEM Lagrange density without indices:

$$\mathfrak{L} = -\frac{m}{V}(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2} - \frac{(q - \sqrt{G}\,m)}{c^2 V}(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$
$$-\frac{1}{2}((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$
$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2)$$

3. Apply:

$$-\frac{(q - \sqrt{G}\,m)}{V} = -\frac{\partial^2 \phi}{c\partial t^2} + c\frac{\partial^2 \phi}{\partial x^2} + c\frac{\partial^2 \phi}{\partial y^2} + c\frac{\partial^2 \phi}{\partial z^2}$$

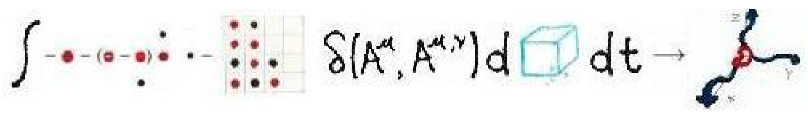
$$\frac{(q - \sqrt{G}\,m)}{V}\frac{\partial x}{c\partial t} = \frac{\partial^2 A_x}{c\partial t^2} - c\frac{\partial^2 A_x}{\partial x^2} - c\frac{\partial^2 A_x}{\partial y^2} - c\frac{\partial^2 A_x}{\partial z^2}$$

$$\frac{(q - \sqrt{G}\,m)}{V}\frac{\partial y}{c\partial t} = \frac{\partial^2 A_y}{c\partial t^2} - c\frac{\partial^2 A_y}{\partial x^2} - c\frac{\partial^2 A_y}{\partial y^2} - c\frac{\partial^2 A_y}{\partial z^2}$$

$$\frac{(q - \sqrt{G}\,m)}{V}\frac{\partial z}{c\partial t} = \frac{\partial^2 A_z}{c\partial t^2} - c\frac{\partial^2 A_z}{\partial x^2} - c\frac{\partial^2 A_z}{\partial y^2} - c\frac{\partial^2 A_z}{\partial z^2}$$

4. Executive summary:

$J^\mu = \Box^2 A^\mu$ where $J^\mu = \frac{(q - \sqrt{G}\,m)}{V}\frac{U^\mu}{c}$



Classical Fields

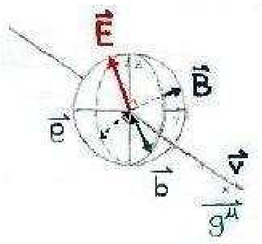
The classical fields $\vec{E}$ and $\vec{B}$ together make up the antisymmetric tensor $(A^{\mu,\nu} - A^{\nu,\mu})$. Introduce three new fields, $\vec{e}$ and $\vec{b}$ which have EM counterparts, and a 4-vector field g^μ for the diagonal components of the symmetric tensor $(A^{\mu,\nu} + A^{\nu,\mu})$.

- $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$ Electric field.
- $\vec{e} = +\frac{\partial \vec{A}}{\partial t} - c\vec{\nabla}\phi$ Symmetric analog to electric field.
- $\vec{B} = c(0, +\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, +\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, +\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}) = c\vec{\nabla} \times \vec{A}$ Magnetic field.
- $\vec{b} = c(0, -\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, -\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, -\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}) \equiv c\vec{\nabla} \boxtimes \vec{A}$

  Symmetric analog to magnetic field.

- $g^{\mu} = A^{\mu,\mu} = (\frac{\partial \phi}{\partial t}, -c\frac{\partial A_x}{\partial x}, -c\frac{\partial A_y}{\partial y}, -c\frac{\partial A_z}{\partial z})$ Diagonal of $A^{\mu,\nu}$.

5 fields, 3+3+3+3+4=16 degrees of freedom.



Classical Fields in Detail

1. Start with the asymmetric unified field strength tensor, $A^{\mu,\nu}$, written as a matrix:

$$
\begin{array}{ccccc}
 & \mu = \phi & \mu = A_x & \mu = A_y & \mu = A_z \\
{}^{,\nu} = \frac{\partial}{\partial t} & \frac{\partial \phi}{\partial t} & \frac{\partial A_x}{\partial t} & \frac{\partial A_y}{\partial t} & \frac{\partial A_z}{\partial t} \\
{}^{,\nu} = -c\frac{\partial}{\partial x} & -c\frac{\partial \phi}{\partial x} & -c\frac{\partial A_x}{\partial x} & -c\frac{\partial A_y}{\partial x} & -c\frac{\partial A_z}{\partial x} \\
{}^{,\nu} = -c\frac{\partial}{\partial y} & -c\frac{\partial \phi}{\partial y} & -c\frac{\partial A_x}{\partial y} & -c\frac{\partial A_y}{\partial y} & -c\frac{\partial A_z}{\partial y} \\
{}^{,\nu} = -c\frac{\partial}{\partial z} & -c\frac{\partial \phi}{\partial z} & -c\frac{\partial A_x}{\partial z} & -c\frac{\partial A_y}{\partial z} & -c\frac{\partial A_z}{\partial z}
\end{array}
$$

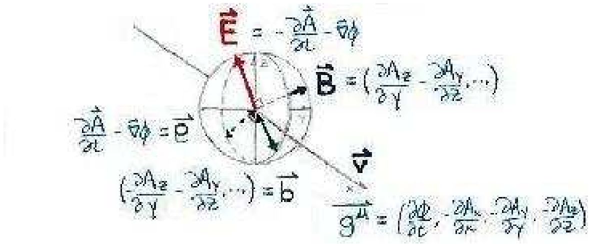
2. An antisymmetric and symmetric sum equal to $A^{\mu,\nu}$:

$$
(A^{\mu,\nu} - A^{\nu,\mu})/2 = \begin{array}{cccc}
0 & \frac{\partial A_x}{\partial t} + c\frac{\partial \phi}{\partial x} & \frac{\partial A_y}{\partial t} + c\frac{\partial \phi}{\partial y} & \frac{\partial A_z}{\partial t} + c\frac{\partial \phi}{\partial z} \\
-c\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t} & 0 & -c\frac{\partial A_y}{\partial x} + c\frac{\partial A_x}{\partial y} & -c\frac{\partial A_z}{\partial x} + c\frac{\partial A_x}{\partial z} \\
-c\frac{\partial \phi}{\partial y} - \frac{\partial A_y}{\partial t} & -c\frac{\partial A_x}{\partial y} + c\frac{\partial A_y}{\partial x} & 0 & -c\frac{\partial A_z}{\partial y} + c\frac{\partial A_y}{\partial z} \\
-c\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} & -c\frac{\partial A_x}{\partial z} + c\frac{\partial A_z}{\partial x} & -c\frac{\partial A_y}{\partial z} + c\frac{\partial A_z}{\partial y} & 0
\end{array}
$$

$$
(A^{\mu,\nu} + A^{\nu,\mu})/2 = \begin{array}{cccc}
\frac{\partial \phi}{\partial t} & \frac{\partial A_x}{\partial t} - c\frac{\partial \phi}{\partial x} & \frac{\partial A_y}{\partial t} - c\frac{\partial \phi}{\partial y} & \frac{\partial A_z}{\partial t} - c\frac{\partial \phi}{\partial z} \\
-c\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} & -c\frac{\partial A_x}{\partial x} & -c\frac{\partial A_y}{\partial x} - c\frac{\partial A_x}{\partial y} & -c\frac{\partial A_z}{\partial x} - \frac{c\partial A_x}{\partial z} \\
-c\frac{\partial \phi}{\partial y} + \frac{\partial A_y}{\partial t} & -c\frac{\partial A_x}{\partial y} - c\frac{\partial A_y}{\partial x} & -c\frac{\partial A_y}{\partial y} & -c\frac{\partial A_z}{\partial y} - c\frac{\partial A_y}{\partial z} \\
-c\frac{\partial \phi}{\partial z} + \frac{\partial A_z}{\partial t} & -c\frac{\partial A_x}{\partial z} - c\frac{\partial A_z}{\partial x} & -c\frac{\partial A_y}{\partial z} - c\frac{\partial A_z}{\partial y} & -c\frac{\partial A_z}{\partial z}
\end{array}
$$

3. $A^{\mu,\nu}$ written in terms of the gravitational, electric, and magnetic fields:

$$
\begin{array}{cccc}
g_t & e_x - E_x & e_y - E_y & e_z - E_z \\
e_x + E_x & g_x & b_z - B_z & b_y + B_y \\
e_y + E_y & b_z + B_z & g_y & b_x - B_x \\
e_z + E_z & b_y - B_y & b_x + B_x & g_z
\end{array}
$$



Gauss' Law and Newton's [Relativistic] Gravitational Field

Method: $\frac{1}{2}$ (EM law $+$ gravitational analog) $+$ diagonal terms $=$ field equations.

$$\frac{q-\sqrt{G}\,m}{V} = \frac{1}{2}(\vec{\nabla}\cdot\vec{E} + \vec{\nabla}\cdot\vec{e}\,) + \frac{\partial\, g_t}{c\,\partial t}$$

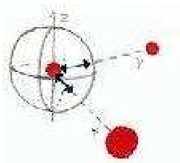
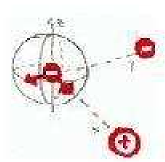
$$= \frac{1}{2}\Big(-\frac{\partial^2 A_x}{\partial x\,\partial t} - c\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2 A_y}{\partial y\,\partial t} - c\frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2 A_z}{\partial z\,\partial t} - c\frac{\partial^2\phi}{\partial z^2}$$

$$+ \frac{\partial^2 A_x}{\partial x\,\partial t} - c\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2 A_y}{\partial y\,\partial t} - c\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2 A_z}{\partial z\,\partial t} - c\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\phi}{c\,\partial t^2}$$

$$= \frac{\partial^2\phi}{c\,\partial t^2} - c\frac{\partial^2\phi}{\partial x^2} - c\frac{\partial^2\phi}{\partial y^2} - c\frac{\partial^2\phi}{\partial z^2} = \Box^2\phi$$

- Newton's [relativistic] gravitational field equation results in the physical situation where there is no electric charge density and no divergence of the field $\vec{E}$.
- Gauss' law results in the physical situation with no mass density and no divergence of the field $\vec{e}$.

Implications for forces: Newton's field law implies an attractive force for mass, while Gauss' law indicates like electric charges repulse.

Ampere's Law and Mass Current

Method: Same as previous.

$$\Big(\frac{q-\sqrt{G}\,m}{V}\Big)\frac{\vec{v}}{c} = \frac{1}{2}\Big(-\frac{\partial\vec{E}}{c\,\partial t} + \frac{\partial\,\vec{e}}{c\,\partial t} + \vec{\nabla}\times\vec{B} + \nabla\boxtimes\vec{b}\,\Big) + \vec{\nabla}_u\, g^u$$

$$= \frac{1}{2}\Big(\frac{\partial^2 A_x}{c\,\partial t^2} + \frac{\partial^2\phi}{\partial t\,\partial x}, \frac{\partial^2 A_y}{c\,\partial t^2} + \frac{\partial^2\phi}{\partial t\,\partial y}, \frac{\partial^2 A_z}{c\,\partial t^2} + \frac{\partial^2\phi}{\partial t\,\partial z}\Big)$$

$$+ \frac{1}{2}\Big(\frac{\partial^2 A_x}{c\,\partial t^2} - \frac{\partial^2\phi}{\partial t\,\partial x}, \frac{\partial^2 A_y}{c\,\partial t^2} - \frac{\partial^2\phi}{\partial t\,\partial y}, \frac{\partial^2 A_z}{c\,\partial t^2} - \frac{\partial^2\phi}{\partial t\,\partial z}\Big)$$

$$+ \frac{c}{2}\Big(-\frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_y}{\partial y\partial x} - \frac{\partial^2 A_x}{\partial z^2} + \frac{\partial^2 A_z}{\partial z\partial x}, -\frac{\partial^2 A_y}{\partial z^2} + \frac{\partial^2 A_z}{\partial z\partial y} - \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_x}{\partial x\,\partial y}, -\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_x}{\partial x\,\partial z} - \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_y}{\partial y\,\partial z}\Big)$$

$$+ \frac{c}{2}\Big(-\frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_y}{\partial y\partial x} - \frac{\partial^2 A_x}{\partial z^2} - \frac{\partial^2 A_z}{\partial z\partial x}, -\frac{\partial^2 A_y}{\partial z^2} - \frac{\partial^2 A_z}{\partial z\partial y} - \frac{\partial^2 A_y}{\partial x^2} - \frac{\partial^2 A_x}{\partial x\,\partial y}, -\frac{\partial^2 A_z}{\partial x^2} - \frac{\partial^2 A_x}{\partial x\,\partial z} - \frac{\partial^2 A_z}{\partial y^2} - \frac{\partial^2 A_y}{\partial y\,\partial z}\Big)$$

$+c(-\frac{\partial^2 A_x}{\partial x^2}, -\frac{\partial^2 A_x}{\partial x^2}, -\frac{\partial^2 A_x}{\partial x^2})$

$= \Box^2 \vec{A}$

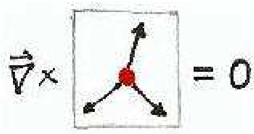
- A pure mass current equation results in the physical situation where there is no electric current density no time change of the field $\vec{E}$ and no curl of the field $\vec{B}$.
- Ampere's law results in the physical situation where there is no mass current density, no gradient of the field g^u and not boxed curl of b.


Homogeneous Equations (Vector Identities)

Vector identities or homogeneous equations are unchanged.

- $\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (c\vec{\nabla} \times \vec{A}) = 0$ No magnetic monopoles.
- $\frac{\partial B}{c\partial t} + \vec{\nabla} \times \vec{E} = \frac{\partial}{\partial t}\vec{\nabla} \times \vec{A} - \vec{\nabla} \times \frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \times c\vec{\nabla}\phi = \vec{0}$ Faraday's law.

No obvious vector identity analogs for gravitational fields found yet.



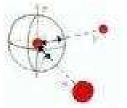
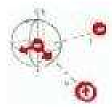
Summary: Field Equations

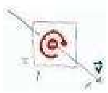
Math:

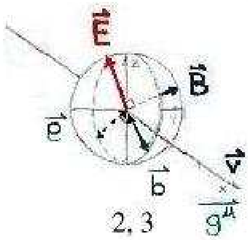
$J^\mu = \Box^2 A^\mu$

Pictures:

1 

4 a 


4 b 




Field Equation Solutions

1. 4D Wave equation solution in a vacuum.

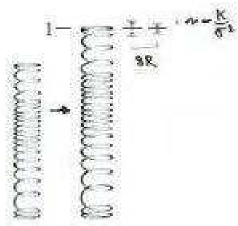
2. Normalization and perturbations.

3. Normalized, perturbation solution to the 4D wave equation.

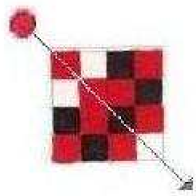
4. Derivative of the normalized, perturbation solution.

5. Only weak gravity.

1



2 - 4



5

4D Wave Equation Solution in a Vacuum

1. Start with 4D wave equation, no source:

   $\Box^2 A^{\mu} = 0$

2. Guess a solution analogous to previous:

   $A^{\mu} = \frac{\sqrt{G}\,h}{c^2}\,((x^2 + y^2 + z^2 - c^2 t^2)^{-1}, 0, 0, 0) = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{\sigma^2}, \vec{0})$

3. Take derivatives:

   $\frac{\partial}{c\partial t}\,(x^2 + y^2 + z^2 - c^2 t^2)^{-1} = +\,2ct\,(x^2 + y^2 + z^2 - c^2 t^2)^{-2}$

   $\frac{\partial}{\partial x}\,(x^2 + y^2 + z^2 - c^2 t^2)^{-1} = -\,2x\,(x^2 + y^2 + z^2 - c^2 t^2)^{-2}$

   $\frac{\partial}{\partial y}\,(x^2 + y^2 + z^2 - c^2 t^2)^{-1} = -\,2y\,(x^2 + y^2 + z^2 - c^2 t^2)^{-2}$

   $\frac{\partial}{\partial z}\,(x^2 + y^2 + z^2 - c^2 t^2)^{-1} = -\,2z\,(x^2 + y^2 + z^2 - c^2 t^2)^{-2}$

4. Take second derivatives:

$$\frac{\partial}{c\partial t}(+2t\sigma^{-4}) = +2(x^2+y^2+z^2-c^2t^2)^{-2} + 8c^2\, t^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

$$\frac{\partial}{\partial x}(-2x\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8x^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

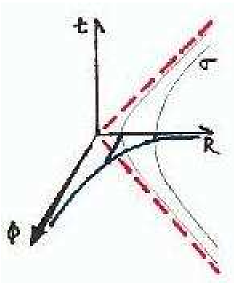
$$\frac{\partial}{\partial y}(-2y\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8y^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

$$\frac{\partial}{\partial z}(-2z\sigma^{-4}) = -2(x^2+y^2+z^2-c^2t^2)^{-2} + 8z^2(x^2+y^2+z^2-c^2t^2)^{-4}$$

5. Sum:

$$\frac{\partial^2 A_0}{c^2\partial t^2} - \frac{\partial^2 A_0}{\partial x^2} - \frac{\partial^2 A_0}{\partial y^2} - \frac{\partial^2 A_0}{\partial z^2} = 0 \quad \text{QED}$$

Practical value: Singularity is the lightcone, $x^2+y^2+z^2-c^2t^2=0$.



Normalization and Perturbations

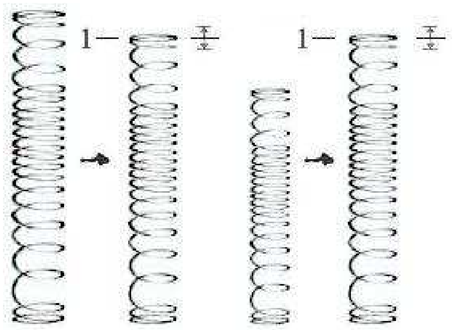
Quantum mechanics cliché: normalize and look at perturbations for a weak field. Gravity is weak.

1. Normalization:

- U(1)xsU(2)xsU(3) Unitary requirement of the standard model.
- $\frac{A^\mu}{|A|} \rightsquigarrow -$ Dimensionless.

2. Perturbations:

- $A \longrightarrow A' = A + k\delta$ Linear restoration.
- k Spring constant (small number).
- δ Variable.



Normalized, Perturbation Solution

1. Start with 4D wave equation solution:

$$A^{\mu} = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{x^2+y^2+z^2-c^2t^2}, \vec{0}) = \frac{\sqrt{G}\,h}{c^2}\,(\frac{1}{\sigma^2}, \vec{0})$$

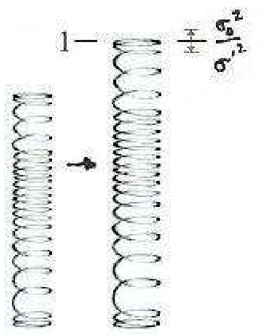
2. Normalize so that the magnitude of A^{μ} is equal to one:

$$A^{\mu} = \frac{A^{\mu}}{|A^{\mu}|} = \frac{c}{\sqrt{G}}\,(\frac{1}{\frac{x^2+y^2+z^2-c^2t^2}{\sigma^2}}, \vec{0}) = (1, \vec{0})$$

3. Perturb $x, y, z,$ and t linearly with a spring constant k:

$$A^{\mu} = \frac{A'^{\mu}}{|A^{\mu}|} = \frac{c}{\sqrt{G}}\,(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0})$$

$$= \frac{c}{\sqrt{G}}\,(\sim 1, \vec{0}) = \frac{c}{\sqrt{G}}\,(\frac{\sigma^2}{\sigma'^2}, \vec{0})$$



Derivative of the Normalized, Perturbation Solution

1. Start with the normalized, perturbation solution:

$$A^{\mu} = \frac{A'^{\mu}}{|A^{\mu}|} = \frac{c}{\sqrt{G}}\,(\frac{1}{(\frac{1}{\sqrt{2}}+\frac{kx}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{ky}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{kz}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{kct}{\sigma^2})^2}, \vec{0}) = \frac{c}{\sqrt{G}}\,(\frac{\sigma^2}{\sigma'^2}, \vec{0})$$

2. Expand:

$$A^\mu = \frac{c}{\sqrt{G}} \left(\frac{1}{(\frac{1}{2} + \frac{k\,x}{\sqrt{2}\,\sigma^2} + \frac{k^2\,x^2}{\sigma^4}) + (\frac{1}{2} + \frac{k\,y}{\sqrt{2}\,\sigma^2} + \frac{k^2\,y^2}{\sigma^4}) + (\frac{1}{2} + \frac{k\,z}{\sqrt{2}\,\sigma^2} + \frac{k^2\,z^2}{\sigma^4}) - (\frac{1}{2} + \frac{k\,c\,t}{\sqrt{2}\,\sigma^2} + \frac{k^2\,c^2\,t^2}{\sigma^4})}, \vec{0} \right)$$

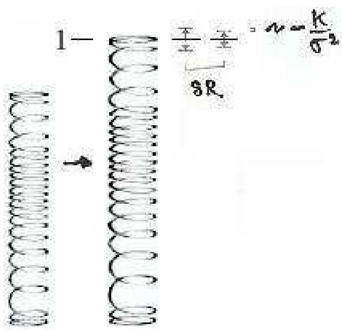
$$= \frac{c}{\sqrt{G}} \left(\frac{\sigma_0^2}{\sigma'^2}, \vec{0} \right)$$

3. Take derivatives:

   $\frac{\partial A^\mu}{c\,\partial t} = \frac{c^2}{\sqrt{G}} \frac{\sigma_0^2}{\sigma'^4} k + O(k^2) \cong \frac{c^2}{\sqrt{G}} \frac{k}{\sigma^2} + O(k^2)$ where the $\frac{1}{\sqrt{2}}$ is now part of k

   $$\frac{\partial A^\mu}{\partial R} = -\frac{c^2}{\sqrt{G}} \frac{\sigma_0^2}{\sigma'^4} k + O(k^2) \cong -\frac{c^2}{\sqrt{G}} \frac{k}{\sigma^2} + O(k^2)$$

- $\frac{1}{\sigma^2}$ An inverse square distance dependence.
- k A small number with units of distance ($\frac{GM}{c^2}$).
- $kx, ky, kz, kt \lll \sigma^2$ Solutions are local, not global.



Only Weak Gravity

A potential that only applies to gravity not EM will have a diagonal field strength tensor.

- The sign of the spring constant k does not effect solving the field equations.
- The sign of the spring constant k does change the derivative of the potential to first order in k.
- Therefore a potential that only has derivatives along the diagonal can be constructed from two potentials that differ by string constants that either constructively interfere to create a non-zero derivative, or destructively interfere to eliminate a derivative.

diagonal SHO $A^\mu = \frac{c^2}{\sqrt{G}}$

$$\left(\frac{1}{(\frac{1}{\sqrt{2}} + \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} + \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{kx}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{ky}{\sigma^2})^2 + (\frac{1}{\sqrt{2}} - \frac{kz}{\sigma^2})^2 - (\frac{1}{\sqrt{2}} + \frac{kct}{\sigma^2})^2}, \right.$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k\,x}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,y}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,z}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{k\,c\,t}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k\,x}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{k\,y}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{k\,z}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{k\,c\,t}{\sigma^2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k\,x}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,y}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,z}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{k\,c\,t}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{k\,x}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,y}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{k\,z}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{k\,c\,t}{\sigma^2})^2},$$

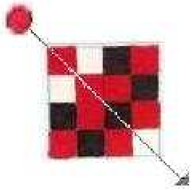
$$\frac{1}{(\frac{1}{\sqrt{2}}+\frac{k\,x}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,y}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,z}{\sigma^2})^2-(\frac{1}{\sqrt{2}}+\frac{k\,c\,t}{\sigma^2})^2}+\frac{1}{(\frac{1}{\sqrt{2}}-\frac{k\,x}{\sigma^2})^2+(\frac{1}{\sqrt{2}}-\frac{k\,y}{\sigma^2})^2+(\frac{1}{\sqrt{2}}+\frac{k\,z}{\sigma^2})^2-(\frac{1}{\sqrt{2}}-\frac{k\,c\,t}{\sigma^2})^2})$$

Notice the pattern for signs of k.

Take the contravariant derivative of this potential, keeping only the terms to first order in the spring constant k. Remember the contravariant derivative flips the sign of the 3-vector.

$$A^{\mu,\nu} \cong \frac{c^2}{\sqrt{G}}\begin{pmatrix} \frac{k}{\sigma 2} & 0 & 0 & 0 \\ 0 & \frac{k}{\sigma 2} & 0 & 0 \\ 0 & 0 & \frac{k}{\sigma 2} & 0 \\ 0 & 0 & 0 & \frac{k}{\sigma 2} \end{pmatrix}$$

This is an identity matrix times $\frac{c^2}{\sqrt{G}}\frac{k}{\sigma 2}$, a simple end result that required much work.

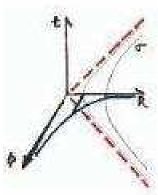


Summary: Field Equation Solutions

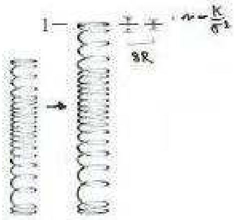
Math:

$$A^{\mu,\nu} \cong \frac{c^2}{\sqrt{G}}\begin{pmatrix} \frac{k}{\sigma 2} & 0 & 0 & 0 \\ 0 & \frac{k}{\sigma 2} & 0 & 0 \\ 0 & 0 & \frac{k}{\sigma 2} & 0 \\ 0 & 0 & 0 & \frac{k}{\sigma 2} \end{pmatrix}$$

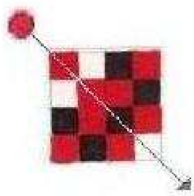
Pictures:



1



2 - 4



5

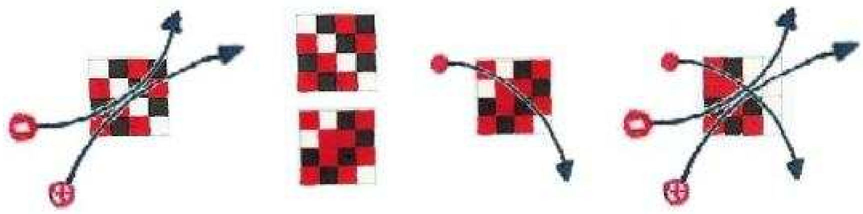
Forces

1. EM Lorentz force.

2. EM to gravity analogy.

3. Gravitational force.

4. GEM force.



1 2 3 4

EM Lorentz Force

The Lorentz force is caused by an electric charge moving in an EM field. The effect is to push particles around.

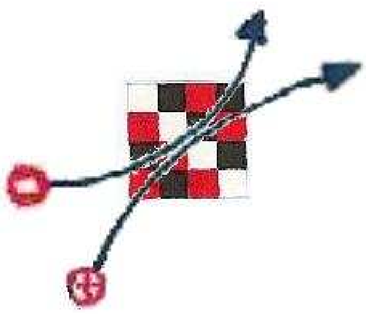
$F^{\mu}_{\text{EM}} = q \, \frac{U_{\nu}}{c}(A^{\mu,\nu} - A^{\nu,\mu}) = \frac{\partial m \, U^{\mu}}{\partial \tau}$

- The cause is electric charge times the velocity contracted with the antisymmetric field strength tensor.

- The effect is to change momentum with respect to the interval τ.

- If the sign of charge is inverted ($q \longrightarrow - q$), F^{μ}_{EM} flips signs, so there are two distinguishable electric charges.

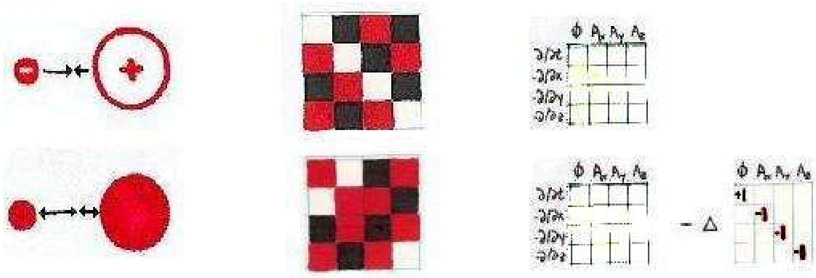
- Like electrical charges are forced away from each other due to the positive sign of the force.



EM to Gravity Analogy

- $-q \longrightarrow +\sqrt{G}\, m$ Electric charge to mass charge.
- Change field strength tensor's symmetry.

  1. $A - A \longrightarrow A + A$ Anti-symmetric to symmetric tensor.
  2. $, \longrightarrow ;$ Derivatives to contravariant derivatives.



Gravitational Force

The gravitational force is caused by a mass charge moving in a gravitational field. The effect is to push particles around.

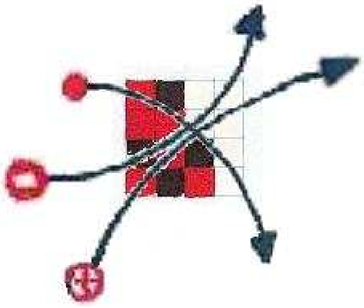
$$F^{\mu}_{G} = -\sqrt{G}\, m \frac{U_{\nu}}{c}(A^{\mu;\nu} + A^{\nu;\mu}) = \frac{\partial m\, U^{\mu}}{\partial \tau}$$

- The cause is mass charge times the velocity contracted with the symmetric field strength tensor.
- The effect is to change momentum with respect to the interval τ.
- If the sign of mass is inverted ($m \longrightarrow -m$), F^{μ}_{G} is invariant so there is one distinguishable mass charge.
- Mass charges are forced toward each other due to the negative sign of the force.

GEM Force

The GEM force is the sum of the gravitational and EM forces.

$F^{\mu}_{\rm GEM} = -\left(\sqrt{G}\,m - q\right)\frac{U_\nu}{c}A^{\mu;\nu} - \left(\sqrt{G}\,m + q\right)\frac{U_\nu}{c}A^{\nu;\mu} = \frac{\partial m U^{\mu}}{\partial \tau}$

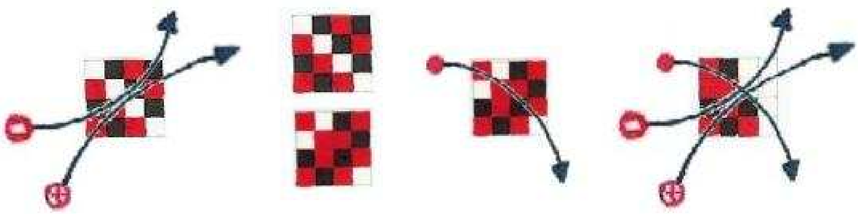


Summary: Forces

Math:

$F^{\mu}_{\rm GEM} = -\left(\sqrt{G}\,m - q\right)\frac{U_\nu}{c}A^{\mu;\nu} - \left(\sqrt{G}\,m + q\right)\frac{U_\nu}{c}A^{\nu;\mu} = \frac{\partial m U^{\mu}}{\partial \tau}$

Pictures:



1 2 3 4

Relativistic Gravitational Force

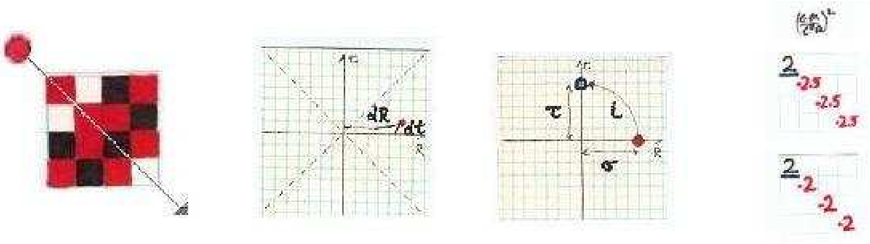
1. Weak field approximation.

2. Exact solution.

3. Exact solution applied.

4. Schwarzschild metric.

5. Schwarzschild versus GEM metric.



1   2   3   4, 5

Weak Field Approximation

1. Start from the gravitational force law:

   $$F_G^{\mu} = -\sqrt{G}\, m\, \frac{U_\nu}{c}(A^{\mu;\nu} + A^{\nu;\mu}) = \frac{\partial m\, U^\mu}{\partial \tau}$$

2. Assume local covariant coordinates (; $\rightarrow$,):

   $$F_G^{\mu} = -\sqrt{G}\, m\, \frac{U_\nu}{c}(A^{\mu,\nu} + A^{\nu,\mu}) = \frac{\partial m\, U^\mu}{\partial \tau}$$

3. Recall weak gravitational field strength tensor:

   $$A^{\mu,\nu} \cong \begin{pmatrix} \frac{k}{\sigma 2} & 0 & 0 & 0 \\ 0 & \frac{k}{\sigma 2} & 0 & 0 \\ 0 & 0 & \frac{k}{\sigma 2} & 0 \\ 0 & 0 & 0 & \frac{k}{\sigma 2} \end{pmatrix}$$

4. Check units of $A^{\mu,\nu}$ to the derivative of the normalized potential:

   $$\sqrt{G}\, A^{\mu,\nu} \rightsquigarrow \frac{\sqrt{L^3}}{t\sqrt{m}}\, \frac{\sqrt{m}}{t\sqrt{L}} = \frac{L}{t^2}$$

$$c\frac{\partial\frac{A'^{\mu}}{|A^{\mu}|}}{\partial t}\rightsquigarrow\frac{L}{t}\frac{1}{t}=\frac{L}{t^2}$$

5. Substitute the normalized potential derivative into the force law, noting the units and the sign flip on the contravariant derivative. Expand the velocities, $U_\nu\to(U_0,-\vec{U})$ and $U^\mu\to(U_0,\vec{U})$:

$$F_G^\mu=-mc^2\left(\frac{U_0}{c},-\frac{\vec{U}}{c}\right)\begin{pmatrix}\frac{k}{\sigma^2} & 0\\ 0 & \frac{k}{\sigma^2}\end{pmatrix}=\left(\frac{\partial m U_0}{\partial\tau},\frac{\partial m\vec{U}}{\partial\tau}\right)$$

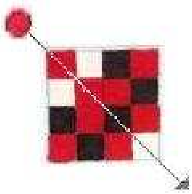
6. Contract the rank-1 velocity tensor with the rank-2 derivative of the potential:

$$F_G^\mu=m\left(-\frac{ck}{\sigma^2}U_0,\frac{ck}{\sigma^2}\vec{U}\right)=\left(\frac{\partial m U_0}{\partial\tau},\frac{\partial m\vec{U}}{\partial\tau}\right)$$

7. Substitute $c^2\tau^2$ for $-\sigma^2$:

$$F_G^\mu=m\left(\frac{k}{\tau^2}\frac{U_0}{c},-\frac{k}{\tau^2}\frac{\vec{U}}{c}\right)=\left(\frac{\partial m U_0}{\partial\tau},\frac{\partial m\vec{U}}{\partial\tau}\right)$$

Warning: The relationship between σ^2 and τ^2 is simple. What gets tricky is the relationship between σ and τ, because there the signs are "free" $(\pm i\sigma\to\pm c\tau)$.



Exact Solution

The gravitational force for the weak field is a first order differential equation that can be solved exactly.

1. Start from the gravitational force for a weak field:

$$F_G^\mu=m\left(\frac{k}{\tau^2}\frac{U_0}{c},-\frac{k}{\tau^2}\frac{\vec{U}}{c}\right)=\left(\frac{\partial m U_0}{\partial\tau},\frac{\partial m\vec{U}}{\partial\tau}\right)$$

2. Apply the chain rule to the cause terms. Assume $U_0\frac{\partial m}{\partial\tau}=\vec{U}\frac{\partial m}{\partial\tau}=0$.

   Collect terms on one side:

$$\left(m\frac{\partial U_0}{\partial\tau}-m\frac{k}{\tau^2}\frac{U_0}{c},m\frac{\partial\vec{U}}{\partial\tau}+m\frac{k}{\tau^2}\frac{\vec{U}}{c}\right)=0$$

3. Assume the equivalence principle. Drop m:

$$\left(\frac{\partial U_0}{\partial \tau} - \frac{k}{\tau^2}\frac{U_0}{c}, \frac{\partial \vec{U}}{\partial \tau} + \frac{k}{\tau^2}\frac{\vec{U}}{c} \right) = 0$$

4. Solve for velocity:

   $(U_0, \vec{U}) = (u_0\, e^{-\frac{k}{c\tau}}, \vec{u}\, e^{+\frac{k}{c\tau}})$ where $(u_0, \vec{u})$ are constant velocities.

5. Contract the velocity solution:

   $$U^\mu U_\mu = u_0^2\, e^{-2\frac{k}{c\tau}} - \vec{u}^2\, e^{+2\frac{k}{c\tau}}$$

6. There are four constraints on the contracted velocity solution for flat spacetime $(k \to 0, \text{or}\, \tau \to \infty)$:

   $$U^\mu U_\mu = (\,u_0, \;\; \vec{u}\,)\,(\,u_0, \;\; -\vec{u}\,)$$

   $$= \left(c\frac{\partial t}{\partial \tau}, \frac{\partial \vec{R}}{\partial \tau}\right)\left(c\frac{\partial t}{\partial \tau}, -\frac{\partial \vec{R}}{\partial \tau}\right) = \frac{c^2\,(\partial t)^2 - (\partial R)^2}{(\partial t)^2 - (\frac{\partial R}{c})^2} = c^2$$

   True if and only if: $u_0 = c\frac{\partial t}{\partial \tau}, \quad \vec{u} = \frac{\partial \vec{R}}{\partial \tau}$

   The integration constants $(u_0, \vec{u})$ are the velocities in flat spacetime.

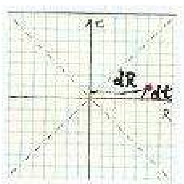
7. Substitute $c\frac{\partial t}{\partial \tau}$ for u_0, $\frac{\partial \vec{R}}{\partial \tau}$ for $\vec{u}$ into the contracted velocity solution. Multiply through by $(\frac{\partial \tau}{c})^2$:

   $$(\partial \tau)^2 = e^{-2\frac{k}{c\tau}}\,(\partial t)^2 - e^{+2\frac{k}{c\tau}}\,(\frac{\partial \vec{R}}{c})^2$$

This is a unique algebraic road to a metric equations. The logic will have to be looked at by mathematicians.

- $k = 0$, or $\tau \to \infty$ Flat spacetime.

- $e^{-2\frac{k}{c\tau}} \neq 1$ Curved spacetime.



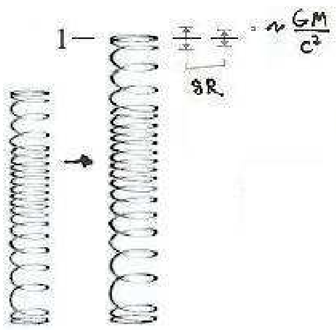
Exact Solution Applied

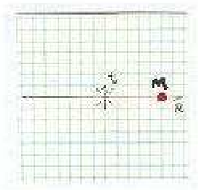
Apply to a weak, spherically symmetric, gravitational system.

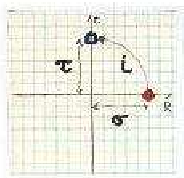
- $k = \frac{GM}{c^2} \rightsquigarrow \frac{L^3}{mt^2}\, m\, \frac{t^2}{L^2} = L$ Gravitational source spring constant.
- $\sigma^2 = R^2 - (ct)^2 = R'^2$ Static field approximated by R'.
- $|\sigma| = |c\tau| = R$ σ and $c\tau$ have the same magnitude.
- $(+i\sigma)^2 = (+c\tau)^2$ To make a real metric, choose σ to be imaginary.

Plug into the exact solution:

$$(\partial\tau)^2 = e^{-2\frac{GM}{c^2 R}}(\partial t)^2 - e^{+2\frac{GM}{c^2 R}}\left(\frac{\partial\vec{R}}{c}\right)^2$$







Schwarzschild Metric

The Schwarzschild metric is a solution of general relativity for a neutral, non-rotating, spherically symmetric source mass (derivation not shown). Write out the Taylor series expansion of the Schwarzschild metric in isotropic coordinates to third order in $\frac{GM}{c^2 R}$.

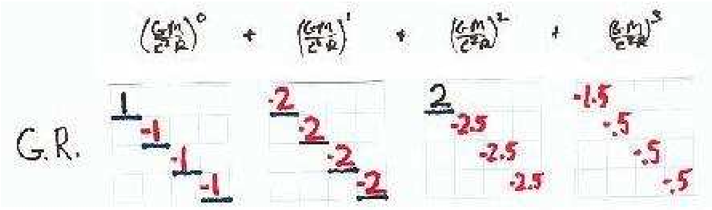
Schwarzschild metric:

$$(\partial\tau)^2 = \left(\underline{1 - 2\frac{GM}{c^2 R} + 2\left(\frac{GM}{c^2 R}\right)^2} - \frac{3}{2}\left(\frac{GM}{c^2 R}\right)^3\right)(\partial t)^2 - \left(\underline{1 - 2\frac{GM}{c^2 R}} + \frac{3}{2}\left(\frac{GM}{c^2 R}\right)^2 + \frac{1}{2}\left(\frac{GM}{c^2 R}\right)^3\right)\left(\frac{\partial\vec{R}}{c}\right)^2$$

The five underlined terms have been confirmed experimentally. Tests include:

- Light bending around the Sun.
- Perihelion shift of Mercury.

- Time delay in radar reflections off of planets.



Compare Metrics: Schwarzschild to GEM

Write out the Taylor series expansion of the Schwarzschild and GEM metrics in isotropic coordinates to third order in $\frac{GM}{c^2 R}$.

1. Schwarzschild metric:

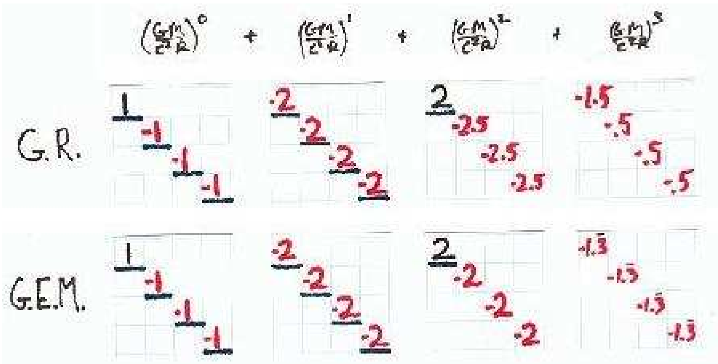
$$(\partial\tau)^2 = (\underline{1 - 2\,\frac{GM}{c^2 R} + 2(\frac{GM}{c^2 R})^2} - \frac{3}{2}(\frac{GM}{c^2 R})^3)(\partial t)^2 - (\underline{1 - 2\frac{GM}{c^2 R}} + \frac{3}{2}(\frac{GM}{c^2 R})^2 + \frac{1}{2}(\frac{GM}{c^2 R})^3)(\partial\vec{R})^2$$

2. GEM metric:

$$(\partial\tau)^2 = (\underline{1 - 2\,\frac{GM}{c^2 R} + 2(\frac{GM}{c^2 R})^2} - \frac{4}{3}(\frac{GM}{c^2 R})^3)(\partial t)^2 - (\underline{1 - 2\,\frac{GM}{c^2 R}} + 2(\frac{GM}{c^2 R})^2 + \frac{4}{3}(\frac{GM}{c^2 R})^3)(\frac{\partial\vec{R}}{c})^2$$

Compare the two metrics:

- Identical for tested terms of Taylor series expansion.
- Different for higher order terms, so can be tested (not easy).
- GEM metric is more symmetric.



Classical Gravitational Force

1. Breaking spacetime symmetry.

2. Newton's gravitational law derivation.

3. Need for new classical solutions:

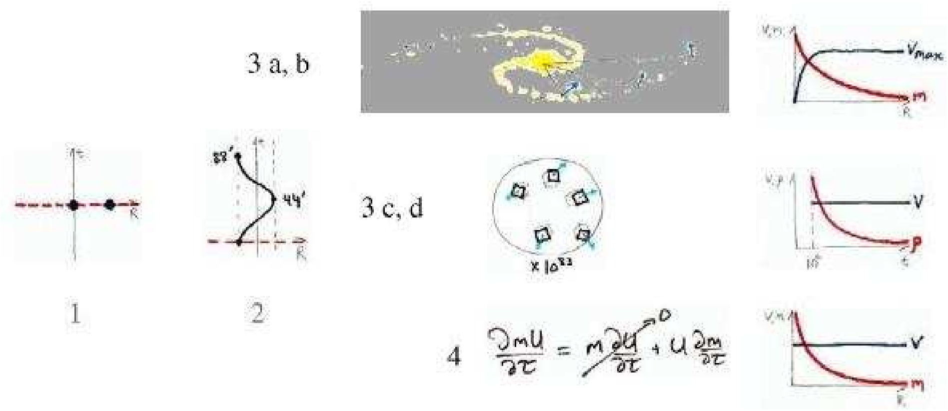
   a) Problem statement for rotation profiles of spiral galaxies.

   b) Solution requirements for rotation profiles.

   c) Problem statement for the big bang.

   d) Solution requirements for the big bang.

4. Constant velocity solutions.

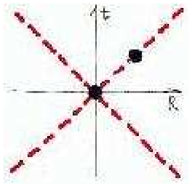


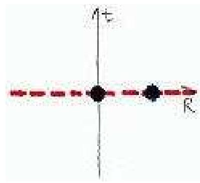
Breaking Spacetime Symmetry

Spacetime symmetry must be broken to go from the relativistic weak gravitational force to a classical force for both cause and effect.

Contrast the relativistic geometry of Minkowski spacetime with the geometry of Newtonian absolute space and time.

Minkowski Spacetime	Geometry	Newtonian Space and Time
True, Elegant	Utility	Accurate, Practical
$(\partial\tau)^2 = (dt)^2 - \frac{(dR)^2}{c^2}$	Interval	$\text{distance}^2 = dR^2 \neq f(t)$
$c = 1$	Speed of Light	$c = \infty$
$(U_0, \vec{U}) = (c\,\frac{\partial t}{\partial\tau}, \frac{\partial \vec{R}}{\partial\tau})$	Velocity	$(\mathbb{U}_0, \vec{\mathbb{U}}) \equiv (\frac{\partial t}{\partial \lvert R\rvert}, c\frac{\partial \vec{R}}{\partial \lvert R\rvert}) = (0, c\,\hat{R})$
$(\frac{\partial U_0}{\partial\tau}, \frac{\partial \vec{U}}{\partial\tau}) = (c\,\frac{\partial^2 t}{\partial\tau^2}, \frac{\partial^2 \vec{R}}{\partial\tau^2})$	Acceleration	$(\frac{\partial \mathbb{U}_0}{\partial\tau}, \frac{\partial \vec{\mathbb{U}}}{\partial\tau}) = (0, c^2\,\frac{\partial^2 \vec{R}}{\partial \lvert R\rvert^2})$





Newton's Gravitational Law Derivation

1. Start from the gravitational force for a weak field:

   $F_G^\mu = m\,(\frac{k}{\tau^2}\,\frac{U_0}{c}, -\frac{k}{\tau^2}\,\frac{\vec{U}}{c}) = (\frac{\partial\, m U_0}{\partial\tau}, \frac{\partial\, m \vec{U}}{\partial\tau})$

2. Apply the chain rule to the cause terms, $\frac{\partial\, m U^\mu}{\partial\tau} = m\frac{\partial\, U^\mu}{\partial\tau} + U^\mu\,\frac{\partial\, m}{\partial\tau}$.

   Assume $\frac{\partial\, m}{\partial\tau} = 0$:

   $F_G^\mu = m\,(\frac{k}{\tau^2}\,\frac{U_0}{c}, -\frac{k}{\tau^2}\,\frac{\vec{U}}{c}) = (m\,\frac{\partial\, U_0}{\partial\tau}, m\,\frac{\partial\, \vec{U}}{\partial\tau})$

3. Break spacetime symmetry:

   - $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\,\hat{R})$ Velocity.
   - $(\frac{\partial U_0}{\partial\tau}, \frac{\partial \vec{U}}{\partial\tau}) \longrightarrow (0, c^2\,\frac{\partial^2 \vec{R}}{\partial \lvert R\rvert^2})$ Acceleration.

   $F_G^\mu = m\,(0, -\frac{k}{\tau^2}\,\hat{R}) = (0, m c^2\,\frac{\partial^2 \vec{R}}{\partial \lvert R\rvert^2})$

4. Assume the gravitational spring constant $(k = \frac{GM}{c^2})$:

   $F_G^\mu = (0, -\frac{GMm}{c^2\tau^2}\,\hat{R}) = (0, m c^2\,\frac{\partial^2 \vec{R}}{\partial \lvert R\rvert^2})$

5. Substitute: σ^2 for $-c^2\tau^2$ in the cause term.

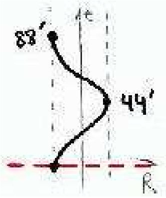
   Substitute: $-(\frac{\partial}{\partial\tau})^2$ for $(\frac{c\partial}{\partial\sigma})^2 \approx (\frac{c\,\partial}{\partial \lvert R\rvert})^2$ in the effect term.

   $F_G^\mu = (0, \frac{GMm}{\sigma^2}\,\hat{R}) = (0, -m\,\frac{\partial^2 \vec{R}}{\partial\tau^2})$

6. Assume the static field approximation: $\sigma^2 = R^2 - t^2 \cong R'^2$.

Assume the low speed approximation: $\frac{\partial^2}{\partial\tau^2} \cong \frac{\partial^2}{\partial t^2}$:

$F_G^{\mu} = (0, \frac{G M m}{R^2} \hat{R}) = (0, -m \frac{\partial^2 \vec{R}}{\partial t^2})$ QED



Problem Statement for the Rotation Profile of Galaxies

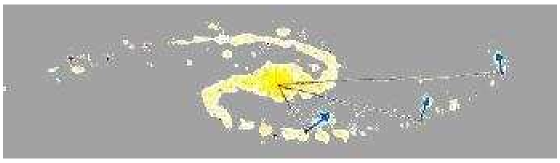
The momentum of stars in thin spiral galaxies has two problems:

- The flat velocity profile problem.

  After attaining a maximal speed consistent with Newton's law of gravity near the core, the velocity profile stays flat with increasing distance. Newton's law predicts a "Keplerian" decline for the velocity of the outer stars.

- The stability problem.

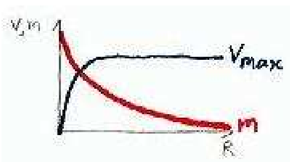
  Thin spiral galaxies are mathematically unstable to small disturbances along the axis which should lead to collapse.



Solution Requirements for Rotation Profiles

Requirements for a solution:

1. Stable mathematically to axial perturbations.
2. Same velocity for all outer stars.
3. Describes the change in mass distribution in spacetime, which falls off exponentially with distance ($2 \times 3 = \Delta$momentum).
4. Fits every observational constraint.



Problem Statement for the Big Bang

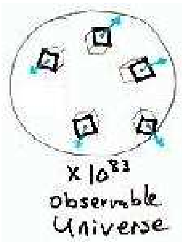
Big bang cosmology has two big problems:

- The horizon problem.

  All $\sim 10^{83}$ separate, independent spacetime volumes of the early Universe must travel at the same velocity to create the uniform black body radiation spectrum seen in the cosmic background radiation.

- The flatness problem.

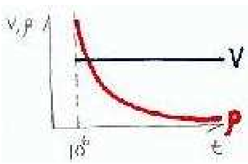
  The initial conditions must be tuned to one part in $\sim 10^{55}$ so the mathematically unstable solution lasts 10^{10} years.



Solution Requirements for the Big Bang

Requirements for a solution:

1. Stable mathematically for initial conditions.
2. Same velocity for all independent regions of spacetime.
3. Describes the change in mass distribution in spacetime, from high density early to lower later ($2 \times 3 = \Delta$momentum).
4. Fits every observational constraint.



Stable Constant Velocity Solutions

1. Start from the gravitational force for a weak field:

   $F^{\mu}_{G} = m\left(\frac{k}{\tau^2}\frac{U_0}{c}, -\frac{k}{\tau^2}\frac{\vec{U}}{c}\right) = \left(\frac{\partial\, mU_0}{\partial\tau}, \frac{\partial m\vec{U}}{\partial\tau}\right)$

2. Apply the chain rule to the cause terms, $\frac{\partial mU^\mu}{\partial\tau} = m\frac{\partial U^\mu}{\partial\tau} + U^\mu\frac{\partial m}{\partial\tau}$.

   Assume constant velocity, $\frac{\partial U^\mu}{\partial\tau} = 0$:

$F^{\mu}_{G} = m\,(\frac{k}{\tau^2}\,\frac{U_0}{c}, -\,\frac{k}{\tau^2}\,\frac{\vec{U}}{c}) = (U_0\,\frac{\partial m}{\partial \tau}, \vec{U}\,\frac{\partial m}{\partial \tau})$

3. Break spacetime symmetry: $(U_0, \vec{U}) \longrightarrow (\mathbb{U}_0, \vec{\mathbb{U}}) = (0, c\,\hat{R})$.

   $F^{\mu}_{G} = m\,(0, -\,\frac{k}{\tau^2}\,\hat{R}) = (0, \frac{\partial m}{\partial \tau}\,c\hat{R})$

4. Assume the gravitational spring constant ($k = \frac{GM}{c^2}$):

   $F^{\mu}_{G} = (0, -\,\frac{GMm}{c^2\,\tau^2}\,\hat{R}) = (0, \frac{\partial m}{\partial \tau}\,c\hat{R})$

5. Collect terms on one side:

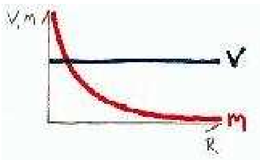
   $(c\,\frac{\partial m}{\partial \tau} + \frac{GMm}{c^2\,\tau^2})(0, \hat{R}) = 0$

6. Solve for m:

   $m = m_o\,e^{\frac{GM}{c^3\tau}}$

7. Substitute: R for $c\tau$ which depends on exactly the same assumptions used in the metric derivation (static field, $|\sigma| = |c\,\tau|$=R, and sigma is imaginary):

   $m = m_0\,e^{\frac{GM}{c^2\,R}}$



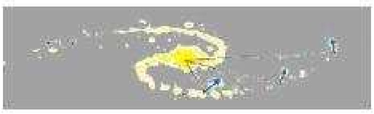
Summary: Classical Gravitational Force

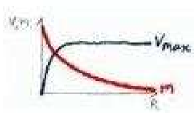
Math:

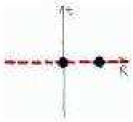
$m = m_0\,e^{\frac{GM}{c^2\,R}}$

Pictures:

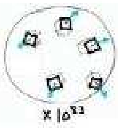
3 a, b

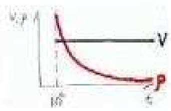






3 c, d

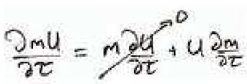


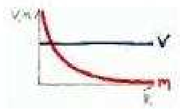


1

2

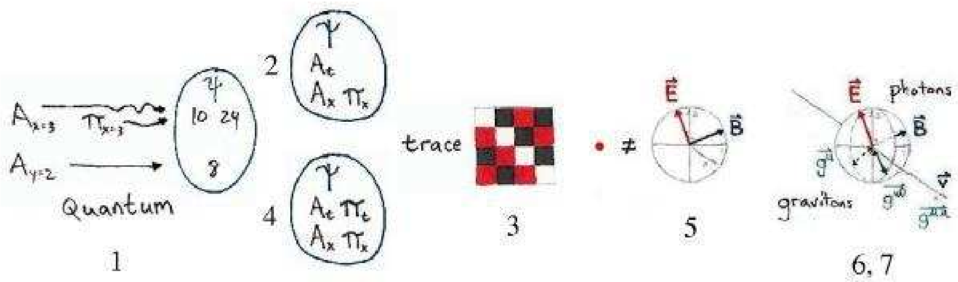
4





Appendix A: Quantization

1. Classical physics versus quantum mechanics.
2. Momentum from classic (not quantum) EM Lagrange density.
3. Quantizing EM fields by fixing the gauge.
4. Quantizing EM by fixing the Lorenz gauge (Gupta/Bleuler method).
5. Skeptical analysis of fixing the Lorenz gauge.
6. Momentum from GEM Lagrange density.
7. GEM quantization.



Classical Physics versus Quantum Mechanics

Classical physics:

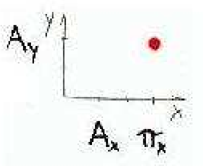
- $A_x = 10 \quad A_y = 8 \quad \pi_x = 24$ Observables are numbers.
- $A_x \pi_x - \pi_x A_x = 0$ All observables are independent.

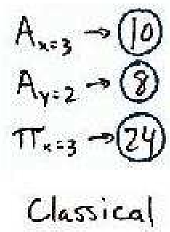
Quantum mechanics:

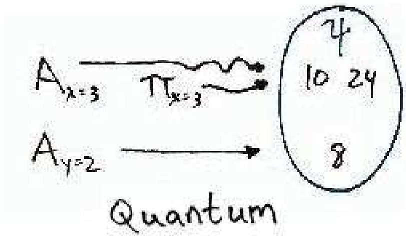
- $A_x|\psi\rangle = 10 \quad A_y|\psi\rangle = 8 \quad \pi_x|\psi\rangle = 24$ Observables are operators that act on the wave function ψ to generate a number.
- $A_x A_y|\psi\rangle - A_y A_x|\psi\rangle = [A_x, A_y]|\psi\rangle = 0$ Most observables are independent. $[A_x, A_y]$ is called the *commutator*.
- $[A_x, \pi_x]|\psi\rangle \neq 0$ Conjugate observables

are *not* independent.

Conjugate observables, like the potential and momentum, must have a non-zero commutator to quantize a field.







Momentum from Classic (Not Quantum) EM Lagrange Density

1. Start with the EM Lagrange density written without indices.

$$\mathfrak{L}_{\rm EM} = -\frac{m}{\gamma V} - \frac{q}{c^2 V}\frac{U_\mu}{\gamma} A^\mu - \frac{1}{4c^2}(A^{\mu,\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$$

$$= -\frac{m}{V}(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2} - \frac{(q - \sqrt{G}\,m)}{c^2 V}(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}((\quad -(\frac{\partial\phi}{\partial x})^2 - (\frac{\partial\phi}{\partial y})^2 - (\frac{\partial\phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 \qquad + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 \qquad + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2$$

$$-2\frac{\partial A_x}{c\partial t}\frac{\partial\phi}{\partial x} - 2\frac{\partial A_y}{c\partial t}\frac{\partial\phi}{\partial y} - 2\frac{\partial A_z}{c\partial t}\frac{\partial\phi}{\partial z} - 2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x})$$

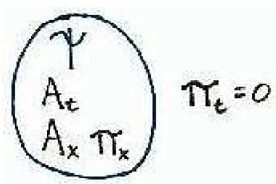
2. Calculate momentum:

$$\pi^\mu = h\sqrt{G}\frac{\partial\mathfrak{L}}{\partial\frac{\partial A^\mu}{c\partial t}} = h\sqrt{G}\,(0, \frac{\partial A_x}{c\partial t} + \frac{\partial\phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial\phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial\phi}{\partial z})$$

Energy-momentum vector.

3. Momentum cannot be made into an operator:

$[A_t, \pi_t]|\psi\rangle = [A_t, 0]|\psi\rangle = 0$ Energy commutes with its conjugate operator.

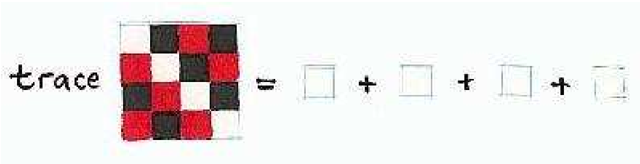


Quantizing EM Fields by Fixing the Gauge

An EM gauge is a relationship between ϕ and $\vec{A}$ that does not change the Maxwell equations. Examples:

- $\text{trace}(A^{\mu,\nu}) = \vec{\nabla} \cdot \vec{A} = 0$ Coulomb gauge.
- $\text{trace}(A^{\mu,\nu}) = \frac{\partial \phi}{c\partial t} + \vec{\nabla} \cdot \vec{A} = 0$ Lorenz gauge.

For EM with no gravity, one is free to assign arbitrary values to the diagonal of the antisymmetric field strength tensor.



Quantizing EM by Fixing the Lorenz Gauge (Gupta/Bleuler method)

Fix the Lorenz gauge in the EM Lagrange density.

1. Start with the Gupta-Bleuler Lagrange density written without indices:

$$\mathfrak{L}_{G-B} = -\frac{m}{\gamma V} - \frac{q}{c^2 V}\frac{U_\mu}{\gamma}A^\mu - \frac{1}{2c^2}(A^\mu{}_{,\mu})^2 - \frac{1}{4c^2}(A^{\mu.\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$$

$$= -\frac{m}{V}\Big(\sqrt{1 - (\frac{\partial x}{c\partial t})^2 - (\frac{\partial y}{c\partial t})^2 - (\frac{\partial z}{c\partial t})^2} - \frac{(q - \sqrt{G}\,m)}{c^2 V}(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z)$$

$$-\frac{1}{2}\Big((\frac{\partial \phi}{c\partial t})^2 - (\frac{\partial \phi}{\partial x})^2 - (\frac{\partial \phi}{\partial y})^2 - (\frac{\partial \phi}{\partial z})^2 - (\frac{\partial A_x}{c\partial t})^2 + (\frac{\partial A_x}{\partial x})^2 + (\frac{\partial A_x}{\partial y})^2 + (\frac{\partial A_x}{\partial z})^2$$

$$-(\frac{\partial A_y}{c\partial t})^2 + (\frac{\partial A_y}{\partial x})^2 + (\frac{\partial A_y}{\partial y})^2 + (\frac{\partial A_y}{\partial z})^2 - (\frac{\partial A_z}{c\partial t})^2 + (\frac{\partial A_z}{\partial x})^2 + (\frac{\partial A_z}{\partial y})^2 + (\frac{\partial A_z}{\partial z})^2$$

$$-2\frac{\partial A_x}{c\partial t}\frac{\partial \phi}{\partial x} - 2\frac{\partial A_y}{c\partial t}\frac{\partial \phi}{\partial y} - 2\frac{\partial A_z}{c\partial t}\frac{\partial \phi}{\partial z} - 2\frac{\partial A_y}{\partial z}\frac{\partial A_z}{\partial y} - 2\frac{\partial A_z}{\partial x}\frac{\partial A_x}{\partial z} - 2\frac{\partial A_x}{\partial y}\frac{\partial A_y}{\partial x}\Big)$$

$$+2\frac{\partial \phi}{c\partial t}\frac{\partial A_x}{\partial x} + 2\frac{\partial \phi}{c\partial t}\frac{\partial A_y}{\partial y} + 2\frac{\partial \phi}{c\partial t}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_y}{\partial y} + 2\frac{\partial A_x}{\partial x}\frac{\partial A_z}{\partial z} + 2\frac{\partial A_y}{\partial y}\frac{\partial A_z}{\partial z}\Big)$$

2. Calculate momentum:

$$\pi^\mu = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^\mu}{c\partial t}} = h\sqrt{G}\,(-\frac{\partial \phi}{c\partial t} - \vec{\nabla} \cdot \vec{A}, \frac{\partial A_x}{c\partial t} + \frac{\partial \phi}{\partial x}, \frac{\partial A_y}{c\partial t} + \frac{\partial \phi}{\partial y}, \frac{\partial A_z}{c\partial t} + \frac{\partial \phi}{\partial z})$$

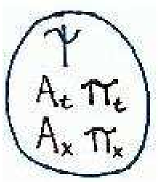
Energy-momentum vector.

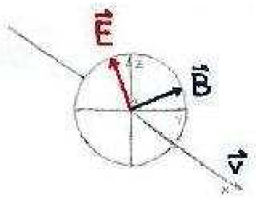
3. Momentum can be made into an operator:

Using the Euler-Lagrange equation [not shown], the equations of motion are identical to those of $\mathfrak{L}_{\text{GEM}}$!

$J^\mu = \Box^2 A^\mu$

Reference: "Theory of longitudinal photons in quantum electrodynamics", Suraj N. Gupta, Proc. Phys. Soc. 63:681-691, 1950.

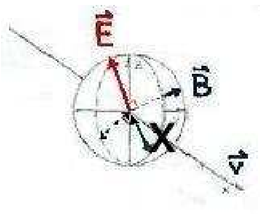




Gupta/Bleuler Quantization Method

Results of quantization method:

- Four modes of transmission:
    1. Two transverse waves.
    2. One longitudinal wave.
    3. One scalar wave.
- Transverse waves are photons for EM.
- Scalar mode of transmission called a "scalar photon".
- "Supplementary condition" imposed to eliminate scalar and longitudinal photons as real particles, so they are always virtual.

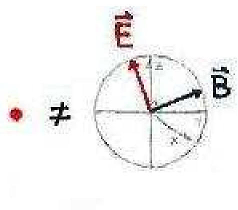


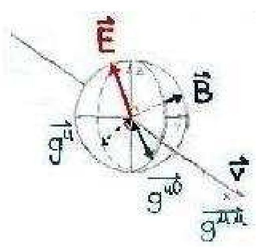
Skeptical Analysis of Fixing the Lorenz Gauge

1. A scalar photon is an oxymoron. Photons must transform like vectors, even if photons happen to be virtual.

2. Eliminating an oxymoron cannot justify the supplementary condition.

3. A better interpretation for the 4D-wave equation of motion may exist.





Momentum from GEM Lagrange Density

1. Start with the GEM Lagrange density written without indices:

$$\mathfrak{L} = -\frac{m}{V}\left(\sqrt{1-\left(\frac{\partial x}{c\partial t}\right)^2-\left(\frac{\partial y}{c\partial t}\right)^2-\left(\frac{\partial z}{c\partial t}\right)^2} - \frac{(q-\sqrt{G}\,m)}{c^2 V}\left(c\phi - \frac{\partial x}{\partial t}A_x - \frac{\partial x}{\partial t}A_y - \frac{\partial z}{\partial t}A_z\right)\right.$$

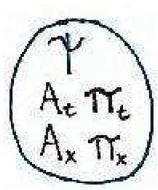
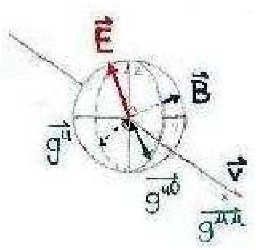
$$-\frac{1}{2}\left(\left(\frac{\partial \phi}{c\partial t}\right)^2 - \left(\frac{\partial \phi}{\partial x}\right)^2 - \left(\frac{\partial \phi}{\partial y}\right)^2 - \left(\frac{\partial \phi}{\partial z}\right)^2 - \left(\frac{\partial A_x}{c\partial t}\right)^2 + \left(\frac{\partial A_x}{\partial x}\right)^2 + \left(\frac{\partial A_x}{\partial y}\right)^2 + \left(\frac{\partial A_x}{\partial z}\right)^2\right.$$

$$\left. - \left(\frac{\partial A_y}{c\partial t}\right)^2 + \left(\frac{\partial A_y}{\partial x}\right)^2 + \left(\frac{\partial A_y}{\partial y}\right)^2 + \left(\frac{\partial A_y}{\partial z}\right)^2 - \left(\frac{\partial A_z}{c\partial t}\right)^2 + \left(\frac{\partial A_z}{\partial x}\right)^2 + \left(\frac{\partial A_z}{\partial y}\right)^2 + \left(\frac{\partial A_z}{\partial z}\right)^2\right)$$

2. Calculate momentum:

$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\left(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t}\right)$$

3. Momentum can be made into an operator:

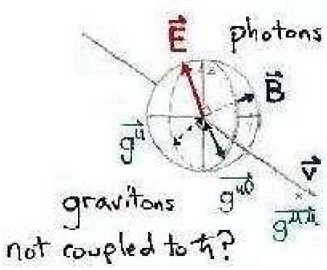
GEM Quantization

- Four modes of transmission:
  1. Two transverse waves.

2. One longitudinal wave.

3. One scalar wave.

- Transverse waves are photons for EM.
- Longitudinal and scalar modes are gravitons of gravity traveling at the speed of light, generated by a symmetric rank-2 field strength tensor.
- General relativity predicts transverse waves, not scalar or longitudinal ones. The LIGO experiment to detect gravitational waves will be looking for transverse gravitational waves. GEM predicts the polarization will not be transverse.
- Gravitational modes are coupled to $\sqrt{G}$ and not h bar. This might get around negative energy problem because gravity quanta are not emitted.

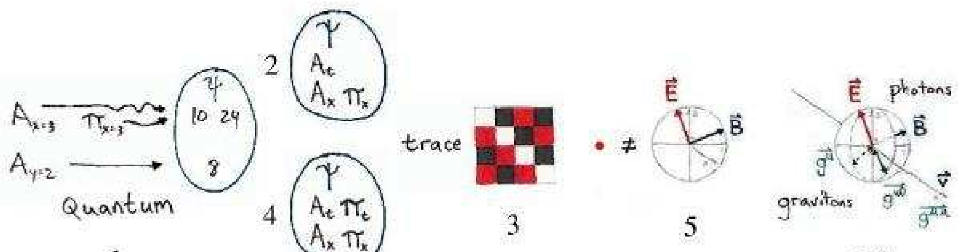


Summary: Quantization

Math:

$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\partial t}} = h\sqrt{G}\left(-\frac{\partial \phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t}\right)$$

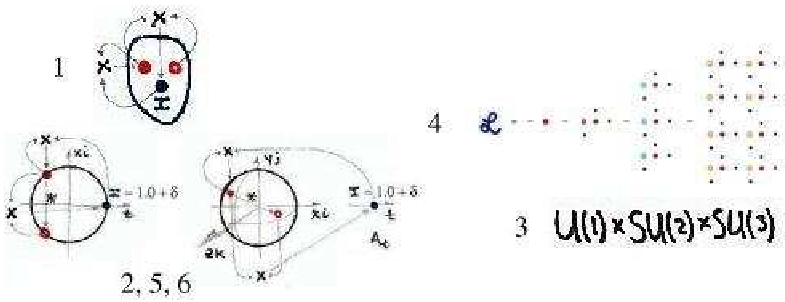
Pictures:



1 2 4 3 5 6, 7

Appendix B: The Standard Model

1. Group theory.
2. Group theory by example.
3. The standard model.
4. The standard model Lagrange density.
5. Defining the multiplication operator.



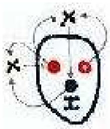
Group Theory

Way to organize symmetry systematically.

Definition: A set S with a binary operation ($\times$ or $+$) such that $s_1 \times s_2 \in S$ for all possible pairs of elements in S. A group has:

- An identity.
- An inverse for every element.
- Associative law holds.

If $s_1 \times s_2 = s_2 \times s_1$, the group is Abelian, otherwise it is non-Abelian.



$S = (\mathbb{R}, +) \quad I = 0 \quad s^{-1} = -s$

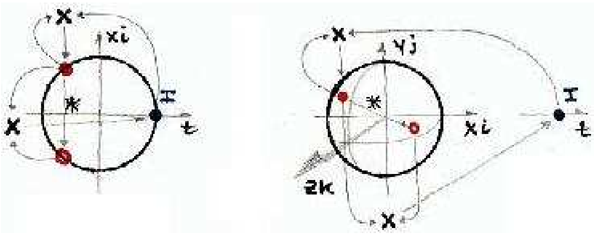
$S' = (\mathbb{R}/\{0\}, \times) \quad I = 1 \quad s^{-1} = \frac{1}{s}$

Group Theory by Example

- U(1), $z \times z^* = 1$, or unitary complex numbers.
  - $I = (1, 0)$ Identity is one.
  - $z^{-1} = z^*$ Inverse is the conjugate.

- $z_1 \times z_2 = z_2 \times z_1$ Abelian.
- $U(\alpha) = e^{\begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix}}$ One number generates all elements.

- SU(2), $q \times q^* = 1$, or unit quaternions (4D analog to complex numbers).
  - $I = (1, 0, 0, 0)$ Identity is one.
  - $q^{-1} = q^*$ Inverse is the conjugate.
  - $q_1 \times q_2 \neq q_2 \times q_1$ Non-Abelian.
  - $U(\alpha, \beta, \gamma) = e^{\begin{pmatrix} 0 & -\alpha & -\beta & -\gamma \\ \alpha & 0 & -\gamma & \beta \\ \beta & \gamma & 0 & -\alpha \\ \gamma & -\beta & \alpha & 0 \end{pmatrix}}$ Three numbers generate all elements.



The Standard Model

Predicts patterns of all subatomic particles and three of four forces in Nature:

- U(1) EM.
- SU(2) Weak force.
- SU(3) Strong force.

Says nothing about gravity.

U(1)×SU(2)×SU(3)

The Standard Model Lagrange Density

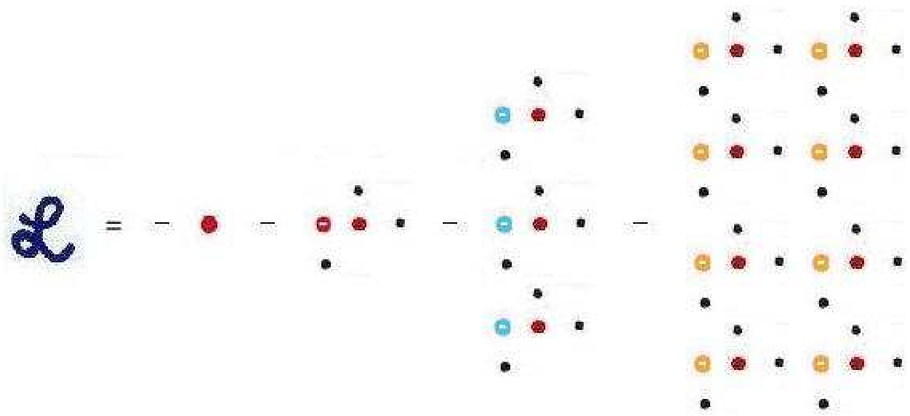
Describes all interactions of all subatomic forces in a volume.

$\mathfrak{L}_{\text{SM}} = \bar{\psi} \gamma^\mu D_\mu \psi$

$D_\mu = \partial_\mu - i g_{\text{EM}} Y A_\mu - i g_{\text{weak}} \frac{\tau^a}{2} W_\mu^a - i g_{\text{strong}} \frac{\lambda^b}{2} G_\mu^b$

- γ^μ Spinor matrix (no details provided here).

- $g_{...}$ Coupling constant to force.
- Y Generator of U(1) symmetry.
- $\tau^{a(1-3)}$ Generator of SU(2) symmetry.
- $\lambda^{b(1-8)}$ Generator of SU(3) symmetry.
- A_μ, W^a_μ, G^b_μ Complex-valued 4-potentials, two with internal symmetries.



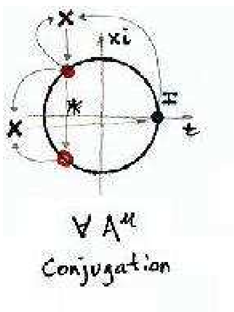
Defining the Multiplication Operator

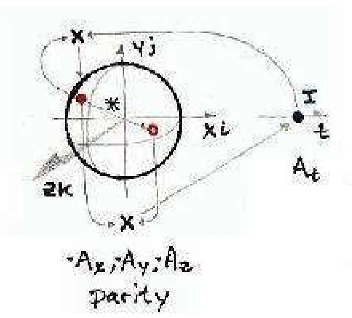
Four components:

1. $(a, bi)^* = (a, -bi)$ Complex conjugation.
2. $(\phi, \vec{A})^p = (\phi, -\vec{A})$ Parity operator.
3. $g_{\mu\nu}$ Metric tensor.
4. $\frac{A^\mu}{|A|}$ Potentials normalized to themselves.

Define multiplication of 4-potentials in the standard model as:

$$\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = \frac{g_{tt}|A_t|^2 - g_{xx}|A_x|^2 - g_{yy}|A_y|^2 - g_{zz}|A_z|^2 - g_{\mu\nu}|A^\mu A^\nu|_{\mu\neq\nu}}{|A|^2}$$





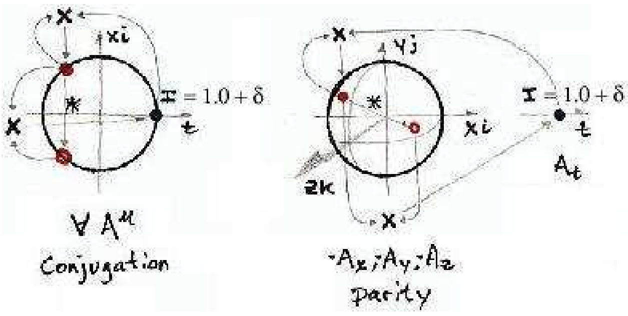
Multiplication Operator in Spacetime

- $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = 1.0$ In flat spacetime.
- $\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = 1.0 + \delta$ In curved spacetime.

In curved spacetime, mass breaks U(1), SU(2), and SU(3) symmetry in a precise way (circles get larger).

Y, τ^a, λ^b and the Higgs particle are not needed.

No new symmetry was added to the standard model, so no new particle can be added. Instead, it may turn out that every pratice can "act like a graviton" when it is involved with a distance measurement of the field.

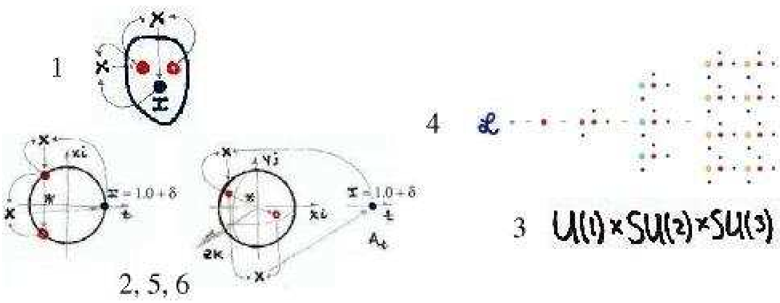


Summary: The Standard Model

Math:

$$\frac{A^\mu}{|A|}\frac{A^{\nu*p}}{|A|}g_{\mu\nu} = \frac{g_{tt}|A_t|^2 - g_{xx}|A_x|^2 - g_{yy}|A_y|^2 - g_{zz}|A_z|^2 - g_{\mu\nu}|A^\mu A^\nu|_{\mu\neq\nu}}{|A|^2}$$

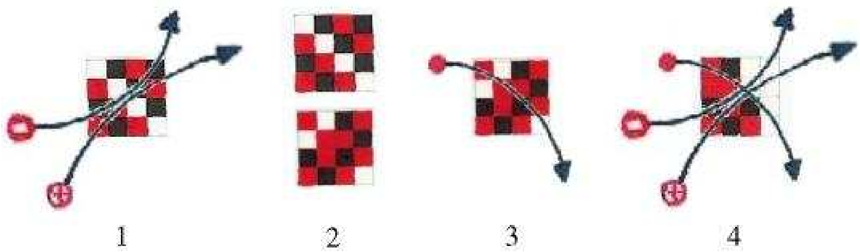
Pictures:



Appendix C: Stresses

1. Stress tensor.

2. Stress tensor of GEM.



Stress Tensor

The rank-2 stress tensor is related to a derivative of a Lagrange density.

1. Start with a Lagrange density:

   $\mathfrak{L} = f(A^{\sigma}, A^{\sigma,\nu})$

2. Take the derivative:

   $\mathfrak{L}^{,\mu} = \frac{\partial \mathfrak{L}}{\partial A^{\sigma}} A^{\sigma,\mu} + \frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}} A^{\sigma,\nu,\mu}$

3. Apply the Euler-Lagrange equation $(\frac{\partial \mathfrak{L}}{\partial A^{\sigma}} = (\frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}})^{,\nu})$ to the first term. Change the order of partial derivatives in the second term:

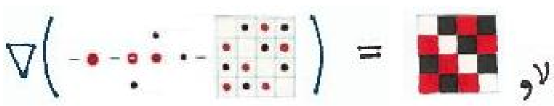
   $\mathfrak{L}^{,\mu} = (\frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}})^{,\nu} A^{\sigma,\mu} + \frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}} A^{\sigma,\mu,\nu}$

4. Apply the chain rule to condense into one term:

   $\mathfrak{L}^{,\mu} = ((\frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}}) A^{\sigma,\mu})^{,\nu}$

5. Define the rank-2 stress tensor as the stuff inside:

   $T^{\mu}{}_{\nu} \equiv (\frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}}) A^{\sigma,\mu}$



Stress Tensor of GEM

1. The stress tensor definition: $T^{\mu}{}_{\nu} = (\frac{\partial \mathfrak{L}}{\partial A^{\sigma,\nu}}) A^{\sigma,\mu}$

2. GEM Lagrange density:

   $\mathfrak{L}_{\mathrm{GEM}} = -\frac{m}{\gamma V} - \frac{(q - \sqrt{G}\, m)}{c^2 V} \frac{U_{\mu}}{\gamma} A^{\mu} - \frac{1}{2\, c^2} A^{\sigma,\nu} A_{\sigma,\nu}$

3. Apply:

   $T^{\mu}{}_{\nu} = -\frac{1}{2} A_{\sigma,\nu} A^{\sigma,\mu}$

4. Write it all out. (oops, 128 differentials, try just one).

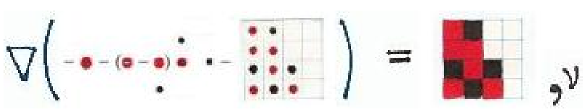
   $$T^{0}{}_{0} = -\frac{1}{2}((\frac{\partial \phi}{\partial t})^2 - (\frac{\partial A_x}{\partial t})^2 - (\frac{\partial A_y}{\partial t})^2 - (\frac{\partial A_z}{\partial t})^2)$$

   $$= \frac{1}{2}(-g_{\mathrm{tt}} + (g_{x0} - E_x)^2 + (g_{y0} - E_y)^2 + (g_{z0} - E_z)^2)$$

Caveats:

- $T^{0}{}_{0} = E^2 + B^2$ in EM. It is unclear what the difference means. The B^2 terms are in a different section of the stress tensor.

- $T^{\mu}{}_{\nu} \longrightarrow F^{\mu}$ There should be a path between the GEM stress tensor and the relativistic force, but I have not figured it out yet.



Appendix D: Geodesics

1. Effects of a geodesic.
2. Cause of curvature in a geodesic.
3. Killing's differential equation.

Effect of a Geodesic

A geodesic is the path of zero external force. Investigate the change in momentum (or effect) term of $F^{\mu}_{\rm GEM}$.

1. Start with the change in momentum set equal to zero. Apply the chain rule to expand:

   $$0=\frac{\partial m U^{\mu}}{\partial\tau}=m\,\frac{\partial U^{\mu}}{\partial\tau}+U^{\mu}\,\frac{\partial m}{\partial\tau}$$

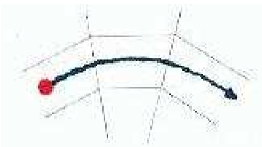
2. Assume $\frac{\partial m}{\partial\tau}=0$. Use the chain rule to expand $\frac{\partial U^{\mu}}{\partial\tau}$:

   $$0=m\,\frac{\partial U^{\mu}}{\partial\tau}=m\,\frac{\partial U^{\mu}}{\partial x^{\nu}}\,\frac{\partial x^{\nu}}{\partial\tau}=m U^{\mu}{}_{;\nu}U^{\nu}$$

3. Apply the definition of a covariant derivative of a contravariant vector (normal derivative + change in the metric, $A^{\mu}{}_{;\nu}=A^{\mu}{}_{,\nu}+\Gamma^{\mu}{}_{\nu\varpi}\,A^{\varpi}$):

   $$0=m U^{\mu}{}_{,\nu}U^{\nu}\;+\;m\,\Gamma^{\mu}{}_{\nu\omega}\,U^{\nu}U^{\omega}\;=m\,\frac{\partial^2 x^{\mu}}{\partial\tau^2}\;+\;m\,\Gamma^{\mu}{}_{\nu\omega}\,U^{\nu}U^{\omega}$$

If any acceleration is seen without a force ($m\,\frac{\partial^2 x^{\mu}}{\partial\tau^2}\neq 0, F^{\mu}_{\rm GEM}=0$), then the effect is entirely due to the curvature of spacetime ($m\,\Gamma^{\mu}{}_{\nu\omega}\,U^{\nu}U^{\omega}\neq 0$).



Cause of Curvature

Every effect must have a cause. Explore the change in potential (or cause) term.

1. Start with force set equal to zero:

   $$0=-\,(\sqrt{G}\,m-q)\,\frac{U_{\nu}}{c}A^{\mu;\nu}-(\sqrt{G}\,m+q)\,\frac{U_{\nu}}{c}A^{\nu;\mu}$$

2. Apply the definition of a contravariant derivative of a contravariant vector (normal derivative - change in the metric, $A^{\mu;\nu}=A^{\mu,\nu}-\Gamma_{\varpi}{}^{\mu\nu}\,A^{\varpi}$):

   $$0=-\,(\sqrt{G}\,m-q)\,\frac{U_{\nu}}{c}A^{\mu,\nu}-(\sqrt{G}\,m+q)\,\frac{U_{\nu}}{c}A^{\nu,\mu}$$

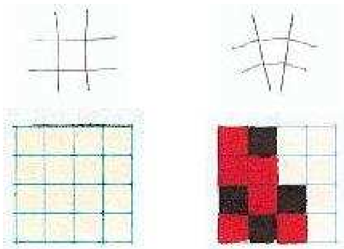
$$- \sqrt{G}\, m \frac{U_\nu}{c} \Gamma_\varpi{}^{\mu\nu} A^\varpi + q \frac{U_\nu}{c} \Gamma_\varpi{}^{\mu\nu} A^\varpi - \sqrt{G}\, m \frac{U_\nu}{c} \Gamma_\varpi{}^{\nu\mu} A^\varpi - q \frac{U_\nu}{c} \Gamma_\varpi{}^{\nu\mu} A^\varpi$$

$$= - (\sqrt{G}\, m - q) \frac{U_\nu}{c} A^{\mu,\nu} - (\sqrt{G}\, m + q) \frac{U_\nu}{c} A^{\nu,\mu} - 2m \frac{U_\nu}{c} \Gamma_\varpi{}^{\mu\nu} A^\varpi$$

Curvature is coupled directly to mass, not to q.

Any curvature of spacetime without a force ($2m \frac{U_\nu}{c} \Gamma_\varpi{}^{\mu\nu} A^\varpi \neq 0, F^\mu_{\text{GEM}} = 0$) is caused by change in the potential which are coupled to both the mass charge and electric charge.

General relativity provides a way to calculate curvature by comparing two nearby geodesics using a tidal effect. Because general relativity lacks a means within the geodesic to calculate the cause of curvature, general relativity is incomplete.



Killing's Differential Equation

If $F^\mu_{\text{GEM}} = 0$, then $\alpha A^{\mu;\nu} + \beta A^{\nu;\mu} = 0$. This is a generalization of Killing's differential equation where $\alpha = \beta = 1$. The solutions are known as Killing vector fields.

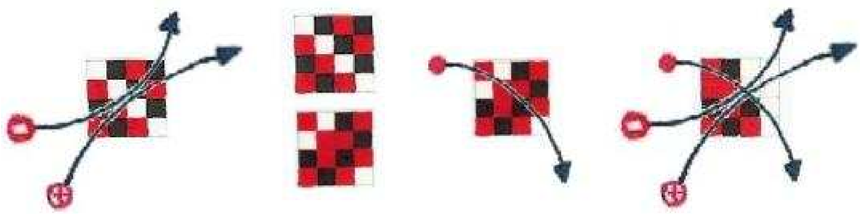
The implications of this observation are not understood by me at this time.

Summary: Stresses, Forces, and Geodesics

Math:

$$F^\mu_{\text{GEM}} = - (\sqrt{G}\, m - q) \frac{U_\nu}{c} A^{\mu;\nu} - (\sqrt{G}\, m + q) \frac{U_\nu}{c} A^{\nu;\mu} = \frac{\partial m U^\mu}{\partial \tau}$$

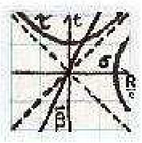
Pictures:

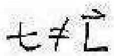


1 2 3 4

Appendix E: Units

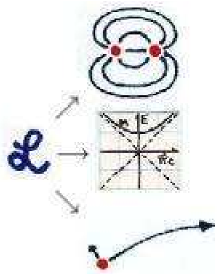
1. Basic units.
2. Units for conversion factors.
3. Units for spacetime.
4. Units for potentials, fields, & charges.
5. Units in action:
   a) Lagrange densities.
   b) Euler-Lagrange equations (fields).
   c) Momentum.
   d) Force.


1, 3


2


4


5a

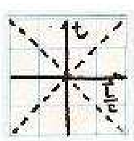

5b, c, d

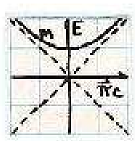
Basic Units

- t Time.
- L Length.
- m Mass.

For EM, Gaussian units will be used. Units of electric charge are found from Coulomb's law:

$F = \frac{qq'}{R^2} \rightsquigarrow \frac{mL}{t^2}$ so $q \rightsquigarrow \frac{\sqrt{mL^3}}{t}$ where " $\rightsquigarrow$ " means "has units of".

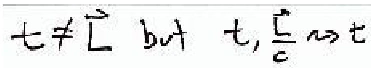




Units for Conversion Factors

For gravity, spacetime, & quantum mechanics.

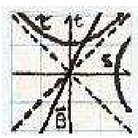
- $G \rightsquigarrow \frac{L^3}{mt^2}$ Gravitational constant.
- $c \rightsquigarrow \frac{L}{t}$ Speed of light.
- $h \rightsquigarrow \frac{mL^2}{t}$ Planck's constant.



Units for Spacetime

Where all events of gravity, EM, and quantum mechanics take place.

- $V \rightsquigarrow L^3$ Volume.
- $\tau^2 = t^2 - \frac{\vec{R}\cdot\vec{R}}{c^2} \rightsquigarrow t^2$ Interval squared.
- $\sigma^2 = \vec{R}\cdot\vec{R} - c^2t^2 = -\,c^2\,\tau^2 \rightsquigarrow L^2$ 4D-distance squared.
- $\gamma = \frac{1}{\sqrt{1-\frac{\vec{v}\cdot\vec{v}}{c^2}}} = \frac{\partial t}{\partial\tau} \rightsquigarrow -$ Stretch factor.
- $\vec{\beta} = \frac{\vec{v}}{c} \rightsquigarrow -$ Relativistic 3-velocity
- $U^\mu = (c\frac{\partial t}{\partial\tau}, \frac{\partial\vec{R}}{\partial\tau}) = (c\gamma, c\gamma\vec{\beta}) = (\frac{E}{mc}, \frac{\vec{\pi}}{mc}) \rightsquigarrow \frac{L}{t}$ Velocity vector.



Units for Potentials, Fields, & Charges

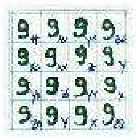
The way to describe where stuff is everywhere, everywhen.

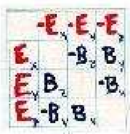
- $A^\mu = (\phi, \vec{A}) \rightsquigarrow \frac{\sqrt{m}}{\sqrt{L}}$ Potential vector.
- $A^{\mu;\nu} \rightsquigarrow \vec{g} \rightsquigarrow \vec{E} \rightsquigarrow \vec{B} \rightsquigarrow \frac{\sqrt{m}}{t\sqrt{L}}$ Derivatives of potential vectors (fields!).

- $q \rightsquigarrow \frac{\sqrt{m L^3}}{t} \rightsquigarrow \sqrt{G}\, m \, [\frac{\sqrt{L^3}}{t\sqrt{m}} m] \rightsquigarrow \sqrt{h\, c}\, [\frac{L\sqrt{m}}{\sqrt{t}} \frac{\sqrt{L}}{\sqrt{t}}]$ Charge.

- $J^\mu = \frac{q}{V} \frac{U^\mu}{\gamma c} = (\frac{q}{V}, \frac{q}{V}\vec{\beta}) \rightsquigarrow \frac{\sqrt{m}}{t\sqrt{L^3}} \rightsquigarrow \frac{\sqrt{G}\, m}{V} [\frac{\sqrt{L^3}}{t\sqrt{m}} m \frac{1}{L^3}] \rightsquigarrow \frac{\sqrt{h\, c}}{V} [\frac{L\sqrt{m}}{\sqrt{t}} \frac{\sqrt{L}}{\sqrt{t}} \frac{1}{L^3}]$

  Current density vector.





Units in Action: Lagrange Density

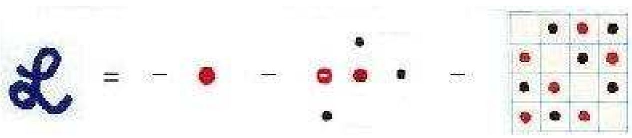
Lagrange Density, where all mass, energy, and interactions are in a volume.

- $\mathfrak{L} \rightsquigarrow \frac{m}{L^3}$ Mass density.

- $\mathfrak{L} \rightsquigarrow \frac{m}{\gamma V} [\frac{m}{L^3}] \rightsquigarrow \frac{1}{c^2} \frac{q}{V} \frac{U^\mu}{\gamma} A_\mu [\frac{t^2}{L^2} \frac{\sqrt{m L^3}}{t} \frac{1}{L^3} \frac{L}{t} \frac{\sqrt{m}}{\sqrt{L}}] \rightsquigarrow$

  $\frac{\sqrt{G}}{c^2} \frac{m}{V} \frac{U^\mu}{\gamma} A_\mu [\frac{\sqrt{L^3}}{t\sqrt{m}} \frac{t^2}{L^2} m \frac{1}{L^3} \frac{L}{t} \frac{\sqrt{m}}{\sqrt{L}}] \rightsquigarrow \frac{1}{c^2} A^{\mu;\nu} A_{\mu;\nu} [\frac{t^2}{L^2} \frac{\sqrt{m}}{t\sqrt{L}} \frac{\sqrt{m}}{t\sqrt{L}}]$

  Equivalent units.



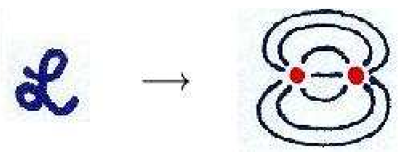
Units in Action: Euler-Lagrange Equations

Euler-Lagrange equations, generates field equations given a Lagrange density.

- $c \frac{\partial \mathfrak{L}}{\partial \phi} = c\, \partial_\mu \frac{\partial \mathfrak{L}}{\partial(\partial_\mu \phi)}$ From principle of least action.

- $c \frac{\partial \mathfrak{L}}{\partial \phi} [\frac{L}{t} \frac{m}{L^3} \frac{\sqrt{L}}{\sqrt{m}}] \rightsquigarrow \frac{q}{V} [\frac{\sqrt{m L^3}}{t} \frac{1}{L^3}] \rightsquigarrow \nabla A^{\mu;\nu} [\frac{1}{L} \frac{\sqrt{m}}{t\sqrt{L}}]$

  $\rightsquigarrow \nabla \vec{g}\, [\frac{1}{L} \frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow \nabla \vec{E}\, [\frac{1}{L} \frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow \nabla \vec{B}\, [\frac{1}{L} \frac{\sqrt{m}}{t\sqrt{L}}] \rightsquigarrow J^\mu [\frac{\sqrt{m}}{t\sqrt{L^3}}]$ Equivalent units.



Units in Action: Momentum

Energy and 3-momentum from a derivative of a Lagrange density.

- $\pi^{\mu} = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\,\partial t}} \rightsquigarrow \frac{m\,L^2}{t^2}$ Derivative of the Lagrange density.

- $\pi^{\mu}{}_{[\frac{m\,L^2}{t^2}]} \rightsquigarrow h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^{\mu}}{c\,\partial t}}{}_{[\frac{m\,L^2}{t}\,\frac{\sqrt{L^3}}{t\sqrt{m}}\,\frac{L}{t}\,\frac{t\sqrt{L}}{\sqrt{m}}\,\frac{m}{L^3}]}$ Equivalent units.

Note: units suggest relativistic (c), quantum (h) gravity (G).

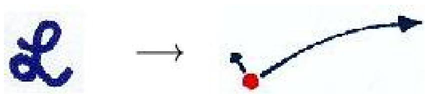
Units in Action: Relativistic Force

- $F^{\mu} = \text{cause} = \frac{\partial m\,U^{\mu}}{\partial \tau}$ Force is a cause which has an effect on momentum.

- $F^{\mu}{}_{[\frac{m\,L}{t^2}]} \rightsquigarrow \frac{1}{c}\,qU_{\mu}A^{\mu;\nu}{}_{[\frac{t}{L}\frac{\sqrt{m\,L^3}}{t}\frac{L}{t}\frac{\sqrt{m}}{t\sqrt{L}}]} \rightsquigarrow \frac{\sqrt{h}}{\sqrt{c}}\,nU_{\mu}A^{\mu;\nu}{}_{[\frac{L\sqrt{m}}{\sqrt{t}}\frac{\sqrt{t}}{\sqrt{L}}\frac{L}{t}\frac{\sqrt{m}}{t\sqrt{L}}]}$

  $\rightsquigarrow \frac{\sqrt{G}}{c}\,mU_{\mu}A^{\mu;\nu}{}_{[\frac{\sqrt{L^3}}{t\sqrt{m}}\,\frac{t}{L}\,m\,\frac{L}{t}\,\frac{\sqrt{m}}{t\sqrt{L}}]}$

  Equivalent units.

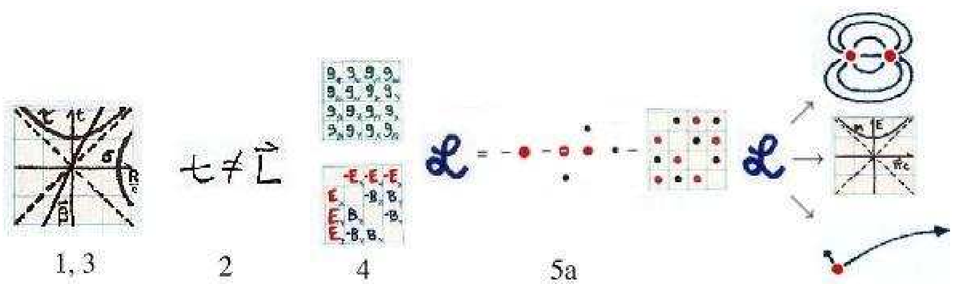


Summary: Units

Math:

$q \rightsquigarrow \frac{\sqrt{m\,L^3}}{t} \rightsquigarrow \sqrt{G}\,m\,{}_{[\frac{\sqrt{L^3}}{t\sqrt{m}}\,m]} \rightsquigarrow \sqrt{h\,c}\,{}_{[\frac{L\sqrt{m}}{\sqrt{t}}\,\frac{\sqrt{L}}{\sqrt{t}}]}$

Pictures:



1, 3 2 4 5a 5b, c, d