Unifying Gravity and EM or GEM  

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Start with the EM action in a (possibly curved) vacuum

$$ S_{\text{EM}} = \int \sqrt{-g} \, d^4x (\nabla^\mu A^\nu - \nabla^\nu A^\mu) (\nabla_\mu A_\nu - \nabla_\nu A_\mu) $$

**EM symmetries**

$$ \delta S_{\text{EM}} = \int \sqrt{-g} \, d^4x (\nabla^\mu A^\nu - \nabla^\nu A^\mu) (\nabla_\mu A_\nu - \nabla_\nu A_\mu) \delta \psi $$

**Vary**  \hspace{1cm} **Conserve**

- $\delta t: t \to t' = t + \delta t$  \hspace{1cm} Energy: $m \frac{dt}{d\tau}$
- $\delta R: R \to R' = R + \delta R$  \hspace{1cm} Momentum: $m \frac{dR}{d\tau}$

Not the complete story of 4-change of a 4-potential

$(\nabla^\mu A^\nu - \nabla^\nu A^\mu)$ has 6 parts of 16 part story

**GEM action in a vacuum**

$$ S_{\text{GEM}} = \int \sqrt{-g} \, d^4x \left( (\nabla^\mu A^\nu - \nabla^\nu A^\mu) (\nabla_\mu A_\nu - \nabla_\nu A_\mu) 
+ (\nabla^\mu A^\nu + \nabla^\nu A^\mu) (\nabla_\mu A_\nu + \nabla_\nu A_\mu) \right) $$

**GEM Symmetry**

$$ \delta S_{\text{GEM}} = \int \sqrt{-g} \, d^4x \ L_{\text{GEM}} \delta \psi $$

**Vary**  \hspace{1cm} **Conserve**

- How 4-change in the 4-potential is measured.
  
  Example: From flat Euclidean spacetime to curved spacetime:

  $$ \delta (\partial^\mu A^\nu) : (\partial^\mu A^\nu) \to (\partial^\mu A^\nu)' = (\partial^\mu A^\nu) - \delta (\Gamma^\nu_{\mu\sigma} A^\sigma) $$

**GEM action in a vacuum, vary $A^\mu$, fix $g_{\mu\nu}$ up to a diffeomorphism.**

$$ \Box^2 A^\mu = 0 $$

**Vacuum Solutions**

$$ A^\mu = (\frac{1}{R}, 0) \quad \text{and} \quad g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{so} \quad \nabla^2 \frac{1}{R} = 0 \quad \checkmark $$

$$ A^\mu = \text{constants} \quad \text{and} \quad g_{\mu\nu} = \begin{pmatrix} \exp(-2GM/c^2R) & 0 \\ 0 & -\exp(2GM/c^2R) \end{pmatrix} \quad \text{static, diagonal} $$

$$ \text{so} \quad 0 = \partial_\mu \Gamma^0_{\sigma\nu} A^\sigma = \nabla g_{00} g^{00} \frac{\dot{R}}{c^2 R} = \nabla^2 \frac{GM}{c^2 R} = 0 \quad \checkmark $$

The Rosen exponential metric = Schwarzschild to 1st order PPN accuracy, not 2nd order PNN, so it is consistent with current first order tests, and could be confirmed or rejected for higher order tests. Example: GEM predicts 0.8 $\mu$arcseconds more bending by the Sun than GR.

**Quantization**

**Gupta/Blueler quantization of a 4D wave equation with a twist.**

Spin 1 field is 2 transverse modes of EM, like charges repel

Spin 2 field is scalar, longitudinal mode of Gravity, like charges attract.