# Gravity by Analogy to EM

## ■ Abstract

A Lagrange density for gravity is proposed based on a strict analogy to the classical Lagrangian for electromagnetism. For local covariant coordinates where the connection is zero, the field equations are a four−dimensionalwave equation. The classic field equations contain both the Maxwell equations and Newton’s field equations under certain conditions. The four−dimensionalwave equation has been quantized before. The scalar and longitudinal modes of emission are interpreted as gravitons, so they can do the work of gravity. If gravitational waves are detected, this proposal predicts scalar or longitudinal polarization. How the proposal integrates with the standard model Lagrangian is worked out.

A force equation is written based on the same strict analogy to the relativistic Lorentz force of electromagnetism. For geodesic motion, the cause of the curvature is due entirely to the gravitational and electric potentials. This is a new type of statement about curvature. A specific, normalized, weak−field potential is investigated. Analysis of small perturbations yields changes in the potential that depend on an inverse distance squared. By breaking spacetime symmetry, Newton’s law of gravity results. By using the chain rule, a stable, constant−velocitysolution is apparent, which may yield insight to the rotation profile of galaxies and early big bang cosmology, since both require stable, constant−velocity solutions. If spacetime symmetry is preserved, the second−orderdifferential equations can be solved exactly. Eliminating the constants and rearrange terms generates an equation that has the form of a metric equation. The Taylor series expansion of the metric equation is identical to the Schwarzschild metric to parameterized−post−Newtonianaccuracy. The Taylor series for the two metrics differ for higher order terms and may be tested experimentally.

## ■ Introduction

The goal of this paper is to create one mathematical structure for gravity and electromagnetism that can be quantized. The difference between gravity and electromagnetism is the oldest core problem facing physics, going back to the first studies of electromagnetism in the seventeenth century. Gravity was the first inverse square law, discovered by Isaac Newton. After twenty years of effort, he was able to show that inside a hollow massive shell, the gravitational field would be zero. Ben Franklin, in his studies of electricity, demonstrated a similar property for an electrically charged hollow sphere. Joseph Priestly realized this meant that the electrostatic force was governed by an inverse square law just like gravity. Coulomb got the credit for the electrostatic force law modeled on Newton’s law of gravity.

Over a hundred years later, Einstein started from the tensor formalism of electromagnetism on the road to general relativity. Instead of an antisymmetric field strength tensor, Einstein used a symmetric tensor because the metric tensor is symmetric. There is a precedence for transforming mathematical structures between gravity and electromagnetism.

The process of transforming mathematical structures from electromagnetism to gravity will be continued. Specifically, the gravitational analog to the classic electromagnetic Lagrange density will be written. There are several consequence of this simple procedure. The Lagrangian contains both terms with a connection and the Fermi Lagrangian of electromagnetism. This makes it reasonable to suppose the Lagrangian can describe both a dynamic geometry required for gravity and the Maxwell equations for electrodynamics. The gravitational field equations are analogues to Gauss’ and Ampere’s laws, and contain the Newton's gravitational field equation. These field equations are not second rank like those used in general relativity. It must be stressed that the field strength tensor is a second order symmetric tensor, so this does not conflict with proofs that at least a symmetric second rank tensor is required to completely describe spacetime curvature. The Maxwell equations result if the gravitational field is zero. The field equations have been quantized before, but new interpretations will flow from the unification effort. A link to the the Lagrangian of the standard model will be detailed.

A weak static gravitational field in a vacuum will be studied using standard modern methods: normalizing the potential and looking at perturbations. The potential will be plugged into a gravitational force equation analogous to the Lorentz force equation of electromagnetism. The force equation leads to a geodesic equation where the potential causes the curvature, something which is missing from general relativity. Newton's law of gravity is apparent if spacetime symmetry is broken. A new class of solutions emerges for the gravitational source where velocity is constant, but the distribution of mass varies with distance. This may provide new ways to look at problems with the rotation profiles of disk galaxies and big bang cosmology. If spacetime symmetry is preserved, solving the force equation and eliminating the constants creates a metric equation similar to the Schwarzschild metric. The metrics are equivalent to first−orderparameterized post−Newtonianaccuracy. Therefore the metric will past all weak field tests. The coefficients are different to second−order,so the proposal can be verify or rejected experimentally.

## ■ Lagrangians

The classic electromagnetic Lagrangian density has three terms: one for kinetic energy, one for a moving change, and a third for the antisymmetric second rank field strength tensor:

$$\mathcal{L}_{EM} = -\frac{m}{\gamma V} - \frac{q}{c^2 V}\frac{U^{\mu}}{\gamma} A_{\mu} - \frac{1}{4 c^2}\left(A^{\mu,\nu} - A^{\nu,\mu}\right)\left(A_{\mu,\nu} - A_{\nu,\mu}\right)$$

An analogous Lagrangian for gravity would also contain these three components, but three changes are required. First, gravity depends on mass, not charge, so where there is an electrical charge $-q$,an inertial mass $m$ will be substituted. The change in sign is required so that like charges attract for gravity. Mass does not have the same units as electric charge, so mass will have to be multiplied by the square root of Newton's gravitational constant $G$ to keep the units identical. Second, because gravity effects metrics which are symmetric, the source of gravity must also be symmetric. Therefore the minus sign that makes the electromagnetic field strength tensor antisymmetric will be made positive. Third, in order that symmetric object transforms like a tensor requires a replacement of the standard derivative (symbolized by a comma) with a covariant derivative (symbolized by a semicolon):

$$\mathcal{L}_{g} = -\frac{m}{\gamma V} + \frac{\sqrt{G}\, m}{c^2 V}\frac{U^{\mu}}{\gamma} A_{\mu} - \frac{1}{4 c^2}\left(A^{\mu;\nu} + A^{\nu;\mu}\right)\left(A_{\mu;\nu} + A_{\nu;\mu}\right)$$

The total Lagrangian will be a merger of these two which only apply if the other force is not in effect. The kinetic energy term is the same as either Lagrangian separately. The moving charge term is a sum. Without loss of generality, the regular derivatives in the electromagnetic Lagrangian (Eq. L_EM) can be written as covariant derivatives. This leads to the unified Lagrangian for gravity and electromagnetism:

$$\mathcal{L}_{gEM} = -\frac{m}{\gamma V} - \frac{q - \sqrt{G}\, m}{c^2 V}\frac{U^{\mu}}{\gamma} A_{\mu} - \frac{1}{2 c^2} A^{\mu;\nu} A_{\nu;\mu}$$

$$= -\frac{m}{\gamma V} - \frac{q - \sqrt{G}\, m}{c^2 V} (\phi - \vec{A} \cdot \vec{v}) -$$
$$\frac{1}{2 c^2} A^{\mu,\nu} A_{\nu,\mu} - \frac{1}{2 c^2} \Gamma^{\omega}{}_{\mu\nu} (\Gamma_{\rho}{}^{\mu\nu} A^{\rho} A_{\omega} - 2 A^{\mu,\nu} A_{\omega})$$

The kinetic energy term is for one particle experiencing both gravity and electromagnetism. The Fermi Lagrangian of electromagnetism is a subset. This establishes a link to electromagnetism. The Christoffel symbols (or connection coefficients) represent derivatives of metrics. Because a dynamic metric is part of the Lagrangian, this Lagrangian could describe the dynamics of the metric, which is a central accomplishment of general relativity. The potential to do both gravity and electromagnetism is here.

In local covariant coordinates, the connect is zero, which leads to a simpler expression of the Lagrangian:

$$\mathcal{L}_{gEM} = -\frac{m}{V} \sqrt{1 - \left(\frac{\partial \vec{R}}{\partial t}\right)^2} - \frac{q - G^{.5}\, m}{c^2 V} (\phi - \vec{A} \cdot \vec{v}) -$$
$$\frac{1}{2 c^2} \left( \left(\frac{\partial \phi}{\partial t}\right)^2 - (\vec{\nabla} \phi)^2 - \left(\frac{\partial \vec{A}}{\partial t}\right)^2 + (\nabla \vec{A})^2 \right)$$

This is almost identical to working with the classical electromagnetic field equation by choosing the Lorenz gauge, the difference being the inclusion of a mass term. Because the gauge was not fixed, there is more freedom for this Lagrangian.

## ■ Classical Field Equations

The field equations can be found by applying the Euler−Lagrangeequations to the Lagrange density (assuming the connection is zero for simplicity):

$$\Box^2 A^{\mu} = \frac{q - \sqrt{G}\, m}{V} \frac{U^{\mu}}{\gamma c}$$

The fields are expressed in terms of the potential. The symmetric and antisymmetric field strength tensors are very simpliar, differeing only in the sign of A^v;u.  The classical fields required to represent the field strength tensors should also be similar.  There is a symmetric analog to the electric E and B fields: To make a connection to the classical fields of gravity and electromagnetism, use the following substitutions:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - c\, \vec{\nabla} \phi$$

$$\vec{e} = \frac{\partial \vec{A}}{\partial t} - c\, \vec{\nabla} \phi$$

$$\vec{B} = c \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial z}, \frac{\partial}{\partial x} - \frac{\partial}{\partial z}, \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) \vec{A} = c\, \vec{\nabla} \times \vec{A}$$

$$\vec{b} = c\left(-\frac{\partial}{\partial y} - \frac{\partial}{\partial z},\ -\frac{\partial}{\partial x} - \frac{\partial}{\partial z},\ -\frac{\partial}{\partial x} - \frac{\partial}{\partial y}\right)\vec{A} \equiv \vec{\nabla} \times \vec{A}$$

The symmetric curl as defined above has all the same differential operators, but all the signs are negative, so it is easier to remember. The symmetric field strength tensor has four more components that lie along the diagonal. Define a field g to represent the diagonal elements:

$$g = \left(\frac{\partial \phi}{\partial t},\ -c\,\frac{\partial A_x}{\partial x},\ -c\,\frac{\partial A_y}{\partial y},\ -c\,\frac{\partial A_z}{\partial z}\right) = \partial^{\mu} A^{\mu}$$

The diagonal of the field strength tensor A^mu ;nu is g. The first row and column of the asymmetric field strength tensor is the sum of the electric field E and its symmetric analog e. The rest of the off−diagonal terms are the sum of the magnetic field B and its symmetric analog b. If the trace of field strength tensor is zero, then the equations are in the Lorentz gauge.

Substitute the classical fields into the field equations, starting with the scalar field equation:

$$\rho_q - \rho_m = \frac{\partial^2 \phi}{\partial t^2} - c^2\,\vec{\nabla}^2 \phi$$

$$= \frac{c}{2}\,(\vec{\nabla} \cdot \vec{E} + \vec{\nabla} \cdot \vec{e}) + \frac{\partial g^0}{\partial t}$$

This eqation combines Gauss’ law and analogous equation for gravity. The two equations are unified, but under certain physical conditions, can be isolated. A relativistic form of the Newtonian gravitational field equation can be seen with the following constraints:

$$\rho_m = c^2\,\vec{\Box}^2 \phi$$

$$\text{iff}\quad \frac{\partial \vec{A}}{\partial t} = -c\,\vec{\nabla}\phi$$

This equation should be consistent with special relativity without modification. The classical Newtonian field equation arises from these physical constraints:

$$\rho_m = c^2\,\vec{\nabla}^2 \phi$$

$$\text{iff}\quad \frac{\partial \vec{A}}{\partial t} = -c\,\vec{\nabla}\phi \quad \text{and} \quad \frac{\partial g^0}{\partial t} = 0$$

Every aspect of classical Newtonian gravity can be represented by this proposal under these constraints.

Gauss’ law appears under the following conditions:

$$\rho_q = \frac{\partial^2 \phi}{\partial t^2} - c^2\,\vec{\nabla}^2 \phi$$

$$\text{iff}\quad \frac{\partial \vec{A}}{\partial t} = c\,\vec{\nabla}\phi$$

Repeat the exercise for the vector equation.

$$-\vec{J}_m + \vec{J}_q = \frac{\partial^2 A}{\partial t^2} - c^2 \nabla^2 \vec{A}$$

$$= \frac{1}{2} \left( -\frac{\partial \vec{E}}{\partial t} + c \vec{\nabla} \times \vec{B} + \frac{\partial \vec{e}}{\partial t} + c \vec{\nabla} \,\underline{\times}\, \vec{b} \right) + c \vec{\nabla} g^u$$

This has Ampere’s law and a symmetric analog for Ampere’s law for gravity.

This proposal for classical gravitational and electromagnetic field equations is expressed with tensors of rank one (vectors). Einstein’s field equations are second rank. Therefore the two approaches are fundamentally different. One must remember that although the field equations are rank one, the field strength tensor is second rank.

With no gravitational field, the Maxwell source equations result. The homogeneous Maxwell equations are vector identities with these choices of maps to the potentials, and are unaffected by the proposal.

## ■ Canonical Quantization

The classical electromagnetic Lagrangian cannot be quantized. One way to realize this is to consider the generalized 4−momentum:

$$\pi^\mu = h \sqrt{G} \frac{\partial \mathcal{L}_{EM}}{\partial \left( \frac{\partial A^\mu}{c\, \partial t} \right)} = -F^{\mu 0}$$

Unfortunately, the energy component of the moment operator is zero. The commutator [x^0, pi^0] will equal zero, and cannot be quantized. The momentum for the unified Lagrangian of gravity and electromagnetism does not suffer from this problem:

$$\pi^\mu = h \sqrt{G} \frac{\partial A^\mu}{c\, \partial t}$$

When expressed with operators, the commutator [x^0, pi^0]  will not be zero, so the field can be quantized. If the connection is zero, L_gEM generates the same field equations as the classical electromagnetic Lagrangian with the choice of the Lorenz gauge. That field has been quantized before, first by Gupta and Bleuler (S. N. Gupta, Proc. Phys. Soc. London, 63:681−691,1950). They determined that there were four modes of transmission: two transverse, one scalar, and one transverse mode. The interpretation of these modes appears internally inconsistent to this author. They discuss "scalar photons", but photons as the quanta of electric and magnetic fields must transform as a vector, not a scalar. They introduce a supplemental condition solely to make the scalar and longitudinal modes virtual. Yet there is no need to make a nonsense particle virtual.

The field in this proposal must represent both gravity and electromagnetism. The two transverse modes are photons that do all the work of electromagnetism. The symmetric second−rankfield strength tensor cannot be represented by a photon because photons transform differently than a symmetric tensor. Whatever particle does the work must travel at the speed of light like the transverse modes of transmissions of the field. These constraints dictate that the scalar and transverse modes of transmission for this proposal are gravitons.

There are efforts underway to detect the transverse gravitational waves predicted by general relativity. This proposal predicts the polarity of a gravitational wave will be either scalar or longitudinal, not transverse, because those are the modes of transmission. The detection of the first gravitational wave polarization will mark either success or failure of this unified field theory.

## ■ Integration with the Standard Model

The standard model does not in an obvious way deal with curved spacetime. A more explicit connection will be attempted by condensing the unitary aspects of the symmetries U(1), SU(2), and SU(3) with the 4−vectorsand a curved metric. Start with the standard model Lagrangian:

$$\mathcal{L}_{SM} = \bar{\Psi}\,\gamma^{\mu}\,D_{\mu}\,\Psi$$

where

$$D_{\mu} = \partial_{\mu} - i g_{EM}\,Y A_{\mu} - i g_{weak}\,\frac{\tau^{a}}{2}\,W_{\mu}^{a} - i g_{strong}\,\frac{\lambda^{b}}{2}\,G_{\mu}^{b}$$

The electromagnetic potential A_mu is a complex−valued4−vector.The only way to form a scalar with a 4−vectoris to use a metric. Since it is complex−valued,use the conjugate like so:

$$A^{\mu}\,A^{\nu *}\,g_{\mu\nu} = |A_0|^2 - |A_1|^2 - |A_2|^2 - |A_3|^2$$

Use the parity operator to flip the sign of the spatial part of a 4−vector:

$$A^{\mu}\,A^{\nu * p}\,g_{\mu\nu} = |A_0|^2 + |A_1|^2 + |A_2|^2 + |A_3|^2$$

Normalize the potential:

$$\frac{A^{\mu}}{|A|}\,\frac{A^{\nu * p}}{|A|}\,g_{\mu\nu} = 1$$

From this,it can be concluded that the normalized 4−vectoris an element of the symmetry group U(1) if the multiplication operator is the metric combined with the parity and conjugate operators. One does not need the Y in standard model Lagrangian, so this simplifies things. The same logic applies to the 4−vector potentials for the weak and the strong forces which happen to have internal symmetries.

In curved spacetime, the previous equation will not equal one. Mass breaks U(1), SU(2), and SU(3) symmetry, but does so in a precise way (meaning one can calculate what the previous equation should equal). There is no need for the Higgs mechanism to give particles mass while preserving U(1)xSU(2)x-SU(3) symmetry, so this proposal predicts no Higgs particle will be found.

## ■ Forces

The Lorentz Force of electromagnetism involves charge, velocity and the anti−symmetricfield strength tensor:

$$F_{EM}^{\mu} = q\,\frac{U_{\nu}}{c}\,\left(A^{\mu,\nu} - A^{\nu,\mu}\right)$$

Form an analogous force for gravity using the same substitutions as before:

$$F_{g}^{\mu} = -\sqrt{G}\,m\,\frac{U_{\nu}}{c}\,\left(A^{\mu;\nu} + A^{\nu;\mu}\right)$$

The gravitational force and the electromagnetic force behave differently under charge inversion. If the mass changes signs, then both side flip signs, so nothing has really changed. If electric change changes signs, the change in momentum will not change signs. The different behavior under charge inversion may explain why gravitational force is unidirectional, but electrical forces can attract or repulse.

The total force is a combination of the two:

$$F^{\mu}_{gEM} = \left(q - \sqrt{G}\, m\right) \frac{U_{\nu}}{c} A^{\mu;\nu} - \left(q + \sqrt{G}\, m\right) \frac{U_{\nu}}{c} A^{\nu;\mu}$$

If q >> G^.5 m, the equation approaches the form of the Lorentz force law of electromagnetism. If the force is zero, the equation has the form of a Killing's equation, which is used to determine the isometries of a metric. Geodesics are defined by examining the left−handside of F_gEM:

$$\frac{\partial m\, U^{\mu}}{\partial \tau} = m \frac{\partial U^{\mu}}{\partial \tau} + U^{\mu} \frac{\partial m}{\partial \tau} = 0$$

Assume dm/dtau = 0. Apply the chain rule, and then the definition of a covariant derivative to form a geodesic equation:

$$0 = m \frac{\partial^2 x^{\mu}}{\partial \tau^2} + \frac{m}{c} \Gamma^{\mu}{}_{\nu\omega} U^{\nu} U^{\omega}$$

This equation says that if there is no force, all the acceleration seen in spacetime is due to spacetime curvature, the Christoffel symbol. The covariant derivatives on the right side of F_gEM can also be expanded:

$$0 = \left(q - \sqrt{G}\, m\right) \frac{\partial x_{\nu}}{c\, \partial \tau} A^{\mu,\nu} - \left(q + \sqrt{G}\, m\right) \frac{\partial x_{\nu}}{c\, \partial \tau} A^{\nu,\mu} - \frac{2}{c} m \Gamma_{\omega}{}^{\mu\nu} U_{\nu} U^{\omega}$$

This equation says that spacetime curvature is caused by the change in the potential if there is no external force. This is a novel statement. In general relativity, one compares two geodesics, and based on an analysis of the tidal forces between the geodesics, determines the curvature. The unified geodesic equation asserts that the curvature can be calculated directly from the potential. Notice that this equation contains terms linked to a mass m and a charge q, so the geodesic equation applies to electromagnetism as well as gravity.

## ■ Gravitational Force for a Weak Field

The total unified force law is relevant to physics because it contains the Lorentz force law of electromagnetism. It must be established that the terms coupled to the mass m are connected to what is known about gravity.

Since the goal in this section is to study gravity, not electromagnetism, work with a potential that can only contribute to gravity, not electromagnetism. Since the first component of the potential does not appear in the antisymmetric field strength tensor, work with a potential with the form: A_mu = (A_0, 0).

The next task is to find a solution to the unified field equations. The Poisson field equation of classical Newtonian gravity can be solved by a 1/R potential. The potential has a point singularity where R = 0. The unified field equations are relativistic, so time must also be incorporated. A 1/distance potential does not solve the field equations in four dimensions. In local covariant coordinates where the connection is zero, the potential A_mu = (1/sigm^2, 0) solves the field equations, where sigma squared is the Lorentz invariant distance, or the negative of the square of the Lorentz invariant interval tau. Distance is used instead of the interval because classical gravity depends on distance, not time. The idea is to consider the time contribution to be very small relative to the distance. Such a potential has as a singularity that is the entire lightcone, where sigma ^2 = 0. This singularity may not be problematic because massless particles are described by the Maxwell equations, but that hope will required a detailed study.

Gravity is a weak effect. It is common in quantum mechanics to normalize to one and study perturbations of weak fields, an approach that will be followed here. Normalizing means there are small steps will be away from one. Only first order terms will be kept. Here is the normalized potential with a linear perturbation:

$$A^{\mu} = \left(\frac{\sqrt{G}\, h}{c^2\, \sigma^2}, \vec{0}\right) \longrightarrow \left(c \Big/ \left(\sqrt{G}\, \left(\left(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}\, x\right)^2 + \left(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}\, y\right)^2 + \left(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}\, z\right)^2 - \left(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}\, t\right)^2\right)\right), \vec{0}\right)$$

This potential solves the 4D wave equation because the shift by the one over root two factor and the rescaling by the spring constant k over sigma square do not effect the differential equation. One interesting aspect is the shift of units from one that depends on h −suggesting quantum mechanics −to the normalized perturbation which appears to be classical because there is no h.

Take the derivative with respect to t, x, y, and z:

$$\frac{\partial \phi}{\partial t} = \frac{c^2\, k}{\sqrt{G}\, \sigma^2} + O\,(k^2)$$

$$c\, \frac{\partial A_x}{\partial x} = -\frac{c^2\, k}{\sqrt{G}\, \sigma^2} + O\,(k^2)$$

$$c\, \frac{\partial A_y}{\partial y} = -\frac{c^2\, k}{\sqrt{G}\, \sigma^2} + O\,(k^2)$$

$$c\, \frac{\partial A_z}{\partial z} = -\frac{c^2\, k}{\sqrt{G}\, \sigma^2} + O\,(k^2)$$

The change in the potential is a function of a spring constant k over sigma squared. The classical Newtonian dependence on distance is an inverse square, so this is promising. One problem is that a potential that appliex exclusively to gravity is saught. The sign of the spring constant k does not effect the solution to the 4D wave field equations. The sign of the spring constant k does change the derivative of the potential. Therefore a potential that only has derivatives along the diagonal of the field strength tensor can be constructed from two potentials that differ by string constants that either constructively interfere to create a non−zeroderivative, or destructively interfere to eliminate a derivative.

$$\texttt{diagonal}\,\mathrm{SHO}\,A^{\mu}=$$

$$
\begin{aligned}
&\frac{c}{\sqrt{G}}\Bigg(1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,x\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,y\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,z\right)^2-\\
&\qquad\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,t\right)^2\Bigg)+1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,x\right)^2+\\
&\qquad\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,y\right)^2+\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,z\right)^2-\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,t\right)^2\Bigg)\;,\\
&\quad 1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,x\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,y\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,z\right)^2-\\
&\qquad\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,t\right)^2\Bigg)+1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,x\right)^2+\\
&\qquad\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,y\right)^2+\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,z\right)^2-\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,t\right)^2\Bigg)\;,\\
&\quad 1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,x\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,y\right)^2+\\
&\qquad\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,z\right)^2-\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,t\right)^2\Bigg)+\\
&\quad 1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,x\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,y\right)^2+\\
&\qquad\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,z\right)^2-\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,t\right)^2\Bigg)\;,\\
&\quad 1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,x\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,y\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,z\right)^2-\\
&\qquad\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,t\right)^2\Bigg)+1\Big/\Bigg(\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,x\right)^2+\\
&\qquad\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,y\right)^2+\left(\frac{1}{\sqrt{2}}+\frac{k}{\sigma^2}\,z\right)^2-\left(\frac{1}{\sqrt{2}}-\frac{k}{\sigma^2}\,t\right)^2\Bigg)\Bigg)
\end{aligned}
$$

Take the contravariant derivative of this potential, keeping only the terms to first order in the spring constant k.  The contravariant derivative flips the sign for the three−vector.

$$A^{\mu,\nu} = \frac{c^2}{\sqrt{G}} \begin{pmatrix} \frac{k}{\sigma^2} & 0 & 0 & 0 \\ 0 & \frac{k}{\sigma^2} & 0 & 0 \\ 0 & 0 & \frac{k}{\sigma^2} & 0 \\ 0 & 0 & 0 & \frac{k}{\sigma^2} \end{pmatrix}$$

All this work to get a multiple of the identity matrix! Plug this into the gravitational force equation:

$$F_g^\mu = m\, c \left(-\frac{k}{\sigma^2}\, \frac{\partial t}{\partial \tau},\ \frac{k}{\sigma^2}\, \frac{\partial \vec{R}}{\partial \tau}\right)$$

This is a relativistic force law for a weak gravitational field for the inverse interval squared diagonal potential. When spacetime symmetry is broken, this equation will lead to Newton’s law of gravity in the next section. If spacetime symmetry is maintained, then solving the force equation and eliminating the constants yields a metric equation for gravity.

## ■ Newton’s Law of Gravity and More

Several assumptions need to be made to apply the weak gravitational force equation to a classical gravitational system. First, assume that the spring constant is due to the source mass, k = GM. Second, assume that the field is static, so that sigma^2 = R^2 −t^2 =R’^2. In this way it does not depend on time.

Newtonian spacetime is different from Minkowski spacetime because the speed of light is infinite. Spacetime symmetry must be broken. A question arises about how to do this in a formal mathematical sense. The Minkowski interval tau is a consequence of the relationship between time t and space R. The functional relationship between time and space must be severed. By the static field approximation, there is a distance R which is the same magnitude as the interval tau. If the interval tau is replaced by the scalar distance R, then that will sever the functional relationship between time and space:

$$\left(\frac{\partial t}{\partial \tau},\ \frac{\partial \vec{R}}{c\, \partial \tau}\right) \longrightarrow \left(\frac{\partial t}{\partial\, |R|},\ \frac{\partial \vec{R}}{c\, \partial\, |R|}\right) = (0,\ \hat{R})$$

Plug these three assumptions into force equation:

$$F_g^\mu = \left(0,\ -\frac{GMm}{R^2}\ \hat{R}\right)$$

This is not quite Newton’s gravitational force law. The reason is that one must consider the left−hand side of the force equation carefully. According to the chain rule:

$$\frac{\partial m\, U^\mu}{\partial \tau} = m\, \frac{\partial U^\mu}{\partial \tau} + U^\mu\, \frac{\partial m}{\partial \tau}$$

An open question is how should spacetime symmetry be broken for the derivatives with respect to the interval tau? An interval is composed of both changes in time and space. For the acceleration term, if the interval is only about time, then one gets back Newtonian acceleration. For logical consistency, one might be tempted to also substitute time in the dm/dtau term. However, the system is presumed to be static, so this would necessarily be zero. If this derivative is to have any chance at being non−zero,it would have to be with respect to the absolute value of R as has been done earlier in the derivation. So the classical force law should look like so:

$$m \frac{\partial^2 \vec{R}}{\partial t^2} + \frac{\partial \vec{R}}{\partial t} \frac{c\, \partial m}{\partial\, |R|} = -\frac{GMm}{R^2} \hat{R}$$

For a point source, the dm/dtau term will not make a contribution, and one gets Newton’s law of gravity. It is only if the inertial mass is distributed over space like for the big bang or galaxies will the term come into play. If the velocity is constant, then the acceleration is zero. The equation describes the distribution of the inertial mass m that makes up the total gravitational source mass M. The solution to the force equation when there is no acceleration is a stable exponential. Big bang cosmology has two problems: all matter is traveling at exactly the same speed even though it is not possible for them to communicate (the horizon problem), and the model require high levels of precision on initial conditions to avoid collapse (the flatness problem). [A. H. Guth, Phys. Rev. D., 23:347−356,1981]  The force equation has a stable, constant velocity solution which may resolve both problems of the big bang without the inflation hypothesis. Their is also a problem with the rotation profile of thin disk galaxies.[S. M. Kent, Astron. J., 91:1301−1327,1986; S. M. Kent, Astron. J., 93:816−832,1987]  Once the maximum velocity is reached, the velocity stays constant. It has been shown that galaxies should not be stable at all.[A. Toomre, Astrophys. J., 139:1217, 1964] Both problems may again be resolved with stable constant velocity solutions. Numerical approaches on the above equation should be conducted.

## ■ A Metric Equation

The weak gravitational force equation is two second−orderdifferential equations. The equation can be simplified to a set of first−orderdifferential equations by substituting (U^0, U) = (c dt/dtau, dR/dtau)

$$\frac{\partial U^0}{\partial \tau} - \frac{k}{\tau^2} U^0 = 0$$

$$\frac{\partial \vec{U}}{\partial \tau} + \frac{k}{\tau^2} \vec{U} = 0$$

The solution involves exponentials:

$$U^\mu = \left( v e^{-\frac{k}{\tau}},\ \vec{V} e^{\frac{k}{\tau}} \right)$$

For flat spacetime, U^mu = (v, V). The constraint on relativistic velocities in flat spacetime is:

$$U^\mu U_\mu = \frac{c^2\, dt^2 - dR^2}{d\tau^2} = c^2 = v^2 - \vec{V} \cdot \vec{V}$$

Solve for the constants, and plug back into the constraint, multiplying through by dtau ^2.

$$d\tau^2 = e^{-2\frac{k}{\tau}}\, dt^2 - e^{2\frac{k}{\tau}} \frac{dR^2}{c^2}$$

Make the same two assumptions as before: the spring constant is due to the gravitational source, k = GM/c^2, and the field is static, so tau ^2= R^2−t^2 = R’^2. There is one more degree of freedom, because the radius R could either be positive or negative. To make the metric consistent with experiment, choose the negative root:

$$d\tau^2 = e^{-2 \frac{GM}{c^2 R}} dt^2 - e^{2 \frac{GM}{c^2 R}} dR^2$$

This equation has the form of a metric equation. Perform a Taylor series expansion to second order in GM/c^2R:

$$d\tau^2 = \left(1 - 2 \frac{GM}{c^2 R} + 2 \left(\frac{GM}{c^2 R}\right)^2\right) dt^2 - \left(1 + 2 \frac{GM}{c^2 R} + 2 \left(\frac{GM}{c^2 R}\right)^2\right) dR^2$$

If one compares this metric to the Schwarzschild metric in isotropic coordinates to parameterized post−Newtonian accuracy, the coefficients are identical. For that reason, this metric is consistent with all experimental tests weak field tests of general relativity. [C. M. Will, "Theory and experiment in gravitational physics: Revised edition", Cambridge University Press, 1993.]

For higher order terms of the Taylor series expansion, the two metric will predict different coefficients. The validity of this proposal can thus be tested experimentally. It will require a great deal of effort and skill to conduct such experiments, since many physical phenomena will have to be accounted for (an example: the quadrupole moment of the Sun for solar tests).

## ■ Conclusion

Using a nineteenth century approach, an effort to unify physics from the twentieth century has been attempted. The description of geodesics by general relativity is not complete because it does not explicitly show how the potential source causes curvature. A dynamic metric equation is found but it uses a simpler set of field equations (a rank one tensor instead of two). In the standard model as elsewhere, combining two 4−vectorsrequires a metric. By normalizing the 4−vectors,the unitary aspect of the standard model can be self−evident.

This theory makes three testable predictions, two subtle, one not. First, the polarity of gravitational waves will be scalar or longitudinal, not transverse as predicted by general relativity. Second, if gravitation effects are measured to secondary post Newtonian accuracy, the coefficients for the metric derived here are different from the Schwarzschild metric in isotropic coordinates. Such an experiment will be quite difficult to do. The third test is to see if the complete relativistic force equation matches all the data for a thin spiral galaxy. It is this test which should be investigated first.